From last time...

- **Galilean Relativity**
  - Laws of mechanics identical in all inertial ref. frames

- **Einstein’s Relativity**
  - All laws of physics identical in inertial ref. frames
  - Speed of light=c in all inertial ref. frames

- **Consequences**
  - *Simultaneity:* events simultaneous in one frame will not be simultaneous in another.
  - *Time dilation:* time interval between events appear different to different observers

Einstein’s principle of relativity

- **Principle of relativity:**
  - All the laws of physics are identical in all inertial reference frames.

- **Constancy of speed of light:**
  - Speed of light is same in all inertial frames
    (e.g. independent of velocity of observer, velocity of source emitting light)

(These two postulates are the basis of the special theory of relativity)

Consequences of Einstein’s relativity

- Many ‘common sense’ results break down:
  - Events that seem to be simultaneous are not simultaneous in different inertial frames
  - The time interval between events is not absolute. It will be different in different inertial frames
  - The distance between two objects is not absolute. It is different in different inertial frames
  -_velocities don’t always add directly

Time dilation

- Laser bounces up and down from mirror on train.
  - Joe on ground measures time interval w/ his clock.
  - Joe watches Jane’s clock on train as she measures the time interval.
  - Joe sees that these two time intervals are different.

Why is this?

- Jane on train: light pulse travels distance 2d.
- Joe on ground: light pulse travels *farther*
- Relativity: both Joe and Jane say light travels at c
  - Joe measures longer travel time of light pulse
  - This is time dilation

Time dilation, continued

- Observer Jane on train: light pulse travels distance 2d.
  - Time = distance divided by velocity = $2d/c$
  - Time in the frame the events occurred at same location called the proper time $\Delta t_p$
**Time dilation**

Time interval in Jane’s frame
\[ \Delta t_{\text{Jane}} = \frac{\text{round trip distance}}{\text{speed of light}} = \frac{2d}{c} \]

Joe measures a longer time
\[ \Delta t_{\text{Joe}} = \frac{2d^2 + (\frac{\gamma \Delta t_{\text{Jane}}}{2})}{c} \]
\[ \Delta t_{\text{Joe}} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} - \gamma \Delta t_{\text{Jane}} \]
\[ \gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} > 1 \]

**The ‘proper time’**

- We are concerned with two time intervals. Intervals between two events.
  - A single observer compares time intervals measured in different reference frames.
- If the events are at the same spatial location in one of the frames...
  - The time interval measured in this frame is called the ‘proper time’.
  - The time interval measured in a frame moving with respect to this one will be longer by a factor of \( \gamma \).

\[ \Delta t_{\text{other frame}} = \gamma \Delta t_{\text{proper}}, \quad \gamma > 1 \]

**Question**

I am on jet traveling at 500 mph. I throw a ball in the air, catch it in my hand, and time the round trip with a clock on my jet. You are on the ground watching my clock, and you also measure the time interval with your own clock. How do these time intervals compare?

A. \( \Delta t_{\text{jet}} = \Delta t_{\text{Earth}} \)
B. \( \Delta t_{\text{jet}} > \Delta t_{\text{Earth}} \)
C. \( \Delta t_{\text{jet}} < \Delta t_{\text{Earth}} \)

Proper time is measured in the jet frame (events occur at same spatial location). Times measured in other frames are longer (time dilation).

**Atomic clocks and relativity**

- In 1971, four atomic clocks were flown around the world on commercial jets.
- 2 went east, 2 went west \( \rightarrow \) a relative speed \( \sim 1000 \text{ mi/hr} \).
- On return, average time difference was 0.15 microseconds, consistent with relativity.

First atomic clock: 1949
Miniature atomic clock: 2003

**Global Positioning System (GPS)**

Network of 24 satellites orbiting earth at 14,000 km/hr, each carrying atomic clocks, At least three visible from any location.

GPS receiver on ground compares time signals from several satellites. Distance from each satellite given by travel time. Position on ground determined by distance from satellites.

Relativity: clocks run slow by 7 microseconds!

**Traveling to the stars**

Spaceship leaves Earth, travels at 0.95c

\[ d = 4.3 \text{ light-years} \]

Spaceship later arrives at star

0.95c
The ship observer’s frame

Earth leaves...

d = 4.3 light-years

..then star arrives

d = 4.3 light-years

Comparing the measurements

- The ship observer measures ‘proper time’
  - Heartbeats occur at the same spatial location (in the astronaut’s chest).
  - On his own clock, astronaut measures his normal heart-rate of 1 second between each beat.
  - Earth observer measures, with his earth clock, a time much longer than the astronaut’s (\( \Delta t_{\text{earth}} = \gamma \Delta t_{\text{astronaut}} \))

\[
\Delta t_{\text{earth}} = \gamma \Delta t_{\text{astronaut}} = \frac{\Delta t_{\text{astronaut}}}{\sqrt{1 - v^2/c^2}} = 3.2 \times \Delta t_{\text{astronaut}} = 3.2 \text{ sec}
\]

Earth observer sees astronaut’s heart beating slow, and the astronaut’s clock running slow.

Earth observer measures 3.2 sec between heartbeats of astronaut.

The twin ‘paradox’

The Earth observer sees the astronaut age more slowly than himself.
- On returning, the astronaut would be younger than the earthling.
- And the effect gets more dramatic with increasing speed!
- All this has been verified - the ‘paradox’ arises when we take the astronaut’s point of view.

Resolution

- Special relativity applies only to reference frames moving at constant speed.
- To turn around and come back, the astronaut must accelerate over a short interval.
- Only the Earthling’s determination of the time intervals using special relativity are correct.
- General relativity applies to accelerating reference frames, and will make the measurements agree.

Question

Both the astronaut and the earthling measure the time interval between heartbeats of the astronaut? Who measures the proper time?

A. The astronaut
B. The Earthling
C. Both
D. Neither
**Total trip time**
Spaceship leaves Earth, travels at 0.95c

\[
\Delta t_{\text{Earth}} = \frac{d}{v} = \frac{4.3 \text{ light-years}}{0.95c} = 4.5 \text{ years}
\]

Time for astronaut passes more slowly by a factor \(\gamma\).

Trip time for astronaut is \(4.5 \text{ yrs/3.2 = 1.4 years}\)

**Relative velocity of reference frames**

Both observers agree on relative speed, hence also \(\gamma\).

\[
\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - (0.95)^2}} = 3.203
\]

**Are there other ‘paradoxes’?**

- Both observer’s agree on the speed (0.95c)
  - Earth observer: ship moving
  - Ship observer: earth and star moving
  - They both agree on the speed
- But they disagree about the total trip time.
- If the time intervals are different, and speed is the same, how can distances be the same?

- **The distances are not the same!**

**Length Contraction**

- People on ship and on earth agree on relative velocity \(v = 0.95 \text{ c}\).
- But they disagree on the time (4.5 vs 1.4 years).
- What about the distance between the planets?

Earth frame \(d_{\text{Earth}} = vt_{\text{Earth}} = 0.95 \times 3 \times 10^8 \text{ m/s} \times 4.5 \text{ years} = 4 \times 10^{16} \text{ m (4.3 light years)}

Ship frame \(d_{\text{Ship}} = vt_{\text{Ship}} = 0.95 \times 3 \times 10^8 \text{ m/s} \times 1.4 \text{ years} = 1.25 \times 10^{16} \text{ m (1.3 light years)}

**Length contraction and proper length**

- Which one is correct?
  - Just like time intervals, distances are different in different frames.
  - There is no preferred frame, so one is no more correct than the other.
- The ‘proper length’ \(L_p\) is the length measured in a frame at rest with respect to objects
  - Here the objects are Earth and star.

- **Length in moving frame**
  \[L_y = L_n \frac{1 - \frac{v^2}{c^2}}{\gamma}
\]

- **Length in object’s rest frame**
  \[L = L_p \frac{1 - \frac{v^2}{c^2}}{c^2}
\]
The space-time continuum

- An event is indicated by a point in this graph.
- The time-dependent motion of a particle would be a string of these points. This is called the particle’s ‘world-line’

Constant velocity motion

- Worldline of an object moving at constant velocity is a line

Scale of the space-time graph

- Worldline of a light beam is a 45° angle
- X axis units are meters
- Y axis units are time to move one meter

Observing from a new frame

- In relativity these events will look different in reference frame moving at some velocity
  - The new reference frame can be represented as same events along different coordinate axes

Is any measurement the same for all observers?

- Relativity seems to say that there are no more absolutes.
  - Distance between objects depends on observer.
  - Time between events depends on observer.
- Immutable character of events suggest that there might be invariant quantities

The big picture

- Views of the same cube from two different angles.
- Distance between corners (length of red line drawn on the flat page) seems to be different depending on how we look at it.
- But clearly this is just because we are not considering the full three-dimensional distance between the points.
- The 3D distance does not change with viewpoint.
The real ‘distance’ between events

- Need a quantity that is the same for all observers
- A quantity all observers agree on is
  \[ x^2 - c^2t^2 = (\text{separation})^2 - c^2(\text{time interval})^2 \]

- Need to look at separation both in space and time to get the full ‘distance’ between events.
- In 4D: 3 space + 1 time
  \[ x^2 + y^2 + z^2 - c^2t^2 \]

- The same or ‘invariant’ in any inertial frame