From Last Time...

Newton’s three laws of motion:

1) Law of inertia
2) \( F = ma \) (or \( a = F/m \))
3) Action and reaction (forces always come in pairs)

Question

If an apple falls toward the Earth, why doesn’t the moon fall toward the Earth?

A. The moon is too big
B. The moon is too far away
C. The moon does fall toward the earth.

Velocity of the moon

What is the direction of the Velocity of the moon?

Acceleration = \[
\frac{\text{change in velocity}}{\text{change in time}}
\]

Earth’s pull on the moon

- The moon continually accelerates toward the earth,
- But because of its orbital velocity, it continually misses the Earth.
- The orbital speed of the moon is constant, but the direction continually changes.
- Therefore the velocity changes with time.

True for any body in circular motion
Experiment

\[ F = m_2 g \]

\[\text{Acceleration of ball } m_1 = \frac{F}{m_1} = m_2 g \div m_1\]

\( m_1 \) accelerates inward in response to force \( m_2 g \)

\[ \text{Acceleration} = \frac{v^2}{r} \text{ for circular motion} \]

Newton’s falling moon

Throwing the ball fast enough results in orbital motion

From Newton’s Principia, 1615

Question

A 2 newton apple falls from a tree. What is the net force on the apple?

2 newtons downward

What is the acceleration?

\( g \approx 10 \text{ m/s}^2 \) downward

Question, part 2

I throw the 2 N apple horizontally. After I throw it, what is the net force on the apple?

2N downward

What is the acceleration of the apple?

\( g \approx 10 \text{ m/s}^2 \) directly downward

This is because, after I throw the apple, gravity is the only force. The only acceleration is due to gravity.

Shoot the monkey

After the dart leaves the gun, the only force is from gravity. The only deviation from straight-line motion is an acceleration directly downward. This is the same as the monkey.

Acceleration of moon

- So the moon is accelerating at \( \frac{v^2}{r} \text{ m/s}^2 \) directly toward the earth!

- This acceleration is due to the Earth’s gravity.

- Is this acceleration different than \( g \), the gravitational acceleration of an object at the Earth’s surface?
  - Can calculate the acceleration directly from moon’s orbital speed, and the Earth-moon distance.
Phy107 Lecture 6

The radius of the earth

• “Originally” from study of shadows at different latitudes by Eratosthenes!
• \(R(\text{earth})=6500 \text{ km}\)

Phy107 Lecture 6

Distance and diam. of moon

• The diameter of the moon is the diameter of its shadow during a solar eclipse. From the diameter \(d\) and angular size \(d/r \approx 5 \text{ deg}\), infer distance \(r \approx 60r(\text{earth})\).

Phy107 Lecture 6

Moon acceleration, cont

• Distance to moon = 60 earth radii = 3.84\times 10^8 \text{ m}
• Speed of moon:
  - Circumference of circular orbit = \(2\pi r\)
  - Speed = orbital distance \(= \frac{2\pi r}{60 \text{ days}} \approx 1023 \text{ m/s}\)
  - Centripetal acceleration = \(0.00272 \text{ m/s}^2\)

This is the acceleration of the moon due to the gravitational force of the Earth.

Phy107 Lecture 6

Distance dependence of Gravity

• The gravitational force depends on distance.
• Moon acceleration is \(9.81 \text{ m/s}^2 / 0.00272 \text{ m/s}^2 = 3600\) times smaller than the acceleration of gravity on the Earth’s surface.
• The moon is 60 times farther away, and \(3600=60^2\)
• So then the gravitational force drops as the distance squared

\[F \sim \frac{1}{d^2}\]

Newton: I thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly.

Phy107 Lecture 6

Equation for force of gravity

\[F_{\text{gravity}} \propto \frac{(\text{Mass of object 1}) \times (\text{Mass of object 2})}{\text{square of distance between them}}\]

\[F = \frac{m_1 \times m_2}{d^2}\]

For masses in kilograms, and distance in meters,

\[F = 6.7 \times 10^{-11} \frac{m_1 \times m_2}{d^2}\]

Phy107 Lecture 6

Example

• Find the acceleration of an apple at the surface of the earth

\[F_{\text{apple}} \sim \frac{m_1 \times m_2}{d^2}\]

\[\text{Force on apple} = F_{\text{apple}} = 6.7 \times 10^{-11} \frac{m_1 \times m_2}{d^2}\]

\[d = \text{distance between center of objects} = \text{radius of Earth}\]

\[\text{Acceleration of apple} = \frac{F_{\text{apple}}}{m_{\text{apple}}} = 6.7 \times 10^{-11} \frac{m_1}{d^2}\]

\[= 6.67 \times 10^{-11} \text{ N} - \text{m}^2/\text{kg}^2 \times \frac{5.98 \times 10^{24} \text{ kg}}{(6.37 \times 10^6 \text{ m})^2} \approx 9.83 \text{ m/s}^2\]

This is also the force on the Earth by the apple!
Gravitational force decreases with distance from Earth

Force on apple = \( F_{\text{apple}} = 6.7 \times 10^{-11} \frac{m_{\text{Earth}} \times m_{\text{apple}}}{d^2} \)

So moving farther from the Earth should reduce the force of gravity

- Typical airplane cruises at ~5 mi = 8000 m
  - \( d \) increases from 6,370,000 m to 6,378,000 m
  - only about a 0.25% change!

- International space station orbits at 350 km = 350,000 m
  - \( d = 6,370,000 \text{ m} + 350,000 \text{ m} = 6,720,000 \text{ m} \)
  - Again \( d \) has changed only a little, so that \( g \) is decreased by only about 10%.

So why is everyone floating around?

The space station is falling...

...similar to Newton’s apple

- In its circular orbit, once around the Earth every 90 minutes, it is continuously accelerating toward the Earth at ~8.8 m/s².
- Everything inside it is also accelerating at that same rate.
- The astronauts are freely falling inside a freely-falling ‘elevator’. They have the perception of weightlessness, since their environment is falling just as they are.

A little longer ride

Supreme Scream - 300 feet of pure adrenaline rush

A freefall ride

\[
\begin{align*}
  d &= \frac{1}{2} a t^2 \\
  t &= \sqrt{\frac{2d}{a}} \\
  = \sqrt{\frac{2 \times 300 \text{ ft}}{32 \text{ ft/s}^2}} \\
  = 4.3 \text{ sec of freefall}
\end{align*}
\]
Gravitational force between small objects

• At the center of our galaxy is a collection of stars found to be in motion about an invisible object.

Gravitational force at large distances:

Stars orbiting our black hole

• But gravitational force is only 2% weaker - Need to move farther from the Earth.

Accel. of gravity on moon

• On the moon, an apple feels gravitational force from the moon.
• Earth is too far away.

\[
\begin{align*}
\text{Force on apple on moon} & = F_{\text{app}} = 6.7 \times 10^{-11} \frac{m_{\text{moon}} \times m_{\text{apple}}}{r_{\text{moon}}} \\
\text{Accel. of apple on moon} & = a_{\text{app}} = 6.7 \times 10^{-11} \frac{m_{\text{moon}}}{r_{\text{moon}}} \\
\text{Compare to accel on Earth} & = 6.7 \times 10^{-11} \frac{m_{\text{Earth}}}{r_{\text{Earth}}} \\
\text{accel. on moon} & = \frac{m_{\text{moon}}}{m_{\text{Earth}}} \left( \frac{r_{\text{moon}}}{r_{\text{Earth}}} \right) \\
\text{accel. on Earth} & = \frac{m_{\text{Earth}}}{m_{\text{Earth}}} \left( \frac{r_{\text{Earth}}}{r_{\text{Earth}}} \right) \\
\end{align*}
\]

Accel. of gravity on moon

\[
\begin{align*}
\text{accel. on moon} & = \frac{m_{\text{moon}}}{m_{\text{Earth}}} \left( \frac{r_{\text{moon}}}{r_{\text{Earth}}} \right) \\
\text{accel. on Earth} & = \frac{m_{\text{Earth}}}{m_{\text{Earth}}} \left( \frac{r_{\text{Earth}}}{r_{\text{Earth}}} \right) \\
\text{Compare} & = \frac{7.4 \times 10^{25} \text{ kg} / 6.0 \times 10^{24} \text{ kg}}{\frac{(1.7 \times 10^3 \text{ m})}{(6.4 \times 10^4 \text{ m})}} \\
& = \frac{0.0123}{0.175} = 0.0706
\end{align*}
\]

Question

Halfway to the moon, what is the acceleration of an apple due to the Earth’s gravity?

A. $g/2$
B. $g/4$
C. $g/900$

Moon is 60 Earth radii from the Earth.
Halfway is 30 Earth radii.
So apple is 30 times farther than when on surface.
Gravitational force is $(30)^2$ times smaller = $g/900$
Orbits obey Newton’s gravity, orbiting around some central mass

- Scientists at the Max Planck Institute for Extraterrestrische Physik has used infrared imaging to study star motion in the central parsec of our galaxy.
- Movie at right summarizes 14 years of observations.
- Stars are in orbital motion about some massive central object

http://www.mpe.mpg.de/www_ir/GC/intro.html

What is the central mass?

- One star swings by the hole at a minimum distance $b$ of 17 light hours (120 A.U. or close to three times the distance to Pluto) at speed $v=5000$ km/s, period 15 years.
- From the orbit we can derive the mass.
- The mass is 2.6 million solar masses.
- It is mostly likely a black hole at the center of our Milky Way galaxy!