From Last Time...

- Hydrogen atom:
  - One electron orbiting around one proton (nucleus)
  - Electron can be in different "quantum states"
  - Quantum states labeled by integer $n=1, 2, 3, 4, ...$
  - In each different quantum state, electron has:
    - Different orbital radius
    - Different energy
    - Different wavelength
  - $n=1$ is lowest energy state, energy depends on state as $\frac{13.6}{n^2} \text{eV}$

But why?

- Why should only certain orbits be stable?
- Bohr had a complicated argument based on "correspondence principle"
  - That quantum mechanics must agree with classical results when appropriate (high energies, large sizes)
- But incorporating wave nature of electron gives a natural understanding of these "quantized orbits"

Resonance

- Most physical objects will vibrate at some set of natural frequencies
  - Ringing bell
  - Wine glass
  - Musical instrument
- The electrons in an atom analogous to sound waves in a musical instrument.
- In instrument, only certain pitches produced, corresponding to particular vibration wavelengths.
- Since the electrons orbiting around the nucleus are waves, only certain wavelengths are allowed.

Resonance on a string

- Easier to think about in a normal wind instrument, or vibrations of a string.
- Wind instrument with particular fingering plays a particular pitch, particular wavelength.
- Guitar string vibrates at frequency, wavelength determined by string length.

\[ f = \frac{v}{\lambda} \]

\[
\begin{align*}
\text{Fundamental,} & \quad \text{wavelength } 2L/1=2L, \quad \text{frequency } f \\
1\text{st harmonic,} & \quad \text{wavelength } 2L/2=4L, \quad \text{frequency } 2f \\
2\text{nd harmonic,} & \quad \text{wavelength } 2L/3, \quad \text{frequency } 3f
\end{align*}
\]

Resonances of a string

String resonant frequency

A string has a fundamental frequency of 440 Hz. I now decreases the length of the string to 1/2 of it’s original length. The fundamental frequency is now

- A. 440 Hz, unchanged
- B. 220 Hz
- C. 880 Hz

Resonant wavelength has decreased by factor of 2. Since $f = v/\lambda$, frequency has gone up by factor of two.
Electron waves in an atom

- Electron is a wave.
- In the orbital picture, its propagation direction is around the circumference of the orbit.
- Wavelength = \( \frac{h}{p} \) (p=momentum, and energy determined by momentum)
- How can we think about waves on a circle?

Waves on a circle

- Here is my ‘Tonehut’
  - Like a flute, but in the shape of a doughnut.
  - Produces particular pitch.
  - Air inside must be vibrating at that frequency
  - Sound wave inside has wavelength \( \lambda = \frac{v}{f} \) (red line).
  - What determines the frequency/wavelength of the sound?

Waves on a ring

- Condition on a ring slightly different.
- Integer number of wavelengths required around circumference.
- Otherwise destructive interference occurs when wave travels around ring and interferes with itself.

Electron standing-waves on an atom

- Electron wave extends around circumference of orbit.
- Only integer number of wavelengths around orbit allowed.
These are the five lowest energy orbits for the one electron in the hydrogen atom.

- Each orbit is labeled by the quantum number \( n \).
- The radius of each is \( n^2a_0 \).
- Hydrogen has one electron: the electron must be in one of these orbits.
- The smallest orbit has the lowest energy. The energy is larger for larger orbits.

Here the electron is in the \( n=3 \) orbit.

- Three wavelengths fit along the circumference of the orbit.
- The hydrogen atom is playing its third highest note.

Here the electron is in the \( n=4 \) orbit.

- Four wavelengths fit along the circumference of the orbit.
- The hydrogen atom is playing its fourth highest note (lower pitch than \( n=3 \) note).

Here the electron is in the \( n=5 \) orbit.

- Five wavelengths fit along the circumference of the orbit.
- The hydrogen atom is playing its next lowest note.
- The sequence goes on and on, with longer and longer wavelengths, lower and lower notes.

Wavelength gets longer in higher \( n \) states, (electron moving slower) so kinetic energy goes down.

But energy of Coulomb interaction between electron (-) and nucleus (+) goes up faster with bigger \( n \).

End result is

\[
E_n = \frac{13.6}{n^2} \text{ eV}
\]
Another question

Here is Donald Lipski’s sculpture ‘Nail’s Tail’ outside Camp Randall Stadium. What could it represent?

A. A pile of footballs
B. “I’m just glad it’s not my money”
   - Ken Kopp (New Orlean’s Take-Out)
C. “I hear it’s made of plastic. For 200 grand, I’d think we’d get granite”
   - Tim Stapleton (Stadium Barbers)

The wavefunction

- Our explanation of the hydrogen atom originated from wave nature of electron.
- The electron wave is a standing wave around the circumference of the orbit.
- For each quantum state, there is a wavefunction associated with the electron.
- This is not unique to the hydrogen atom.

General aspects of Quantum Systems

- System has set of quantum states, labeled by an integer (n=1, n=2, n=3, etc)
- Each quantum state has a particular frequency and energy associated with it.
- These are the only energies that the system can have: the energy is quantized
- Analogy with classical system:
  - System has set of vibrational modes, labeled by integer fundamental (n=1), 1st harmonic (n=2), 2nd harmonic (n=3), etc
  - Each vibrational mode has a particular frequency and energy.
  - These are the only frequencies at which the system resonates.

Example: ‘Particle in a box’

Particle confined to a fixed region of space e.g. ball in a tube- ball moves only along length L

- Classically, ball bounces back and forth in tube.
- No friction, so ball continues to bounce back and forth, retaining its initial speed.
- This is a ‘classical state’ of the ball.
- Could label each state with a speed, momentum=(mass)x(speed), or kinetic energy.
- Any momentum, energy is possible.
- Can increase momentum in arbitrarily small increments.

Quantum Particle in a Box

- In Quantum Mechanics, ball represented by wave
  - Wave reflects back and forth from the walls.
  - Reflections cancel unless wavelength meets the standing wave condition: integer number of half-wavelengths fit in the tube.

\[ \lambda = 2L \]

One half-wavelength

\[ p = \frac{h}{\lambda} = \frac{h}{2L} = p_n \]

\[ \lambda = L \]

Two half-wavelengths

\[ p = \frac{h}{\lambda} = \frac{h}{L} = 2p_n \]

Particle in box question

A particle in a box has a mass m. It’s energy is all energy of motion = \( p^2/2m \).
We just saw that it’s momentum in state n is \( np_n \).
It’s energy levels

A. are equally spaced everywhere
B. get farther apart at higher energy
C. get closer together at higher energy.
**Quantized energy levels**

- Quantized momentum
  \[ p = \frac{\hbar}{\lambda} = n\frac{\hbar}{2L} \]
- Energy = kinetic
  \[ E = \frac{p^2}{2m} = \left(\frac{np}{L}\right)^2 = n^2E_o \]
- Or Quantized Energy
  \[ E_n = n^2E_o \]

**The wavefunction of a particle**

- We use a probabilistic interpretation
  - The wavefunction \( \Psi(x) \) (psi) of a particle describes the extended, wave-like properties.
  - The square magnitude of the wavefunction \( |\Psi|^2 \) gives the probability of finding the particle at a particular spatial location.
- Similar to the interpretation used for light waves
  - Square of the electric field gives light intensity = number of photons / second.

**Particle in a box: Wavefunctions**

**Wavefunction**

- Ground state wavefunction and probability.
- Height of probability curve represents likelihood of finding particle at that point.

**Next highest energy state**

**Wavefunction**

- Now here is something unusual.
  - In the middle of the box, probability of finding the particle is ZERO!
  - How can we understand this?

**Understanding Probability**

- **Heads** vs **Tails**

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**Discrete vs continuous**

- **“Continuous”** probability distribution
Particle in a box: Wavefunctions

Wavefunction  Probability

Third state

Next higher state

Lowest energy state

Probability of finding electron

- Classically, equally likely to find particle anywhere
- QM - true on average for high n

Quantum Corral

- 48 Iron atoms assembled into a circular ring.
- The ripples inside the ring reflect the electron quantum states of a circular ring (interference effects).