Exam Review: Topics by chapter

- Chap. 13: Principles of quantum mechanics
  - Light quantization, matter waves.
- Chap. 14: Uses of quantum mechanics
  - Hydrogen atom, the wavefunction, uncertainty principle, entanglement.
- Addl. Topics: More quantum mechanics
  - Spin, indistinguishability & symmetry, solids, superconductivity
- Chap. 15: Physics of the nucleus
  - Structure of nucleus, radioactive decay, half-life
- Chap. 16: Fission and fusion
  - Fission, fusion, nuclear weapons, reactors

Hour Exam 3 Review

- Hour Exam 3: Wednesday, Nov. 30
- In-class (2241 Sterling Hall)
- Twenty multiple-choice questions
- Will cover: 13 (Basic Quantum Mechanics), 14 (Uses of Quantum Mechanics), Addl. Lecture Material (Symmetry, solids, superconductivity) 15, 16.1-16.5 (The nucleus, fission & fusion)
- This is Lectures 22-33
- You should bring
  - Your student ID
  - 1 page notes, written double sided
  - Calculator
  - Pencil for marking answer sheet

Chap. 13: Quantum Mechanics

- Quantization of light
  - Light comes in discrete clumps (photons)
  - Light shows both particle and wave-like properties
  - Evidence for particle properties is photoelectric effect & black-body radiation
- Matter waves
  - Matter shows both particle and wave-like properties
  - deBroglie wavelength = Planck’s constant / momentum
  - Evidence for wave properties is interference and diffraction

Photoelectric effect summary

- Light is made up of photons, individual ‘particles’, each with energy $hf = \frac{hc}{\lambda}$
- One photon collides with one electron - knocks it out of metal.
- If photon doesn’t have enough energy, cannot knock electron out.
- Intensity ( = # photons / sec) doesn’t change this.

Minimum frequency (maximum wavelength) required to eject electron

<table>
<thead>
<tr>
<th>A. 300 nm</th>
<th>B. 500 nm</th>
<th>C. 700 nm</th>
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An electron is bound inside copper by a ‘binding energy’ of 4 eV.
Which wavelength will eject electrons from copper?

Minimum photon energy to eject electron is 4 eV. Corresponding photon energy is given by

$4.0 \text{ eV} = \frac{hc}{\lambda} = \frac{1240 \text{ eV \cdot nm}}{\lambda \text{ nm}}$

So $\lambda_{\text{max}} = 310 \text{ nm}$

Topic: Wave properties of matter

- All objects show both wave-like properties and particle-like properties.
- Electromagnetic radiation (e.g. light) shows interference effects (wave-like properties), but also comes in discrete photons of energy $hf$ (particle-like properties)
- Matter clearly shows particle-like properties, but also shows interference and diffraction effects (wave-like properties).
Matter waves

- If light waves have particle-like properties, matter should have wave properties.
- de Broglie postulated that the wavelength of matter is related to momentum as $\lambda = \frac{h}{p}$
- This is called the de Broglie wavelength.

Matter wave question

A neutron has almost 2000 times the rest mass of an electron. Suppose they both have 1 ev of energy. How do their wavelengths compare?

A. both same
B. neutron wavelength < electron wavelength
C. neutron wavelength > electron wavelength

Wavelength depends on momentum, as $h/p$. Same momentum -> same wavelength. $Momentum = \sqrt{2mE}$, depends on energy AND mass

Chap. 14: Using Quantum Mechanics

- Quantum states in a hydrogen atom
  - Models of the hydrogen atom
  - Absorption and emission of light (line spectra)
- The wavefunction of a quantum state
  - The ground state and excited states
  - Probabilistic interpretation of the wavefunction.
- Heisenberg uncertainty principle
  - Position and momentum cannot be known simultaneously
  - Consequence of wave properties
- Quantum jumps and entanglement

Topic: The hydrogen atom

- Hydrogen atom:
  - One electron orbiting around one proton (nucleus)
  - Electron can be in different “quantum states”
  - Quantum states determined by wave condition
  - Quantum states labeled by integer $n$
  - In each different quantum state, electron has
    - Different orbital radius
    - Different energy
    - Different wavelength
  - $n=1$ is lowest energy state, energy depends on state as $E_n = \frac{13.6}{n^2} \text{ eV}$

Energy levels

- Instead of drawing orbits, we can just indicate the energy an electron would have if it were in that orbit.
**Emitting and absorbing light**

Emitting a photon of correct energy makes electron jump to higher quantum state.

\[
E_{21} = E_2 - E_1
\]

Absorbing a photon of correct energy makes electron jump to lower quantum state.

\[
E_{21} = E_2 - E_1
\]

**Line spectra**

- This says that gases such as Hydrogen emit light only at certain frequencies, wavelengths.
- The photon energies correspond to separations between the energy levels.

**Spectral Question**

Compare the wavelength of a photon produced from a transition from \( n=3 \) to \( n=1 \) with that of a photon produced from a transition \( n=2 \) to \( n=1 \).

A. \( \lambda_{31} < \lambda_{21} \)

B. \( \lambda_{31} = \lambda_{21} \)

C. \( \lambda_{31} > \lambda_{21} \)

Eenergetic \( E_{31} > E_{21} \) so \( \lambda_{31} < \lambda_{21} \)

**Topic: The wavefunction**

- Particle can exist in different quantum states, having
  - Different energy
  - Different momentum
  - Different wavelength

- The quantum wavefunction describes wave nature of particle.
- Square of the wavefunction gives probability of finding particle.
- Zeroes in probability arise from interference of the particle wave with itself.

**Particle in a box: Wavefunctions**

- Ground state wavefunction and probability.
- Height of probability curve represents likelihood of finding particle at that point.

**Probability of finding electron**

- Classically, equally likely to find particle anywhere
- QM - true on average for high \( n \)

Quantum-mechanical distribution

Classical distribution

\[ P = \frac{1}{L} \quad 0 < x < L \]

Zeroes in the probability! Purely quantum, interference effect
Quantum Corral

- 48 iron atoms assembled into a circular ring.
- The ripples inside the ring reflect the electron quantum states of a circular ring (interference effects).

Wavefunction: Particle in a Box

- In Quantum Mechanics, ball represented by wave
  - Wave reflects back and forth from the walls.
  - Reflections cancel unless wavelength meets the standing wave condition:
    integer number of half-wavelengths fit in the tube.

\[ \lambda = L \]
Two half-wavelengths

\[ \lambda = 2L \]
One half-wavelength

\[ p = \frac{h}{\lambda} \]

Topic: Uncertainty Principle

- Heisenberg Uncertainty principle
- Arises from wave nature of particles.
- Precise position & momentum cannot be measured at the same time.
- Highly accurate momentum (wavelength) means position is uncertain
- Can localize particle by superimposing many wavelengths, so momentum is uncertain.
- Quantum mechanical tunneling.

Uncertainty in Quantum Mechanics

Position uncertainty = \( L \)

(Since \( \lambda = 2L \))

Momentum uncertainty from

\[ \frac{h}{\lambda} \] \to \frac{h}{\lambda} + \frac{h}{\lambda} = \frac{2h}{\lambda} \]

Reducing the box size reduces position uncertainty, but the momentum uncertainty goes up!

The product is constant:

(position uncertainty)(momentum uncertainty) \( \approx h \)

Quantum-mechanical tunneling

Atomic clock question

Suppose we changed the ammonia molecule so that the distance between the two stable positions of the nitrogen atom INCREASED. The clock would

- A. slow down.
- B. speed up.
- C. stay the same.
**Topic: Quantum jumps & entanglement**
- ‘Philosophical’ effects in quantum mechanics
- Interpretation of the wave function:
  - Calculation using the basic premises of quantum mechanics give highly accurate results...
  - ...but what does it mean?
- Superposition of quantum states
- Entanglement, action at a distance
- Copenhagen interpretation
  - ‘collapse of the wavefunction’

**Additional Lecture Material**
- Spin
  - An additional quantum property of a particle
- Indistinguishability and symmetry
  - Fermions and Bosons
  - Pauli exclusion principle
- Physics of solids
  - Energy bands in a solid
  - Metals, insulators, and semiconductors
  - Superconductors

**Topic: spin**
- Free electron, by itself in space, not only has a charge, but also acts like a bar magnet with a N and S pole.
- Since electron has charge, could explain this if the electron is spinning.
- Then resulting current loops would produce magnetic field just like a bar magnet.
- But...
  - Electron in NOT spinning.
  - As far as we know, electron is a point particle.

**Topic: Indistinguishability & symmetry**
- Several important conceptual aspects of quantum mechanics
- Indistinguishability
  - particles are absolutely identical
  - Leads to Pauli exclusion principle (one Fermion / quantum state).
- Symmetry
  - Charactizes the wavefunctions
  - Leads to different energy levels.

**Spin and symmetry**
- In both cases the probability is preserved, since it is the square of the wavefunction.
- Can be shown that
  - Integer spin particles (e.g. photons)
    have symmetric wavefunctions
    These types of particles are called **Bosons**
  - Half-integer spin particles (e.g. electrons)
    have antisymmetric wavefunctions
    These types of particles are called **Fermions**

**Topic: Physics of solids**
- Solids are large numbers of atoms arranged in a regular **crystal structure**.
- Each atom has electron quantum states, but interactions shift the energies.
- End result is each type atomic electron state (e.g. 1s) corresponds to a broadened ‘band’ of energy levels in a solid.
- Band filling determines electrical properties
  - Partially full bands = metal
  - Bands completely full or empty = insulator / semiconductor
- Substitutional doping of a semiconductor leads to a material useful in electronic devices.
A six-atom molecule

Wavefunctions

Energy levels

Can see different wavelengths for the different molecular states

Energy levels in a solid

• Solids consist of $\sim 10^{24}$ atoms
• Energy levels spaced extremely close together

3-atom molecule 6-atom molecule $10^{24}$-atom ‘molecule’

Solid sodium (metal)

Na = [Ne]3s

$3p$

$3s$

1 electron

$2p$

6 electrons

$2s$

2 electrons

$1s$

2 electrons

Sodium atom Sodium metal

Metals, insulators, semiconductors

Metal (at least one partially full band)

Insulator (all bands completely full or empty)

Semiconductor (insulator with small energy gap)

• Only partially full bands carry current
• Completely full, or completely empty bands, carry no current

Doped semiconductors

• Semiconductors become useful when they are doped.
• Different atom is substituted for Si.
• Possibilities: one extra electron, one fewer electron.

Here an atom with an extra electron is substituted for Si

Topic: Superconductivity

• Superconductor = zero-resistance material
• Many elements are superconducting
• Meissner effect = exclusion of magnetic field
  - Leads to superconducting levitation
• Critical temperature, critical current, critical magnetic field - range of usefulness.
• Most critical temperatures far below room T.
• High-temperature superconductors discovered with transition temp near liquid nitrogen.
Superconductivity

- Superconductors are materials that have exactly zero electrical resistance.
- But this only occurs at temperatures below a critical temperature, $T_c$.
- In most cases this temperature is far below room temperature.

Critical current

- If the current is too big, superconductivity is destroyed.
- Maximum current for zero resistance is called the ‘critical’ current.
- For larger currents, the voltage is no longer zero, and power is dissipated.

Critical magnetic field

- Magnetic field is screened out by screening current.
- Larger fields require larger screening currents.
- Screening currents cannot be larger than the critical current.
- This says there is a critical magnetic field which can be screened.

Chap. 15: Nucleus and radioactivity

- Structure of the nucleus
  - Nucleus has small size, large energy scale
  - Strong force holds nucleus together
  - Isotope: different neutron #, same proton #
  - Nuclear binding energy different for different nuclei
- Radioactive decay
  - ‘Unstable’ nuclei decay by emitting radiation
  - Alpha, beta, gamma decay
- Decay half-life and carbon-dating
  - Decay is a random process
  - Half-life characterizes decay rate

Topic: Structure of the nucleus

- Nucleus is small, tightly bound system of protons & neutrons.
- Proton number determines the element.
- Different isotopes have different # neutrons.
- Some isotopes unstable, radioactively decay
- Nucleus held together by the strong nuclear force
  - Stronger than coulomb force,
  - But much shorter range than coulomb force.
- Strong force actually between quarks, internal constituents of the neutron/proton.
  ‘Leaks’ out to appear as an attractive force.

Size & structure of nucleus

- Nucleus consists of protons and neutrons densely combined in a small space (~$10^{-14}$ m)
  - Protons have a positive electrical charge
  - Neutrons have zero electrical charge (are neutral)
- Spacing between these nucleons is ~ $10^{-15}$ m
- Size of electron orbit is $5 \times 10^{-11}$ m
- Nucleus is 5,000 times smaller than the atom!
**Isotopes**

- Both $^{12}\text{C}$ and $^{14}\text{C}$ have same chemical properties.
- This is why they are both called carbon. Same number of electrons and hence same number of protons in nucleus.
- But the nuclei are different. They have different number of neutrons. These are called isotopes.
- Difference is most easily seen in the binding energy.
- Nuclei that are bound more tightly are less likely to ‘fall apart’.
- In fact $^{14}\text{C}$ is radioactive. It is unstable to emission of an electron.

**Nuclear Binding Energy**

- Mass of nucleus is less than mass of isolated constituents.
- The difference is the binding energy.

**Alpha and beta decay**

- Alpha decay: chunk of nucleus ejected
- Beta decay: electron emitted, neutron changes to proton
- Beta* and gamma decay
- Nucleus emits positron. Proton changes to neutron
- Nucleus changes quantum state from high energy to low energy

**Topic: Radioactive Decay**

- Some nuclei spontaneously emit radiation: alpha, beta, gamma
- Radioactive half-life
- Decay type understood in terms of number of neutrons, protons.
- Understand in terms of weak interaction, Quark internal structure.

**Beta and gamma decay**

- Beta* and gamma decay
- Nucleus emits positron. Proton changes to neutron
- Nucleus changes quantum state from high energy to low energy

**Topic: Radioactive half-life**

- Example of random decay.
- Start with 8 identical radioactive nuclei
- Suppose probability of decaying in one second is 50%.

- Every second, half the atoms decay
- The half-life is one second

- Undecayed nuclei
- T=0 sec  T=1 sec  T=2 sec  T=3 sec
**Carbon Dating**

- $^{14}\text{C}$ has a half-life of ~6,000 years, continually decaying back into $^{14}\text{N}$.
- Steady-state achieved in atmosphere, with $^{14}\text{C}:^{12}\text{C}$ ratio of 1:1 trillion (1 part in $10^{12}$).

As long as biological material alive, atmospheric carbon mix ingested (as CO$_2$), ratio stays fixed.

After death, no exchange with atmosphere. Ratio starts to change as $^{14}\text{C}$ decays.

**Chapter 16: Fission & fusion**

- The fission process
  - Some heavy nuclei split apart after absorbing a neutron.
  - Energy is released according to binding energy
- The fusion process
  - Light nuclei can fuse together under high temperature, pressure
  - Energy is released: binding energy differences
- Fission and fusion weapons
- Fission and fusion reactors

**Differences between nuclei**

- Schematic view of previous diagram
- $^{56}\text{Fe}$ is most stable
- Move toward lower energies by fission or fusion.
- Energy released related to difference in binding energy.

**Topic: Nuclear fission**

- In some cases, the effect of neutron bombardment is more dramatic.
- Leads to nuclear fission, where a heavy nucleus is split apart into two smaller ones.

**Fission chain reaction**

- Neutrons are released in this process, leading to more fission events
- Chain reaction can result

**Uranium isotopes**

- $^{235}\text{U}$ will fission.
- However after $^{235}\text{U}$ absorbs neutron to become $^{236}\text{U}$, it beta decays (neutron changes to proton) to $^{236}\text{Np}$, $t_{1/2}=23$ min
- This quickly beta decays to $^{236}\text{Pu}$, $t_{1/2}=2.3$ days

1941: discovered that Pu will fission.

Fission limited to $^{235}\text{U}$, $^{239}\text{Pu}$
**Controlled Fission Reactors**

- The reactor in a nuclear power plant does the same thing that a boiler does in a fossil fuel plant – it produces heat.
- Basic parts of a reactor:
  - Core (contains fissile material)
  - Moderator (slows neutrons down to enhance capture)
  - Control rods (controllably absorb neutrons)
  - Coolant (carries heat away from core to produce power)
  - Shielding (shields environment from radiation)

**Topic: Nuclear Fusion**

- 'Opposite' process also occurs, where nuclei are fused to produce a heavier nucleus, but requires large initial energy input.
- Called nuclear fusion.

**Terrestrial fusion reactions**

- Deuterium = nucleus of (1 proton & 1 neutron)
- Tritium = nucleus of (1 proton & 2 neutrons)
- Two basic fusion reactions:
  - deuterium + deuterium → ³He + n
  - deuterium + tritium → ⁴He + n

Energy is released as result of fusion:

\[
D + T \rightarrow {}^{3}\text{He} (3.5 \text{ MeV}) + n (14.1 \text{ MeV})
\]