From Last Time...

• Position, velocity, and acceleration
  - velocity = time rate of change of position
  - acceleration = time rate of change of velocity
  - Particularly useful concepts when
    • velocity is constant (undisturbed motion)
    • acceleration is constant (free falling object)
• Momentum and conservation of momentum
  - Descartes was able to characterize changes in motion with momentum.
  - Momentum measures the amount of ‘motion’ of an object.
  - Total momentum of a system is conserved: momentum is just transferred between bodies.

Momentum conservation: equal masses

Before collision:
- \( m = 1 \, \text{kg} \)
- \( v = 1 \, \text{m/s} \)
- \( p = mv = 1 \, \text{kg} \cdot \text{m/s} \)
- \( m = 1 \, \text{kg} \)
- \( v = 0 \, \text{m/s} \)
- \( p = 0 \)
- Total momentum = \( 1 \, \text{kg} \cdot \text{m/s} + 0 \, \text{kg} \cdot \text{m/s} = 1 \, \text{kg} \cdot \text{m/s} \)

After collision:
- \( m = 1 \, \text{kg} \)
- \( v = 0 \, \text{m/s} \)
- \( p = 0 \)
- \( m = 1 \, \text{kg} \)
- \( v = 1 \, \text{m/s} \)
- \( p = 1 \, \text{kg} \cdot \text{m/s} \)

Total momentum = \( 0 \, \text{kg} \cdot \text{m/s} + 1 \, \text{kg} \cdot \text{m/s} = 1 \, \text{kg} \cdot \text{m/s} \)

Momentum transfer

• Ball 2 increased velocity from 0 m/s to 1 m/s.
• Ball 2 increased its momentum by 1 kg-m/s. This momentum came from ball 1, which reduced its momentum by 1 kg-m/s.
• We normally say that 1 kg-m/s of momentum was transferred from ball 1 to ball 2.
• The total momentum of the system (ball 1 + ball 2) is the same before and after the collision.

Are we done yet?

• Where does Newton come in? What is left to do?
• Like Galileo and Descartes, Newton has a law of inertia.
• Newton’s first law:
  Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.
  • The ‘force’ is the ‘external disturbance’ of Galileo and Descartes

Newtonian Forces

• Newton made a definition of force that described how momentum was transferred.
• He viewed it as a continuous process rather than the immediate transfer as Descartes and Galileo.
• This makes a connection with our intuitive understanding of ‘force’ as a push or a pull.

Newton’s second law

• The change in motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

\[
\text{Change in momentum} = \Delta p = F \Delta t \\
\text{Applied force} = F \\
\text{Time interval} = \Delta t
\]
Momentum = \((\text{mass}) \times (\text{velocity})\)

\[
\text{Change in momentum} = (\text{mass}) \times (\text{change in velocity})
\]

\[
\Rightarrow \frac{\text{change in momentum}}{\text{change in time}} = \text{applied force}
\]

\[
\text{Momentum} = \text{(mass)} \times (\text{velocity})
\]

\[
\text{Change in momentum} = \text{(mass)} \times (\text{change in velocity})
\]

\[
\Rightarrow \frac{\text{mass} \times \text{change in velocity}}{\text{change in time}} = \text{applied force}
\]

Newton’s second law

\[
\text{Force} = \text{(mass)} \times (\text{acceleration})
\]

\[
\text{acceleration} = \frac{\text{force}}{\text{mass}}
\]

More than one force...

- Total force determines acceleration
- If \(F_1\) and \(F_2\) balance, acceleration is zero.

What do you think?

When the vectron hovers near the ceiling, the propeller force compared to hovering near the floor is

A. Greater.
B. Less.
C. The same.

Gravity exerts a force downward. When the vectron hovers, its velocity is constant, so the acceleration is zero. This means the net force is zero. The propeller force balances the gravitational force.

Types of forces

1. Strong nuclear force
2. Electromagnetic force
3. Weak nuclear force
4. Gravity

- Only gravity and electromagnetic forces are relevant in classical mechanics (motion of macroscopic objects).

The Four Forces
Force and acceleration

- Larger force gives larger acceleration
- Directly proportional: \( a \propto F \)
- But clearly different bodies accelerate differently under the same force.
  - Heavier objects are harder to push.
  - Proportionality constant may depend on weight?

Inertia again

- But we already said that inertia characterizes a body’s tendency to retain its motion (i.e. to not change its velocity), We say a heavier object has more inertia.
- But inertia and weight are different
  - A body in space is weightless,
  - but it still resists a push

Mass

- Define mass to be ‘the amount of inertia of an object’.
- Can also say mass characterizes the amount of matter in an object.
- Symbol for mass usually \( m \)
- Unit of mass is the kilogram (kg).
- Said before that \( a \propto F \)
- Find experimentally that 

\[
\text{Acceleration} = \frac{\text{Force}}{\text{Mass}}
\]

\[
a = \frac{F}{m}
\]

Force, weight, and mass

\[
F = ma \Rightarrow F = (kg) \times (m/s^2)
\]

\[
= kg \cdot m/s^2 = \text{Newton}
\]

- 1 Newton = force required to accelerate a 1 kg mass at 1 m/s^2.

But then what is weight?
- Weight is a force, measured in Newton’s
- It is the net force of gravity on a body.
- \( F = mg, \ g = F/m \)

What do you think?

Suppose you are on the moon instead of on earth

A. Your weight is less but your mass is the same.
B. Both your weight and mass are less than on earth.
C. Your weight is less and your mass is zero.

Mass is an intrinsic characteristic of a body. The force of gravity on the body (weight) will depend on the other bodies around it.

Is ‘pounds’ really weight?

- In the English system (feet, pounds, seconds), pounds are a measure of force.
- So it is correct to say my weight is 170 pounds.
- Then what is my mass?

\[
m = \frac{F}{g} = \frac{170 \text{ lbs}}{32 \text{ ft/s}^2} = 5.3 \text{ slugs!!}
\]
**Momentum conservation**
- We said before that an impressed force changes the momentum of an object.
- We also said that momentum is conserved.
- This means the momentum of the object applying the force must have decreased.
- According to Newton, there must be some force acting on that object to cause the momentum change.

**Newton’s third law**
- This is the basis for Newton’s third law: *To every action there is always opposed an equal reaction.*

This is momentum conservation in the language of forces.

**Colliding balls again**

Before collision:

During collision:
- Force on ball 1 decelerates it to zero velocity
- Force on ball 2 accelerates it

After collision:

**Examples**

- **Example**
  - A force $F$ acting on a mass $m_1$ results in an acceleration $a_1$. The same force acting on a different mass $m_2$ results in an acceleration $a_2 = 2a_1$. What is the mass $m_2$?
  - $F = m_1a_1 = m_2a_2 = m_2(2a_1)$
  - Therefore, $m_2 = m_1/2$
  - Or in words...*twice the acceleration means half the mass*

- **Example**

  - $M = 10$ kg, $F_1 = 200$ N
  - Find $a$
  - \[ a = \frac{F_{\text{net}}}{M} = \frac{200\text{N}}{10\text{kg}} = 20 \text{ m/s}^2 \]

- **Example**

  - $M = 10$ kg, $F_1 = 200$ N, $F_2 = 100$ N
  - Find $a$
  - \[ a = \frac{F_{\text{net}}}{M} = \frac{(200\text{N}-100\text{N})}{10\text{kg}} = 10 \text{ m/s}^2 \]