Hour Exam 2: Wednesday, March 8
- In-class, covering waves, electromagnetism, and relativity
- Will cover: March Chap 6-12
- Griffith Chap 12, 15, 16
- All lecture material
- You should bring:
  - 1 page notes, written double sided
  - #2 Pencil and a Calculator
  - Review Monday March 6
  - Review questions online under “Review Quizzes” link

**From last time...**
- Einstein’s Relativity
  - All laws of physics identical in inertial ref. frames
  - Speed of light=c in all inertial ref. frames
- Consequences
  - Simultaneity: events simultaneous in one frame will not be simultaneous in another.
  - Time dilation
  - Length contraction
  - Relativistic invariant: \( x^2 - c^2 t^2 \) is ‘universal’ in that it is measured to be the same for all observers

**Time dilation, length contraction**
- \( t = \gamma t_{\text{proper}} \)
  - \( t_{\text{proper}} \) measured in frame where events occur at same spatial location
- \( L = \frac{L_{\text{proper}}}{\gamma} \)
  - \( L_{\text{proper}} \) measured in frame where events are simultaneous (or object is at rest)
  - \( \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \)
- \( \gamma \) always bigger than 1
- \( \gamma \) increases as \( v \) increases
- \( \gamma \) would be infinite for \( v = c \)
- Suggests some limitation on velocity as we approach speed of light

**Question**
A ship is parked at a street corner 12 meters long. It then cruises around the block and moves at 0.8 \( c \) past someone standing on that street corner. The street corner observer measures the ship to have a length of:
- A. 20.0 m
- B. 5.7 m
- C. 0.6 m
- D. 7.2 m

**‘Separation’ between events**
- Views of the same cube from two different angles.
- Distance between corners (length of red line drawn on the flat page) seems to be different depending on how we look at it.
- But clearly this is just because we are not considering the full three-dimensional distance between the points.
- The 3D distance does not change with viewpoint.

**The real ‘distance’ between events**
- Need a quantity that is the same for all observers
- A quantity all observers agree on is \( x^2 - c^2 t^2 = (\text{separation})^2 - c^2 (\text{time interval})^2 \)
- Need to look at separation both in space and time to get the full ‘distance’ between events.
- In 4D: 3 space + 1 time
  - \( x^2 + y^2 + z^2 - c^2 t^2 \)
- The same or ‘invariant’ in any inertial frame
Space travel example, earth observer

- Event #1: leave earth
- Event #2: arrive star

\[ d = 4.3 \text{ light-years (LY)} \]

\[ \Delta t_{\text{earth}} = \frac{d}{c} = \frac{4.3 \text{ light-years}}{0.95c} = 4.526 \text{ yrs} \]

Astronaut observer

Ship stationary, Earth and star move

Astronaut measures proper time

- Earth observer time is dilated, longer by factor gamma

- Event 1: ship and earth together
- Event 2: ship and star together
- Spatial separation in astronaut’s frame is zero!

A relativistic invariant quantity

<table>
<thead>
<tr>
<th>Earth Frame</th>
<th>Ship Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event separation</td>
<td>Event separation</td>
</tr>
<tr>
<td>4.3 LY</td>
<td>0 LY</td>
</tr>
<tr>
<td>Time interval</td>
<td>Time interval</td>
</tr>
<tr>
<td>4.526 yrs</td>
<td>1.413 yrs</td>
</tr>
</tbody>
</table>

\[ (\text{separation})^2 - c^2(\text{time interval})^2 \]

- The quantity \( (\text{separation})^2 - c^2(\text{time interval})^2 \) is the same for all observers
- It mixes the space and time coordinates

Question

Which events have a space-time separation of zero?

A. A & B
B. A & C
C. C & D
D. None of them

Universal space-time distance

\[ (\text{space separation})^2 - c^2(\text{time interval})^2 = (\text{space-time distance})^2 \]

Think of all the different observers measuring different spatial separations, different time intervals.

- Suppose the universal (space-time distance)\(^2 < 0\)
- Minimum time interval: space separation is zero \(\rightarrow\) events occur at same spatial location
  - This was our condition for measuring the proper time.

- In another frame, spatial separation not zero measured time interval must be longer
  - Time dilation!
Newton again

- Fundamental relations of Newtonian physics
  - \( \text{acceleration} = \frac{\text{(change in velocity)}}{\text{(change in time)}} \)
  - \( \text{acceleration} = \frac{\text{Force}}{\text{mass}} \)
  - \( \text{Work} = \text{Force} \times \text{distance} \)
  - \( \text{Kinetic Energy} = \frac{1}{2} (\text{mass}) \times (\text{velocity})^2 \)
  - \( \text{Change in Kinetic Energy} = \text{net work done} \)

- Newton predicts that a constant force gives
  - Constant acceleration
  - Velocity proportional to time
  - Kinetic energy proportional to \((\text{velocity})^2\)

Forces, Work, and Energy in Relativity

- What about Newton’s laws?
  - Relativity dramatically altered our perspective of space and time
    - But clearly objects still move, spaceships are accelerated by thrust, work is done, energy is converted.
  - How do these things work in relativity?

Applying a constant force

- Particle initially at rest, then subject to a constant force starting at \( t=0 \),
  \( \Delta \text{momentum} = \text{momentum} = (\text{Force}) \times (\text{time}) \)

- Using momentum = \((\text{mass}) \times (\text{velocity})\), Velocity increases without bound as time increases

  Relativity says no.

  The effect of the force gets smaller and smaller as velocity approaches speed of light

Relativistic speed of particle subject to constant force

- At small velocities (short times) the motion is described by Newtonian physics
- At higher velocities, big deviations!
- The velocity never exceeds the speed of light

Momentum in Relativity

- The relationship between momentum and force is very simple and fundamental
  \( \frac{\text{change in momentum}}{\text{change in time}} = \text{Force} \)

  Momentum is constant for zero force

This relationship is preserved in relativity

Relativistic momentum

- Relativity concludes that the Newtonian definition of momentum \( (p_{\text{Newton}} = m\text{velocity}) \) is accurate at low velocities, but not at high velocities

  \[ p_{\text{relativistic}} = \\
  \gamma m v \]

  \( \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \)
Was Newton wrong?

- Relativity requires a different concept of momentum
  \[ p_{\text{relativistic}} = \gamma mv \]
  \[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \]
- But not really so different!
- For small velocities \(<\) light speed \(v \approx c\), and so \(p_{\text{relativistic}} = mv\)
- This is Newton’s momentum
- Differences only occur at velocities that are a substantial fraction of the speed of light

How can we understand this?

- acceleration = \( \frac{\text{change in velocity}}{\text{change in time}} \)
- much smaller at high speeds than at low speeds
- Newton said force and acceleration related by mass.
- We could say that mass increases as speed increases.
  \[ p_{\text{relativistic}} = \gamma mv = \gamma m_0 v = m_{\text{relativistic}} v \]
- Can write this
  \[ p_{\text{relativistic}} = \gamma m_0 v = (\gamma m_0) v = \gamma m_0 \]
  \[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \]
  \[ m_0 = m_{\text{rest}} \]
- \( m_0 \) is the rest mass.
- relativistic mass \( m \) depends on velocity

Example

- An object moving at half the speed of light relative to a particular observer has a rest mass of 1 kg. What is its mass measured by the observer?
  \[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} = \frac{1}{\sqrt{1 - (0.5c/c)^2}} = \frac{1}{\sqrt{1 - 0.25}} \]
  \[ = \frac{1}{0.75} = 1.33 \]
  So measured mass is 1.15 kg

Relativistic Momentum

- Momentum can be increased arbitrarily, but velocity never exceeds \( c \)
- We still use
  \[ \frac{\text{change in momentum}}{\text{change in time}} = \text{Force} \]
- For constant force we still have
  \[ \text{momentum} = \text{Force} \times \text{time} \]
- But not really so different!
- Momentum has been redefined
  \[ p_{\text{relativistic}} = \gamma mv = \frac{mv}{\sqrt{1 - (v/c)^2}} \]

Relativistic mass

- The the particle becomes extremely massive as speed increases ( \( m = \gamma m_0 \) )
- The relativistic momentum has new form ( \( p = \gamma m_0 v \) )
- Useful way of thinking of things remembering the concept of inertia

Question

A object of rest mass of 1 kg is moving at 99.5% of the speed of light. What is its measured mass?

A. 10 kg
B. 1.5 kg
C. 0.1 kg
Work and Energy

- Newton says that Work = Force x Distance, and that net work done on an object changes the kinetic energy.
- Used this to find the classical kinetic energy:

\[ KE_{\text{classical}} = \frac{1}{2} m v^2 \]

Relativistic Kinetic Energy

- Might expect this to change in relativity.
- Can do the same analysis as we did with Newtonian motion to find:

\[ KE_{\text{relativistic}} = (\gamma - 1)m c^2 \]

- Doesn’t seem to resemble Newton’s result at all.
- However for small velocities, it does reduce to the Newtonian form:

\[ KE_{\text{relativistic}} \approx \frac{1}{2} m v^2 \quad \text{for} \quad v \ll c \]

Relativistic Kinetic Energy

- Can see this graphically as with the other relativistic quantities.
- Kinetic energy gets arbitrarily large as speed approaches speed of light.
- Is the same as Newtonian kinetic energy for small speeds.

Mass-energy equivalence

- This results in Einstein’s famous relation:

\[ E = \gamma m c^2, \quad \text{or} \quad E = mc^2 \]

- This says that the total energy of a particle is related to its mass.
- Even when the particle is not moving it has energy.
- We could also say that mass is another form of energy.
  - Just as we talk of chemical energy, gravitational energy, etc, we can talk of mass energy.

Total Relativistic Energy

- The relativistic kinetic energy is:

\[ KE_{\text{relativistic}} = \frac{1}{2} m v^2 - m c^2 \]

- Write this as:

\[ m c^2 = KE_{\text{relativistic}} + m c^2 \]

Example

- In a frame where the particle is at rest, its total energy is \( E = m c^2 \).
- Just as we can convert electrical energy to mechanical energy, it is possible to tap mass energy.
- A 1 kg mass has \((1\text{kg})(3\times10^8\text{m/s})^2=9\times10^{16}\) J of energy.
  - We could power 30 million 100 W light bulbs for one year! (~30 million sec in 1 yr)
Nuclear Power

- Doesn’t convert whole protons or neutrons to energy
- Extracts some of the binding energy of the nucleus
- $^{139}\text{Rb}$ and $^{143}\text{Cs} + 3n$ have less rest mass than $^{235}\text{U} + 1n$: $E = mc^2$

Energy and momentum

- Relativistic energy is $E = \gamma m_0 c^2$
- Since $\gamma$ depends on velocity, the energy is measured to be different by different observers
- Momentum also different for different observers
  - Can think of these as analogous to space and time, which individually are measured to be different by different observers
- But there is something that is the same for all observers:
  - $E^2 - c^2 p^2 = (m c^2)^2$ = Square of rest energy
- Compare this to our space-time invariant $x^2 - c^2 t^2$

A relativistic perspective

- The concepts of space, time, momentum, energy that were useful to us at low speeds for Newtonian dynamics are a little confusing near light speed
- Relativity needs new conceptual quantities, such as space-time and energy-momentum
- Trying to make sense of relativity using space and time separately leads to effects such as time dilation and length contraction
- In the mathematical treatment of relativity, space-time and energy-momentum objects are always considered together