Closed strings, Branes and Holes

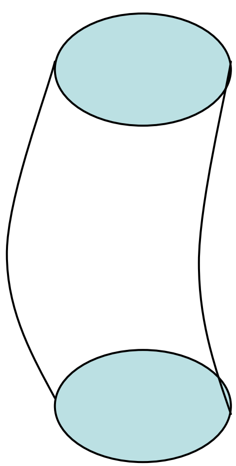
N. Itzhaki

- Based on: hep-th/0304192,
  hep-th/0307221.

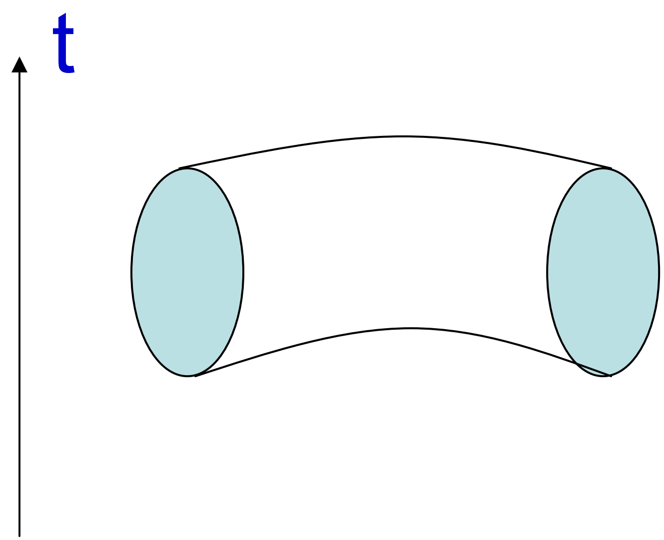
With D. Gaiotto and L.Rastelli

Introduction

The open/closed duality is a simple yet fundamental idea in string theory:



Closed string



One-loop open string

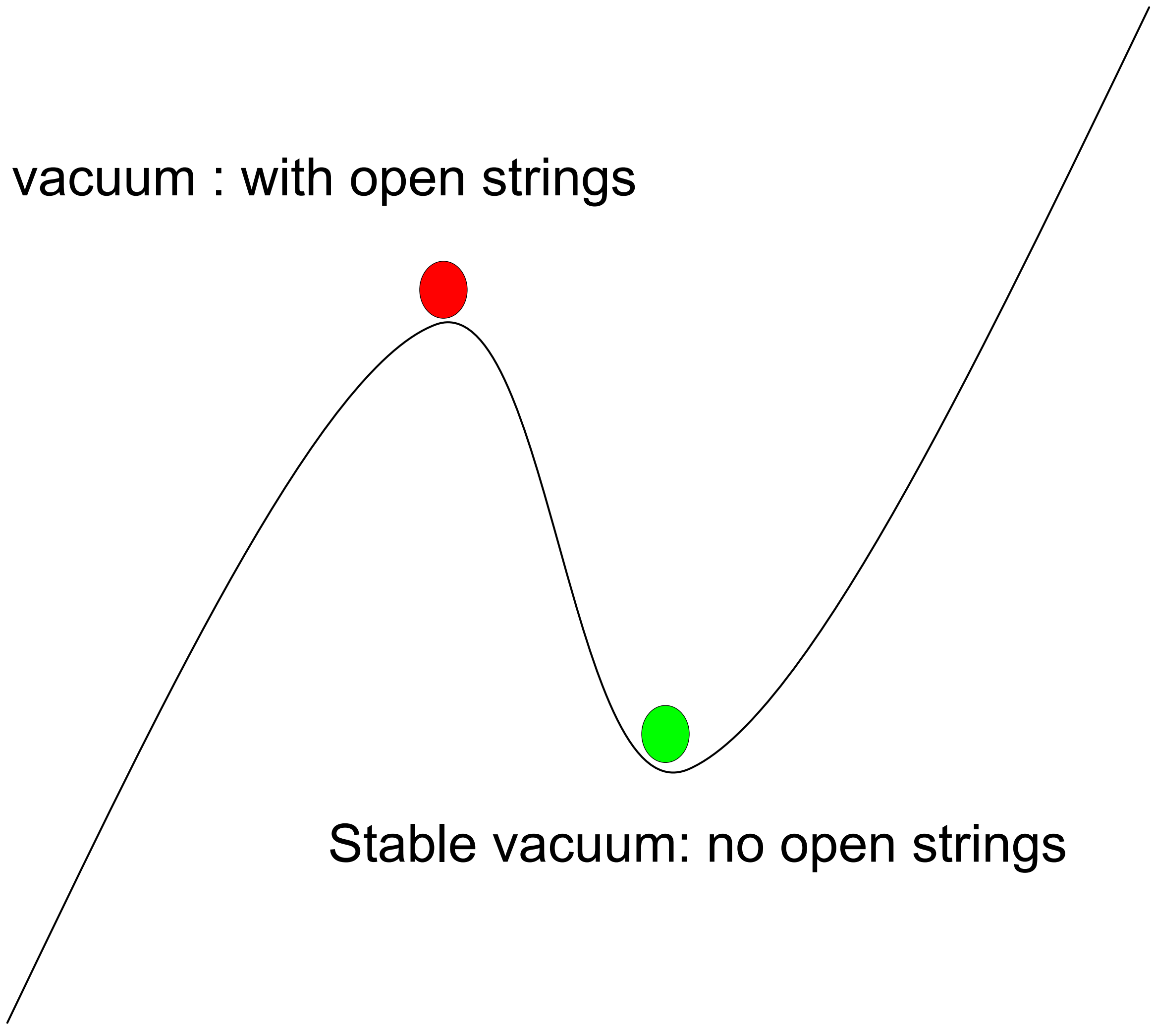
Closely related to ‘t Hooft large N limit.

Most recent developments in string theory are related to the open/closed string duality:

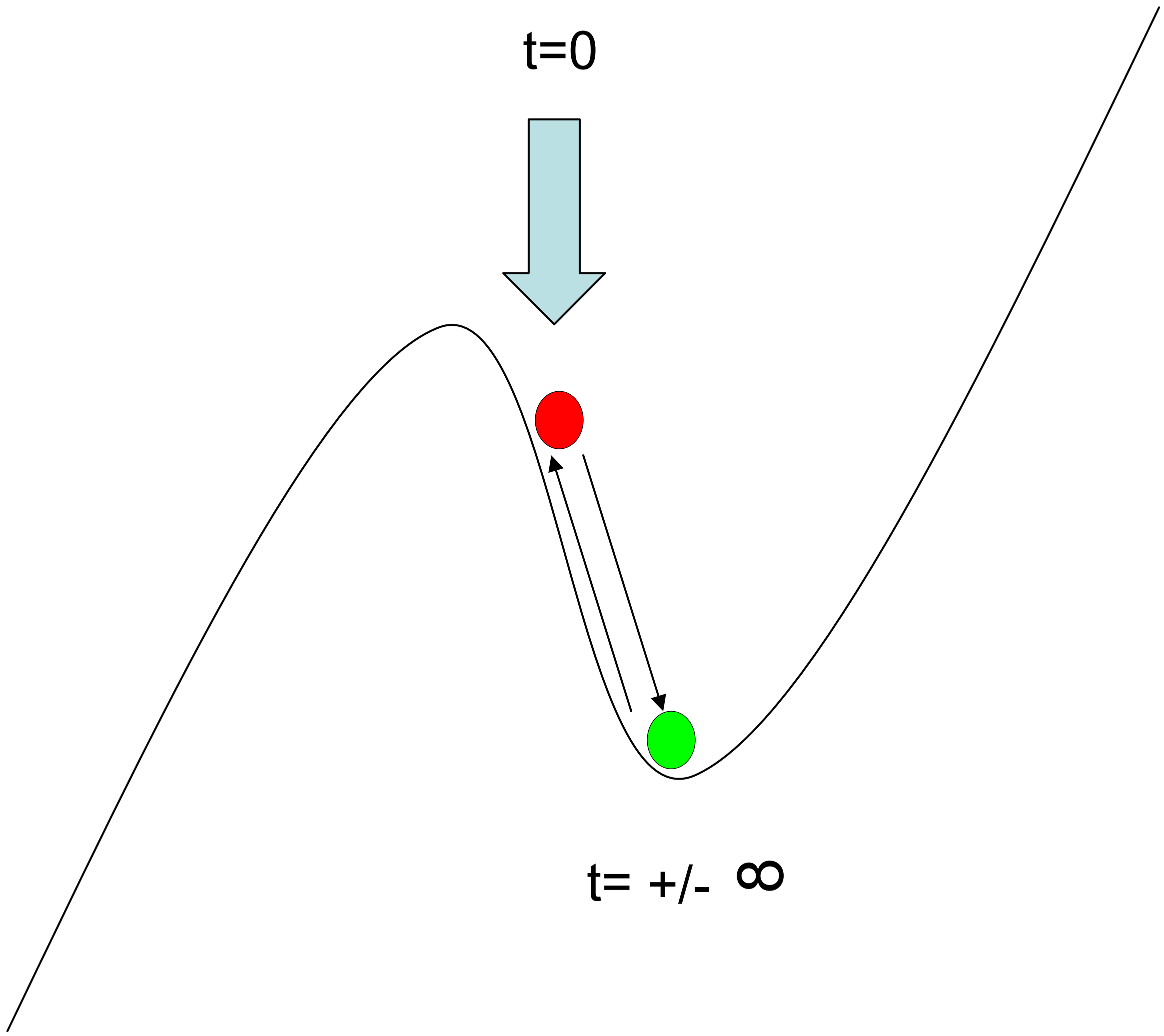
Old and new (c=1, DV, BFSS) matrix models, AdS/CFT and Open SFT.

Usually we need SUSY.

Open string tachyons



Sen's Boundary CFT



The boundary deformation is:

$$\lambda \int dt \cosh(X^0(t))$$

Life time: $\tau = -\log(\sin(\pi\lambda))$

Energy : $T = T_p \cos^2(\pi\lambda)$

$$\lambda = 1/2$$

- Life time = 0.
- Energy = 0.

No brane at all !!!

In particular, no open string degrees of freedom.

CFT question: What happened to the boundary?

To get a better understanding recall where Sen got is idea from:

Back in 94 Callan et. al showed that the boundary deformation

$$\lambda \int dt \cos(X(t))$$

Interpolates between Dp-branes $\lambda = 0$

and an array of D(p-1) branes $\lambda = 1/2$

Located at : $X = 2\pi(n + 1/2)$

So if we Wick rotate:

$$X \rightarrow iX^0$$

We get that Sen's boundary deformation

$$\lambda \int dt \cosh(X^0(t))$$

Is equivalent to an array of D-branes located in imaginary time:

$$X^0 = i2\pi(n+1/2)$$

Is this, indeed, the vacuum?

Sen showed that formally the boundary state vanishes.

Should we conclude that $\lambda = 1/2$ corresponds to nothing?

Hard to believe since the open string vacuum contains closed strings.

Indeed the norm of the boundary state is infinite so it might be that zero x infinity = finite .

Naïve argument gives 0

- Suppose we scatter n closed string off a single brane and get A(p,…) .
- For the array

$$X = \mathrm{a}\,(n+1/2)$$

we'll get

$$S(p,\ldots) = \sum_{n=-\infty}^{\infty} A(p,\ldots)\exp(i\ pa\,(n+1/2)) =$$

$$A(p,\ldots)\,2\pi \sum_{n=-\infty}^{\infty} (-1)^n \delta(Pa - 2\pi\ n)$$

- Wick rotation gives

$$S(E,\ldots) = A(iE,\ldots)\, 2\pi \sum_{n=-\infty}^{\infty} (-1)^n \delta(iE\, \mathrm{a} - 2\pi\, n) = 0$$

But we have to be careful because:

1- A(iE,…) might blow up at some points.

2- Delta function is non-analytic. So how can we analytically continue?

Non-zero example

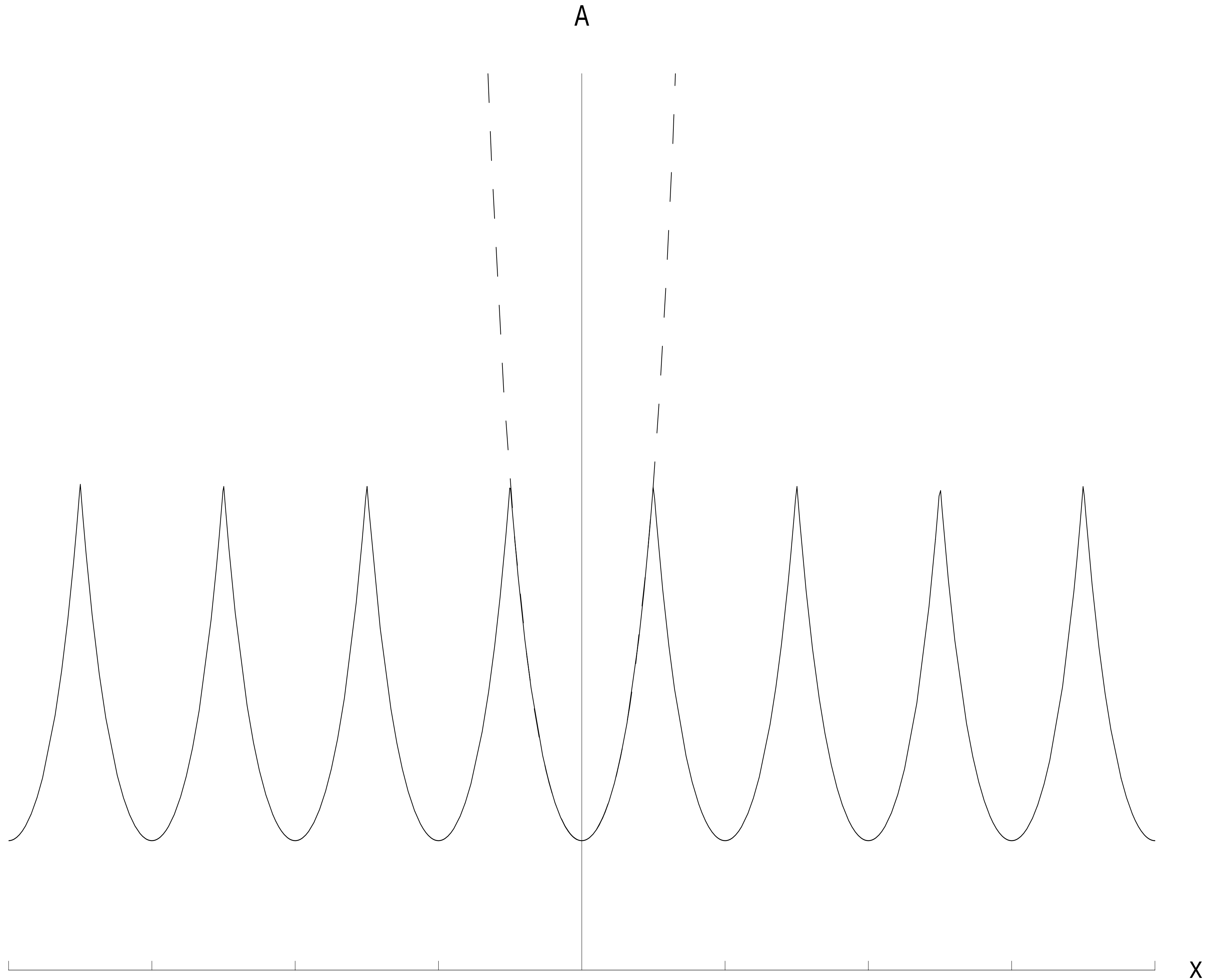
- Suppose: $$\widetilde{A}(P,\ldots) = \frac{1}{P^2 + c^2},$$
- In position space we get $$\frac{\pi}{c} e^{-c|X|}$$
- So we can sum for the array to find: $$\frac{\pi}{c} \sum_{n=-\infty}^{\infty} e^{-c|a(n+\frac{1}{2})+} = \frac{\pi \cosh(cX)}{c \sinh(\frac{ca}{2})}$$
- After Wick rotation we get $$\frac{\pi \cos(cX^0)}{c \sinh(\frac{ca}{2})}$$
- Fourier transform back $$\frac{\pi}{2c \sinh(\frac{ca}{2})} \left(\delta(E-c) + \delta(E+c)\right)$$

Comments

- Non-zero only at poles before the Wick rotation (E=- i p=+/- c).

Change in the dimension of moduli space.

- In the Wick rotation we had to choose a branch since the function was not analytic:


A
X

General Prescription

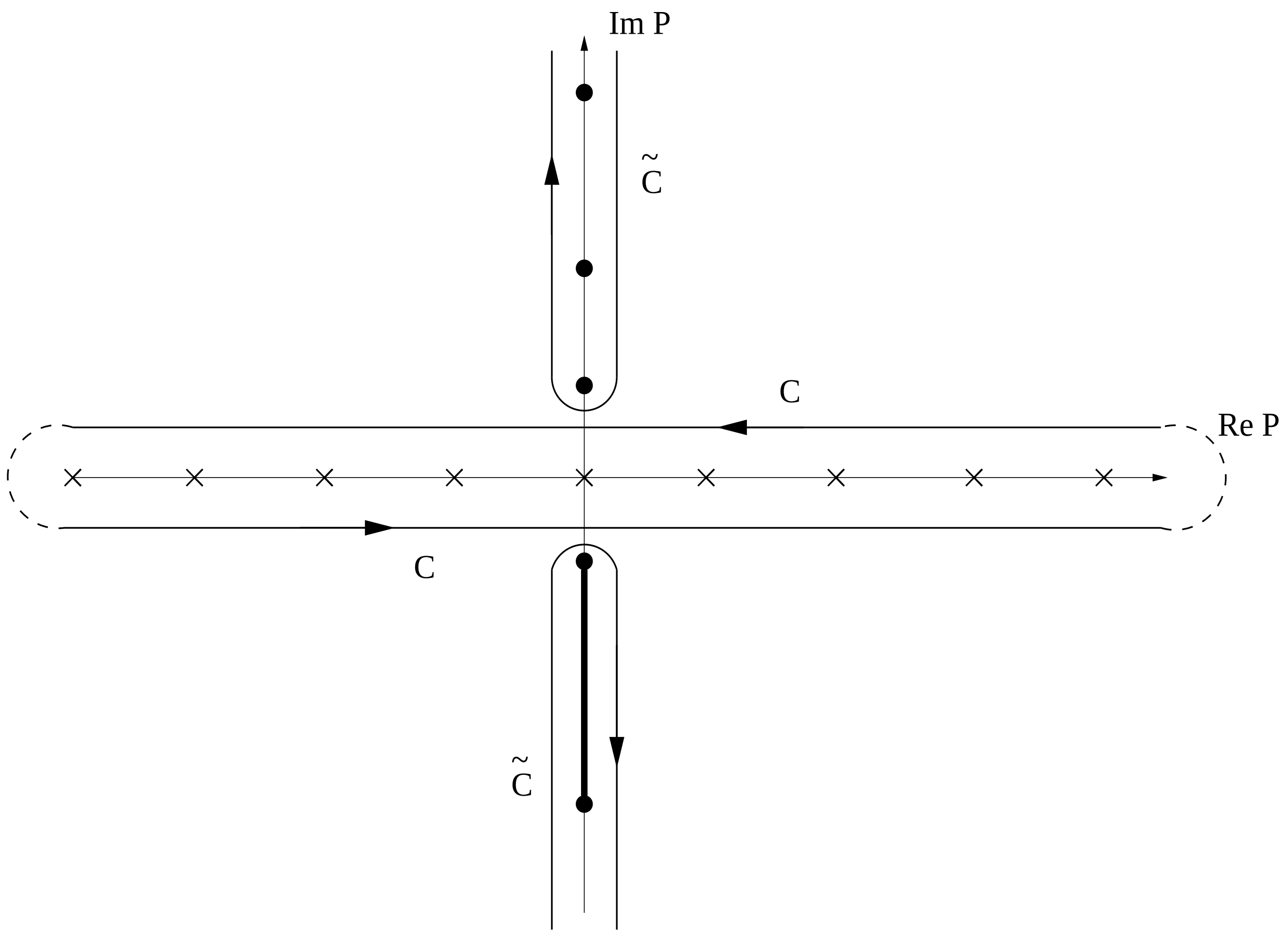
- For a generic case we have:

$$\int_{-\infty}^{\infty} dP\, e^{iPX} \sum_{n=-\infty}^{\infty} (-1)^n\, 2\pi\delta(aP - 2\pi n)\tilde{A}(P, \ldots).$$

- With the residues theorem we get

$$\frac{1}{2i} \oint_C dP\, e^{iPX} \frac{\tilde{A}(P, \ldots)}{\sin(\frac{aP}{2})},$$

Now we change the contour:



So finally we get in momentum space:

$$S(E) = F(E)\,\mathrm{Disc}_E[\widetilde{A}(iE)]$$

Where

$$F(E) = \frac{1}{2\sinh\left(\frac{aE}{2}\right)}\,.$$

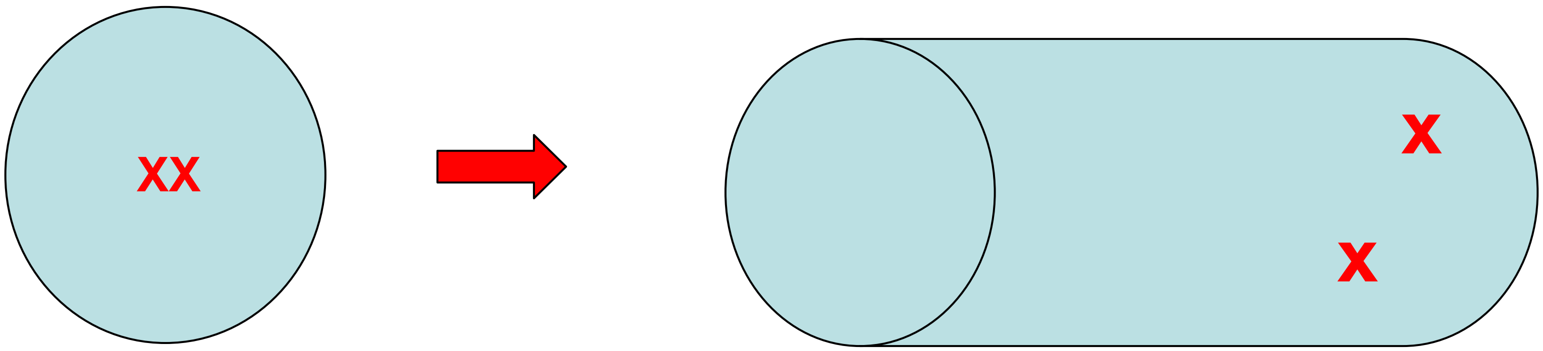
Let's see what happens when we apply this for a disk amplitude.The simplest one is of tachyon two-point amplitude: Before the Wick rotation we have

$$\widetilde{A}(p_1, p_2) = \frac{\Gamma(t/4-1)\Gamma(s-1)}{2\,\Gamma(t/4+s-2)}\,.$$

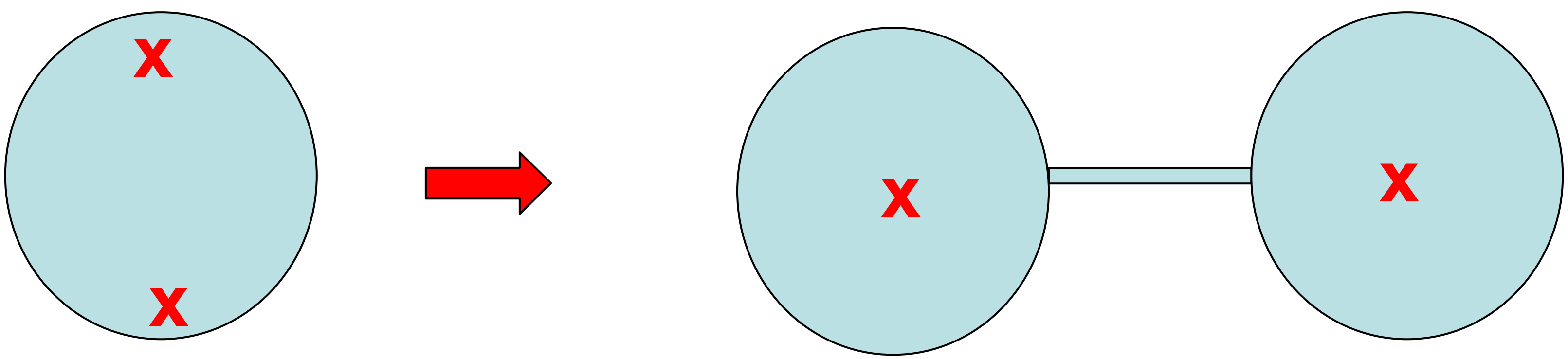
Where $s = p_{1||}^2 = p_{2||}^2$ and $t = (p_1 + p_2)^2$

Note (Hashimoto and Klebanov, 96):

Poles in t are due to closed strings.



Poles in S are due to open strings.



To apply our eq. we note that the disc. comes only from t (and not from s) and we find;

$$S(p_1, p_2) = \frac{1}{2\sinh\left(\frac{a|E|}{2}\right)} \sum_{k=0}^{\infty} f_k(s)\,\delta(t/4 - 1 + k)\,,$$

where

$$f_k(s) = \frac{(-1)^k \Gamma(s-1)}{2\,k!\,\Gamma(s-k-1)} = \frac{(2-s)(3-s)\cdots(1+k-s)}{2\,k!}\,.$$

All comes from the closed string channel !!!

There are no open string DOF!!!

In fact we can write it as a sphere amplitude with an extra closed string:

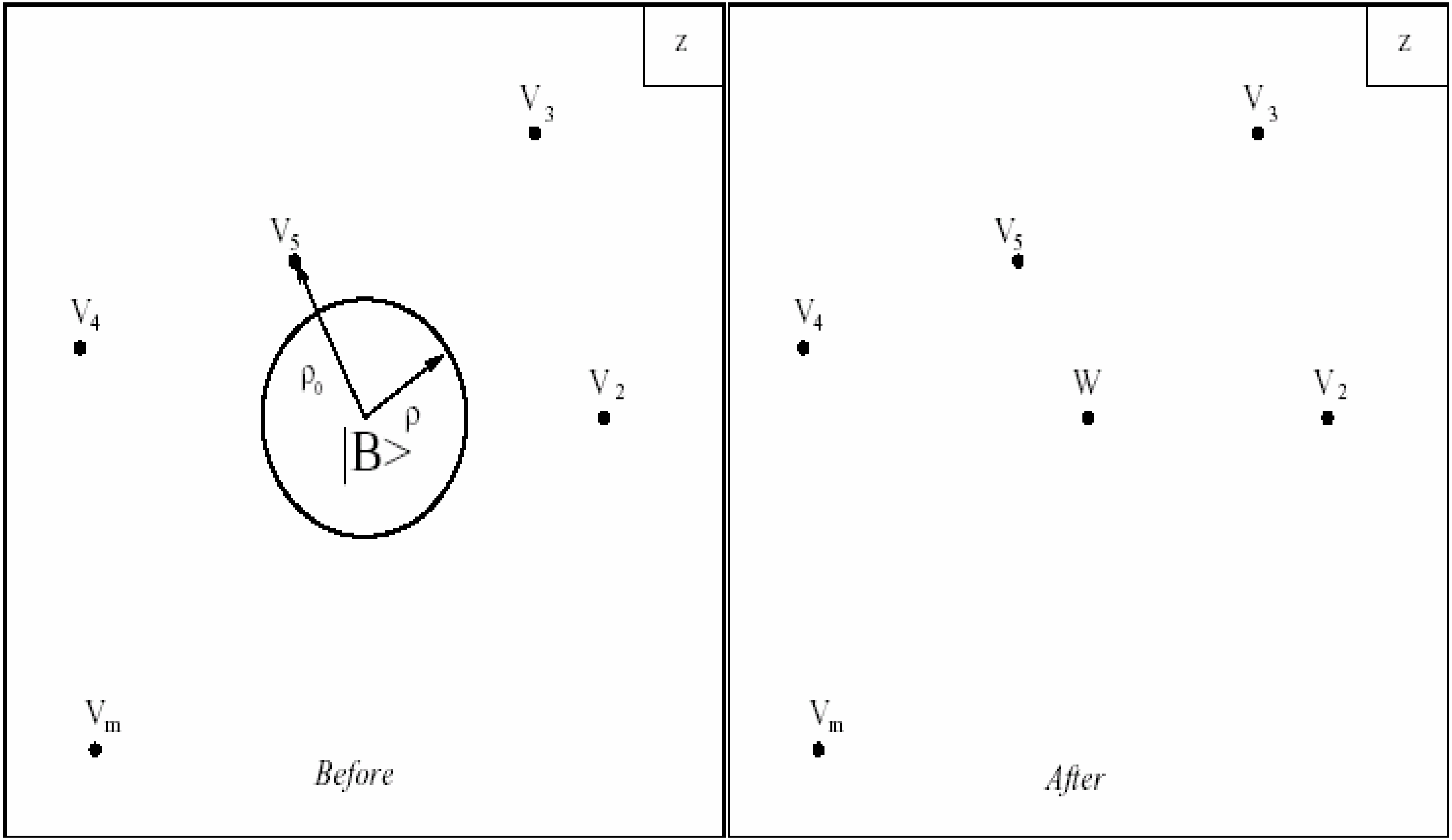
$$S(p_1, p_2) = \langle | \, V(p_1; \infty, \infty) \, V(p_2; 1, 1) \, |W\rangle \,,$$

So we see explicitly how the boundary shrinks and how

Disk → sphere

Is this special to two point function?

Higher points function:



The extra closed string is related to the original D-brane boundary state by:

$$|W\rangle \equiv \int dk \sum_i |k,i\rangle \frac{\delta(k^2/2 + 2l(i))}{2\sinh\left(\frac{a|E|}{2}\right)} \langle k,i| (b_0 + \bar{b}_0)|\mathcal{B}^{p-1}\rangle_{|z|=1}$$

$$= \frac{\delta(L_0 + \bar{L}_0)}{2\sinh\left(\frac{a|E|}{2}\right)} (b_0 + \bar{b}_0)|\mathcal{B}^{p-1}\rangle_{|z|=1} \,.$$

Note:

$$(Q_B + \bar{Q}_B)|W\rangle = 0\,, \quad (b_0 - \bar{b}_0)|W\rangle \,.$$

So this is truly a closed string state.

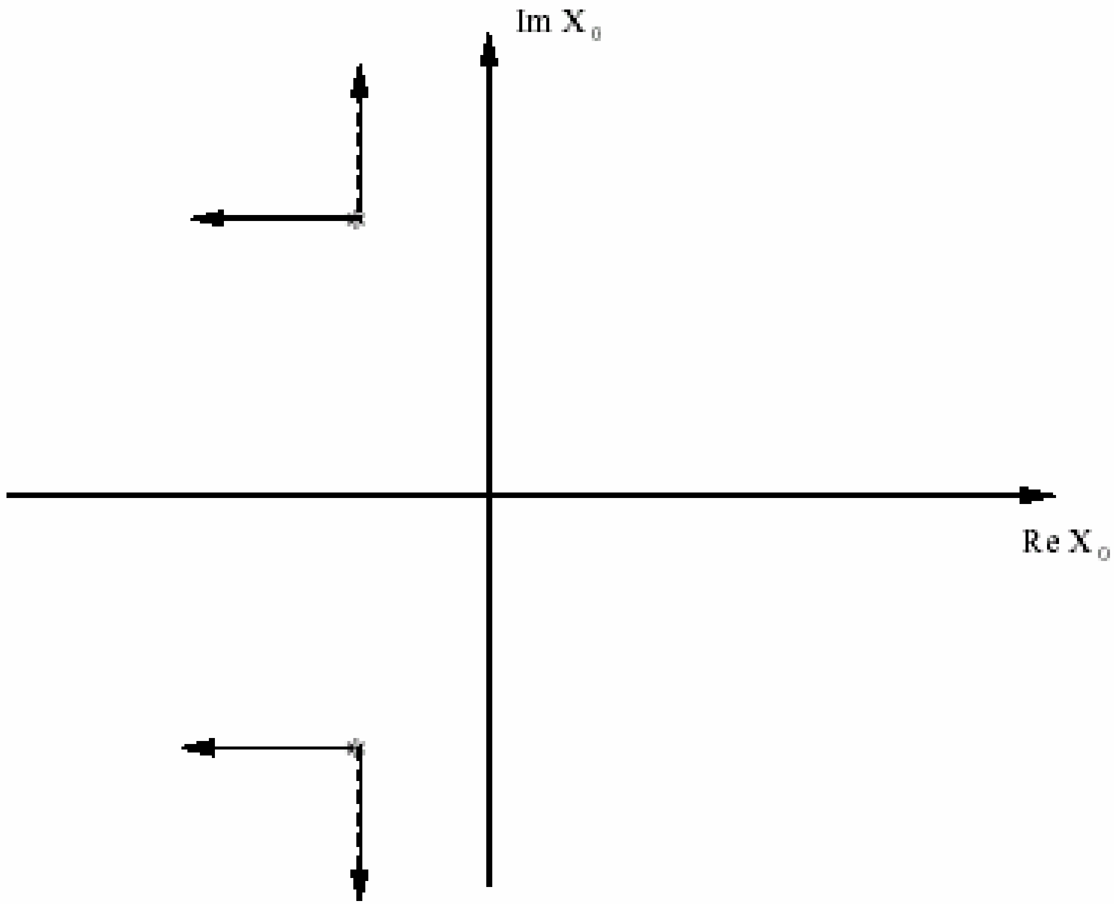
What can we say about W?

- Norm is not 1 in general.
- Due to massive closed strings modes the norm (and energy) blows up as we approach the critical point.
- Massless sector: The dilaton at infinity as if we had a D-brane.

We still can insert open string operators at the boundary.

Q: What's their role after the Wick rotation?

A: They move around the D-branes in Imaginary time according to the closed strings reality condition:


Im X_0
Re X_0

In fact we can do more than that:

We can also multiply the wave function in a non-trivial way that keeps the closed strings real after the Wick rotation.

↓

Reality condition does not commute with the Wick rotation.

Back to the infinite energy issue:

At the critical point a new branch opens up:
There are new on- shell open string DOF.

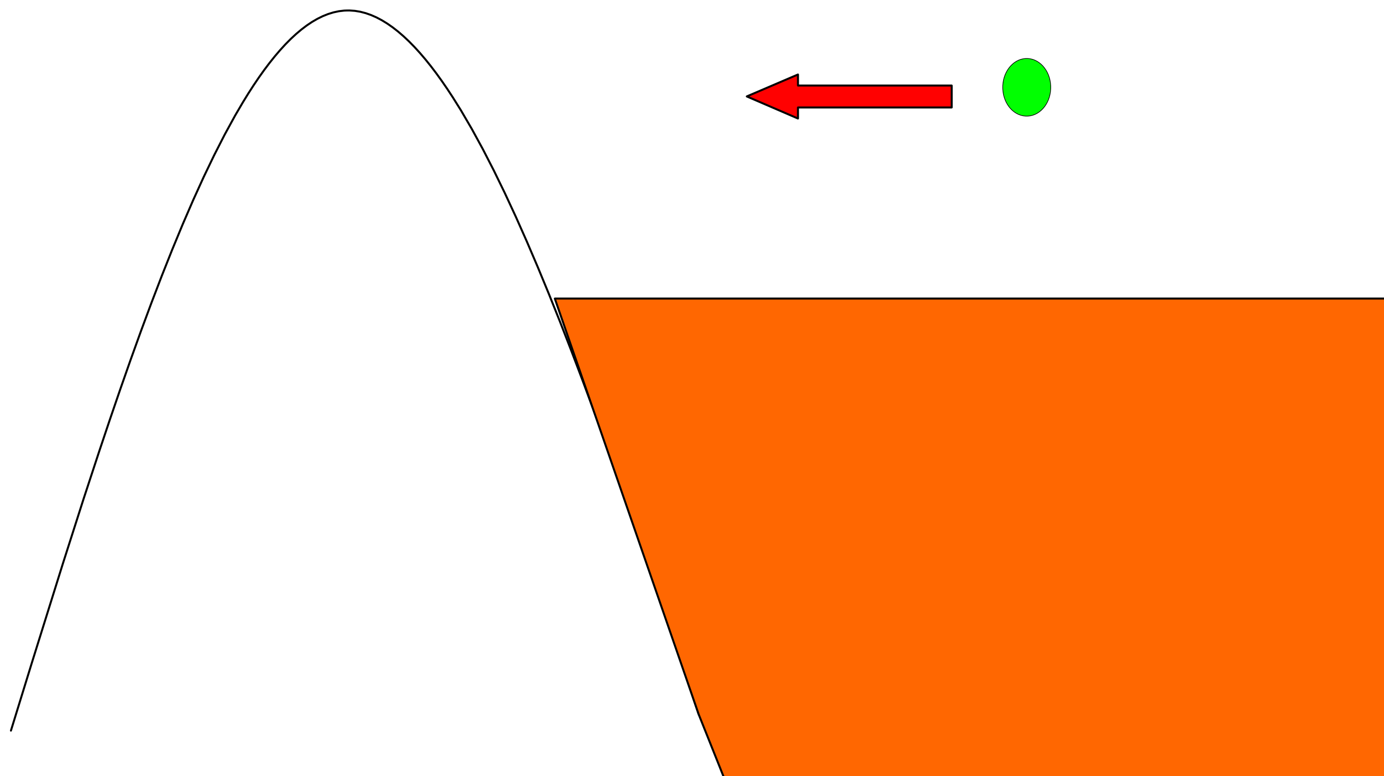
$$\cosh(X^0)$$

These will change the boundary deformation:

$$\lambda \int dt \cosh(X^0(t))$$

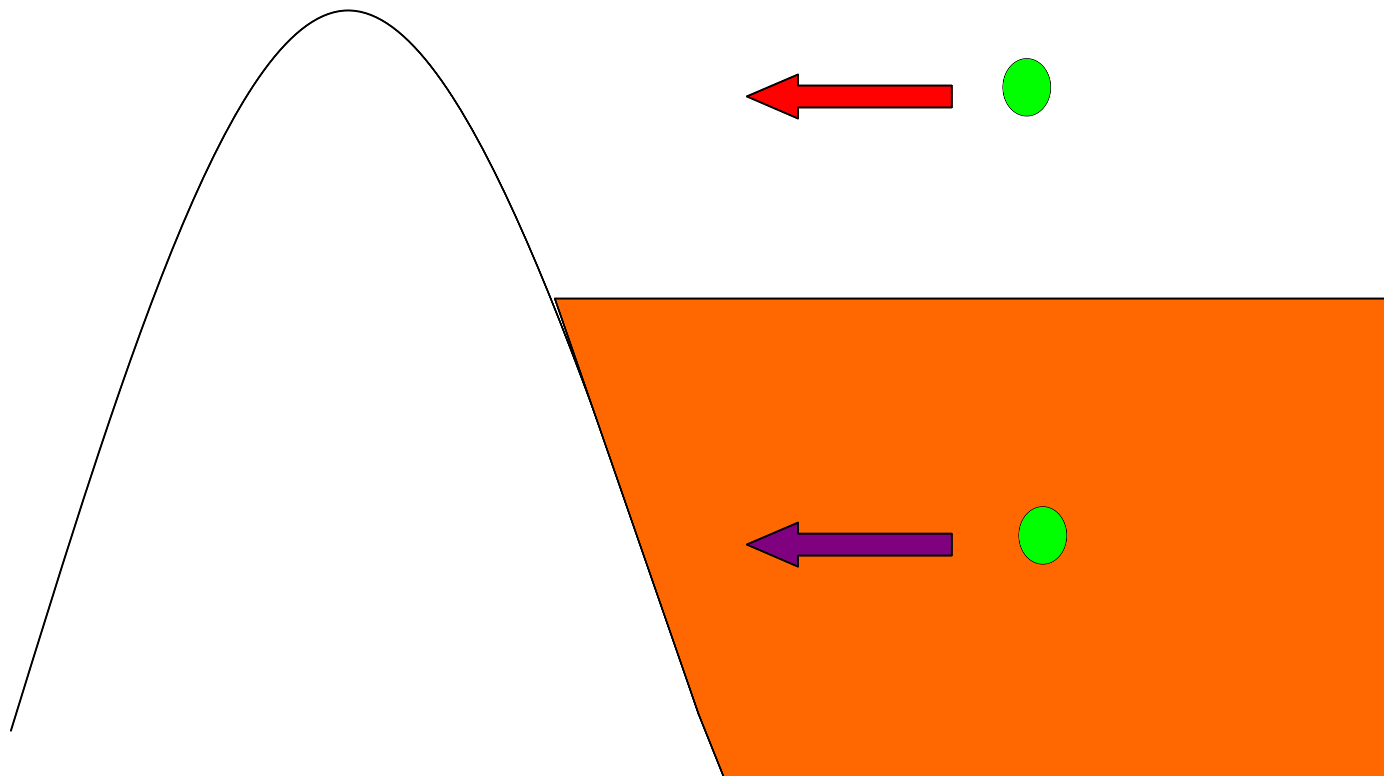
For any value of the deformation we still get an infinite amount of energy.

In the context of the c=1 matrix models KMS showed that taking a wave function of the open string deformation gives the correct energy!



At strings 2003 Sen raised the question:

What about the holes?



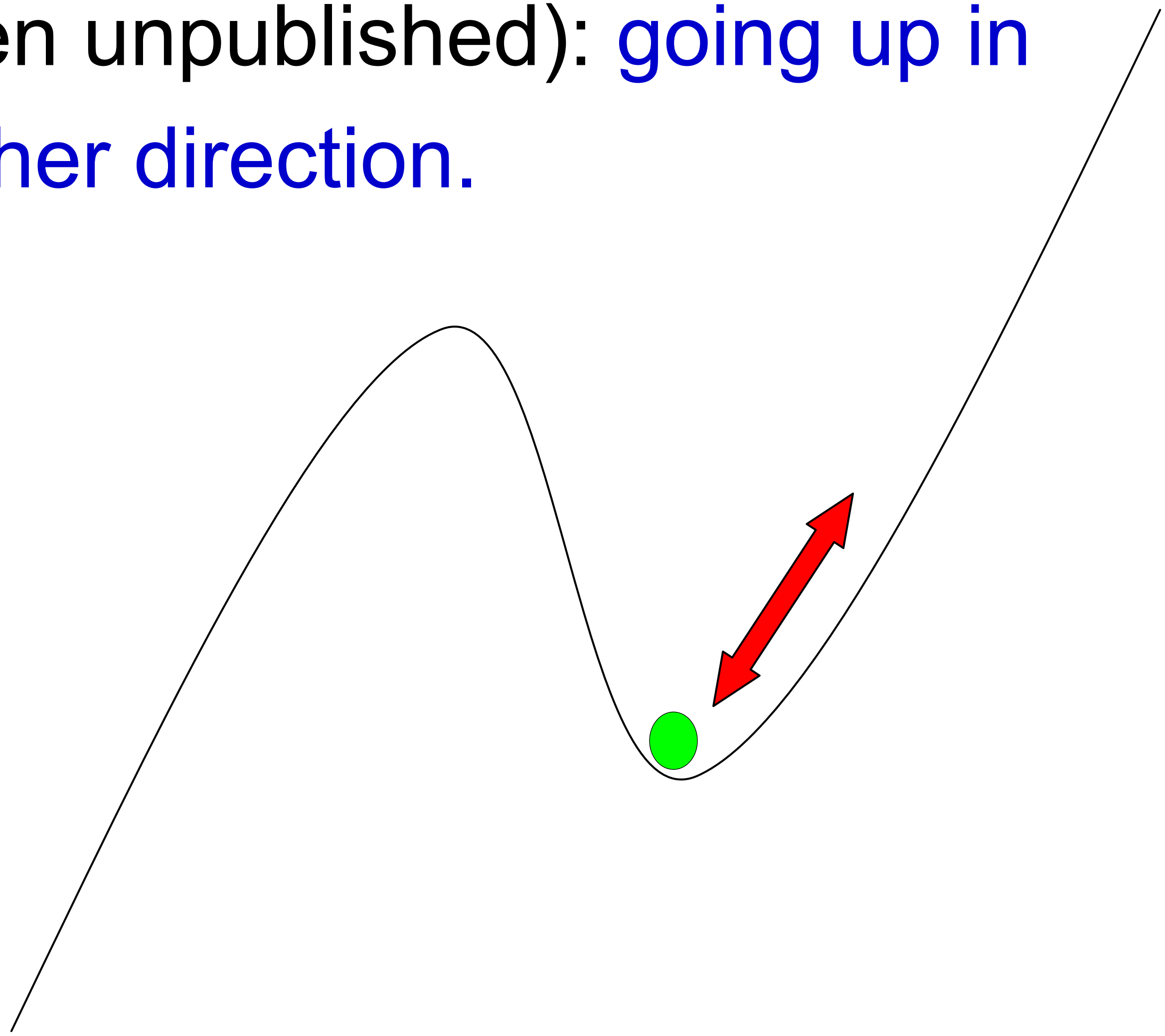
The solution again involves imaginary numbers at places where they are not expected:

$$\lambda \int dt \, \cosh(t) \quad \text{with} \quad \lambda = \frac{1}{2} + i\alpha,$$

Gives negative life time (expected) but also negative energy (unexpected).

This is fixed by multiplying the boundary state by -1.

- Q: what about 10D?
- A (Sen unpublished): going up in the other direction.



Conclusions:

- New relation between D-branes and strings.
- Is there a relation between the Wick rotation and S-dulaity?
- Can we get all closed strings that way?
- Is this a useful point of view on closed strings dynamics?
- Space like AdS/CFT ?

Thank You