Exam 2

- Hour Exam 2: Wednesday, Oct. 26
- In-class
- Twenty multiple-choice questions
- Will cover: 8.1-8.6 (Light and E&M)
  9.1-9.5 (E&M waves and color)
  10, 11 (Relativity)
  All Lecture Material
- You should bring
  - 1 page notes, written double sided
  - #2 Pencil and a Calculator
  - Review Monday 24rd
  - Review questions online under “Review Quizzes” link

General Relativity and Gravity

- Special Relativity deals with inertial reference frames, frames moving with a constant relative velocity.
- It has some rather unusual predictions
  - Time dilation
  - Length contraction
  - Mass-energy equivalence
- These were motivated by thought experiments based on simple postulates
- General Relativity also motivated by some simple thoughts about gravity and free-fall

The Equivalence Principle

- Led Einstein to postulate the Equivalence Principle

Equivalence principle

Accelerating reference frames are indistinguishable from a gravitational force

Try some experiments

Constant velocity

Constant accel.

Floor accelerates upward to meet ball

Cannot do any experiment to distinguish accelerating frame from gravitational field

Light follows the same path

Path of light beam in our frame

Path of light beam in accelerating frame

Velocity = v+at₀

Velocity = v+2at₀
Is light bent by gravity?
• If we can’t distinguish an accelerating reference frame from gravity...
• and light bends in an accelerating reference frame...
• then light must bend in a gravitational field
But light doesn’t have any mass. How can gravity affect light?

Light travels in a straight line
• In some ways, the path of light defines a straight line
• Surveyors use laser to set heights
• Can use a laser level to put up shelves
• Seems to move in exactly a straight line

Maybe we are confused about what a straight line is

What is a straight line?
• Claim that these are all straight lines

Straight is shortest distance
• They are the shortest distances determined by wrapping string around a globe. On a globe, they are called ‘great circles’. In general, geodesics.
• This can be a general definition of straight, and is in fact an intuitive one on curved surfaces
• It is the one Einstein used for the path of all objects in curved space-time
• The confusion comes in when you don’t know you are on a curved surface.

Differently curved surfaces
• Can quantify an object by its local curvature
• Each surface above has constant curvature
• Zero, positive, and negative

Shortest distance on other surfaces
• Shortest distance on different surfaces

Shortest path is minimum length geodesic.
Mass and curvature

- General relativity says that any mass will give space-time a curvature
- Motion of objects in space-time is determined by that curvature

Einstein’s solution

- Einstein guessed that the curvature functions (units of $1/m^2$) are proportional to the local energy and momentum densities (units of $kg/m^3$)
- The proportionality constant from comparison with Newtonian theory is $\frac{8\pi G}{c^2}$, where $G$ is Newton’s constant

Near the Earth

- The ratio of the curvature of space on the surface of the Earth to the curvature of the surface of the Earth is $\sim 7 \times 10^{-10}$
- The curvature of space near Earth is so small as to be usually unnoticeable.
- But does make objects accelerate toward the Earth!

A test of General Relativity

- Can test to see if the path of light appears curved to us
- Local massive object is the sun
- Can observe apparent position of stars with and without the sun
- But need to block glare from sun
**Eddington’s Eclipse Expedition 1919**

- Eddington, British astronomer, went to Principe Island in the Gulf of Guinea to observe solar eclipse.
- After months of drought, it was pouring rain on the day of the eclipse.
- Clouds parted just in time, they took photographic plates showing the location of stars near the sun.
- Analysis of the photographs back in the UK produced a deflection in agreement with the GR prediction.

**Gravitational Lensing**

- Bending light around a massive object from a distant source.
- The orange arrows show the apparent position of the background source. The white arrows show the path of the light from the true position of the source.

**Einstein ring: a prototypical example**

- Optical image of the radio source B1938+666, taken with the Hubble Space Telescope, shows the Einstein ring most prominently. The central bright spot is the lensing galaxy.
- Credit: L.J.King (U. of Manchester), NICMOS, HST, NASA
- An Einstein ring is a special form of a gravitational lens in which source (such as a quasar) and lens (such as a galaxy) are exactly lined up, so a circular halo is observed.

**Lensing with a complicated ‘lens’**

- Gravitational lensing
- Hubble space telescope image of Abell 1689, large galaxy cluster.
- The gravity of the cluster’s stars, plus dark matter, acts as a 2-million-light-year-wide ‘lens’ in space.
- The most prominent lensing effect is the long blue arc, just above, and to the left of, center.
- Makes Astronomy more complex.

**Gravitational lensing simulation**

The Castle on the Mall in Washington, D.C., as viewed from the Natural History museum.

Now we place a black hole with the mass of Saturn over the middle of the Mall, and view the Castle through the resulting gravitational lens.

**A little more about gravity**

- Mass tells space-time how to curve.
- Space tells mass how to move.
- This naturally explains the Universality of Free Fall Acceleration (i.e. objects fall with constant acceleration, independent of mass).
  - Gravity is a property of the geometry of spacetime.
  - All objects move along the same geometrical distortions.
- Can we understand this in a more intuitive way?
Space vs Space-Time

- This seems unusual
- Says that path in space-time of ball held in my hand is only slightly different than ball accelerating in free-fall toward the Earth
- These two motions seem dramatically different considering the space coordinate
- But in fact these paths are almost the same in space-time

All motion is in space-time

- Zero velocity (a stationary particle ball in my hand)
- Constant velocity (no "forces")
- Constant acceleration

The path of the objects in space-time are only slightly different.

Planet and sun

- Two different world lines for planet and sun

Space is Curved

- Einstein said to picture gravity as a warp in space
- Kepler’s Laws can all be explained by movement around these “puckers”
- Everything moving is affected, regardless of mass

A Contemporary View

- Curved Spacetime forms a stage on which other physics happens
- General Relativity (GR) is a very successful description of the interaction between spacetime and objects
- Einstein’s Field Equation: (just for fun...)
  \[ G_{\alpha\beta} = 8\pi T_{\alpha\beta} + \Lambda \]
- Curvature Tensor describing how spacetime is curved
- Stress-Energy Tensor describing distribution of mass and energy

Other Consequences of GR

- Gravitational Radiation!
  - Laser Interferometric Gravitational Observatory (LIGO)
  - One in Washington state, and one in Louisiana
  - LISA: space-based gravitational wave interferometer
- Black Holes
- Expanding Universe (although Einstein missed the chance to predict it!)
Gravitational time dilation
• Gravity warps both space and time!
• At 10,000 km above the Earth’s surface, a clock should run 4.5 parts in $10^{10}$ faster than one on the Earth.
• Comparing timing pulses from atomic oscillator clocks confirms the gravitational time dilation in 1976 to within 0.01%.
• Corrections are now standard in the synchronizing satellites.
• This correction needed in addition to the special relativity correction for GPS.

Mechanics in general relativity
• Einstein’s theory reproduces Newtonian mechanics when curvature is small.
• General relativity gives an approximate $1/r^4$ correction to Newton’s $1/r^2$ force law, largest at small $r$ near the mass.
• Observed in the motions of satellites and planets.

Precession of Mercury
• Deviations from closed elliptical orbits result from deviations from Newton’s $1/r^2$ law.
• The axis of the orbit of Mercury (closest to the Sun so most sensitive) precesses.
• Time dilates near the sun causing the orbit to take longer.
• Relativity explains an excess of 40 arcsec per century as calculated by Einstein.

Gravomagnetism
• Relativity theory requires a velocity dependent component to gravitational interaction.
• This is like magnetism in electromagnetic theory, where moving charges produce a magnetic field.
• A spinning mass behaves as a gravomagnet and two spinning masses interact through gravomagnetism.

Gravitational radiation
• When a mass is moved, the curvature of space-time changes.
• If a mass is oscillated, ripples of space-time curvature carry the signal.
• Gravitational radiation carries energy and momentum and wiggles mass in its path.

Evidence for gravity waves
• In 1974, Joseph Taylor and his student Russell Hulse discovered a binary neutron star system losing energy as expected from gravitational radiation.
Gravity Waves

- Two black holes spiraling in to collide with each other, emitting gravity waves.

Direct detection of gravity waves

LIGO is a collection of large laser interferometers searching for gravity waves generated by exploding stars or colliding black holes.

Curved spacetime in this room

- Earth curves its local spacetime.
  - Curvature at surface = $\frac{2GM}{R^2}$, roughly $1.4 \times 10^{-9}$
  - This means would curve 1.4 meters over 10^9 meters.
- Travel one second in time (300,000,000 meters for light); get 0.4 meters of “curve”
- Ball in one second falls $d = \frac{1}{2}gt^2$, with $t=0.5$ s
  - 1.25 meter-high arc.
  - 0.4 meters is roughly departure from straight line.

\[ \begin{align*}
  \text{one-second arc} & \quad \text{1.25 m} \\
  \text{0.4 m} & \quad \text{0.4 m}
\end{align*} \]