Aspects of hadronic physics

in the gauge/gravity correspondence

Based on:

– *Hadronic Density of States from String Theory* ,
Phys.Rev.Lett. 91(2003)111602, hep-th/0306107
with `D. Vaman`.

– *Regge Trajectories Revisited in the Gauge/Gravity Correspondence*, hep-th/0310nnn
with `J. Sonnenschein and D. Vaman`

UW–Madison October, 2003

Motivation (Wall breaker) :

• AdS/CFT: Beyond the Supergravity Approximation.
Sugra modes $\leftrightarrow$ Protected Operators.

• Sectors of Large Charge:

BMN: Large R-charge in $\mathcal{N}=4$ SYM $\leftrightarrow$ Strings in RR plane wave Background.

GKP: Twist-two operators $\leftrightarrow$ folded string spinning in AdS_5.

PS: Scattering can be treated semiclassically by convolution of wave functions.

Sectors of large charges can be described by semiclassical string configurations.

What is a good universal quantum number?

Can we find out anything about $\mathcal{N}=1$ SYM? $U(1)_R$ is broken!

Can the density of States be computed without full knowledge of the spectrum (without fully solving string theory)? [(confining) `Kutasov`]

High spin states, Regge trajectories. Hadronic states in Gauge/Gravity.

Outline

- Conceptual Framework
- Story in Supergravity
- Hagedorn Density of States in String Theory
- Annulons and their Thermal Partition Function
- Semiclassical calculation of Thermal Partition Function
(For Sugra Backgrounds dual to Confining Gauge Theories)
- (II) Regge trajectories revisited in the Gauge/Gravity Correspondence

Conceptual Take Home

Heroes of the Day:

Genus expansion: The intuition maker

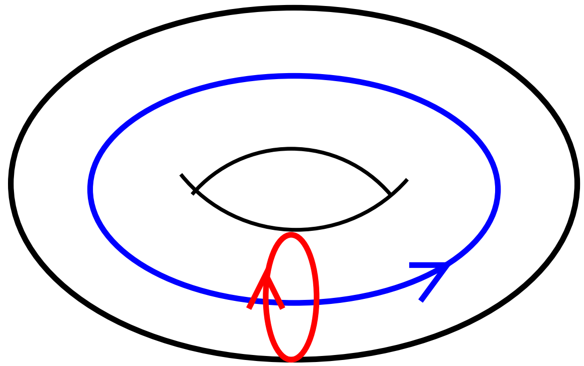
$$Z_{string} = e^{-2\Phi_0} Z_0 + Z_1 + e^{2\Phi_0} Z_2 + \ldots$$

$$Z_{string} = N^2\, Z_0 + N^0\, Z_1 + \frac{1}{N^2} Z_2 + \ldots$$

- Conformal Theories: Main Contribution is N^2.
- Confining Theories: Main Contribution is N^0.

A proposal for a semiclassical evaluation of Z_1:

The Solitonic Object for Torus Topology World Sheet



$$X^0 = n\,\beta\,\sigma_1 + m\,\beta\,\sigma_2, \qquad \text{“Completion”}$$

Include quantum fluctuations around this soliton to compute Z_1:

$$Z \approx Z_{soliton} Z_{quantum}$$

The talk (I): Motivation and Justification for this proposal.

Supergravity Story Predates AdS/CFT [Klebanov]

AdS/CFT Correspondence

$AdS_5 \times S^5 \Longleftrightarrow \mathcal{N} = 4\ SU(N)$ SYM

The Sugra limit $N \gg (g_{YM}^2 N)^{1/4} \gg 1$

$$ds^2 = h^{-1/2}(r)[-f(r)dt^2 + dx^i dx_i] + h^{1/2}[f(r)^{-1}dr^2 + r^2 d\O_5^2]$$

$$h(r) = \frac{R^4}{r^4}, \qquad f(r) = 1 - \frac{r_0^4}{r^4}$$

Temperature: $T = 1/\beta = r_0^2/\pi R^2$

$$S_{BH} = \frac{A_h}{4G} = \frac{\pi^2}{2} N^2 V_3 T^3 + \dots$$

Free $U(N)$ $\mathcal{N} = 4$ Supermultiplet

Content: Gauge Field, $6N^2$ massless scalars, $4N^2$ Weyl Fermions

$$S_0 = \frac{2\pi^2}{3} N^2 V_3 T^3.$$

The Famous 3/4

$$S = N^2 f(g_{YM}^2 N) V T^3$$

Why is a Hagedorn Density of States So Generic?

- Large N theories might be related to strings ('t Hooft, Polyakov, Maldacena).

- Should we expect generic properties of strings to be manifest in the field theory?

String states and Virasoro algebra: Given a primary state $|h\rangle$

$$L_0|h\rangle = h|h\rangle$$
$$L_m|h\rangle = 0, \quad m > 0.$$

States

$$L_{-k_1}L_{-k_2}\ldots L_{-k_m}|h\rangle.$$

L_0 eigenvalue $h_j + \Sigma_i k_i$.
The n^{th} level is spanned by vectors with $\Sigma_i k_i = n$

Q: How many such states?

A: $p(n)$ – Number of ways of writing n as the sum of positive integers.

Hardy-Ramanujan:

$$p(n) \sim \exp\left(\pi\sqrt{2n/3}\right)$$

For strings: $\alpha' m^2 \sim n \Longrightarrow S \sim E$ [Hagedorn] (!!!!) Nonperturbative effects, finite N

Motivating the Proposal: Compactified Boson on a Torus

- Configurations with nonzero winding number Torus $T^2 = \mathbf{C}/\Gamma : z \sim z + \omega_1 \sim z + \omega_2$

$$\Phi(z + k\omega_1 + k'\omega_2) = \Phi(z) + \beta(km + k'n), \qquad k, k' \in \mathbf{Z}$$

(m, n) Specifies a Topological configuration.

$$\Phi = \Phi^{cla}_{m,n} + \phi, \qquad \Phi^{cla}_{m,n} = \beta\left(\frac{z}{\omega_1}\frac{m\bar{\tau} - n}{\bar{\tau} - \tau} - \frac{\bar{z}}{\omega_1^*}\frac{m\tau - n}{\bar{\tau} - \tau}\right) \tag{1}$$

ϕ– periodic.

$$\begin{aligned} S[\Phi^{cla}_{m,n}] &= \frac{1}{2\pi}\int dz d\bar{z}\ \partial\Phi^{cla}_{m,n}\ \bar{\partial}\Phi^{cla}_{m,n} \\ &= \beta^2\frac{|m\tau - n|^2}{8\pi\,\tau_2}. \end{aligned}$$

Modular Invariance $\longrightarrow$ Sum over all sectors (m, n)

$$Z = \sum_{m,n} Z_{m,n} = \frac{\beta}{\sqrt{8}\pi}\ \frac{1}{\tau_2^{1/2}|\eta(\tau)|^2}\sum_{m,n}\exp\left(-\frac{\beta^2}{8\pi\tau_2}|m\tau - n|^2\right).$$

$$Z = Z_{quantum} Z_{soliton}$$

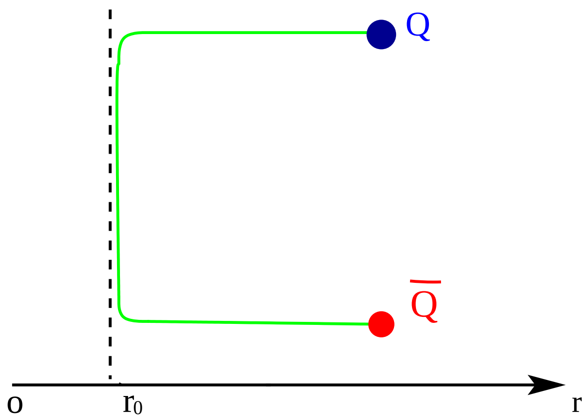
Q: Was the factorization an artifact of flat space?

Q: How to generalize for curved background?

Is there a solvable string theory of hadronic states?

Using a Penrose-Güven limit in Confining backgrounds.
Generic properties of AdS Dual of confining theories

- End of Space.
- Wilson Loop shows confining behavior.



$$T_s = \frac{1}{2\pi\alpha'} g_{tt}(r_0)$$

- $g_{tt}(r_0) \neq 0$.

- $g_{tt}(r_0)$ has a minimum (`J. Sonnenschein et al.`)

The Maldacena-Núñez background

- N D5 branes wrapped on S^2.
- IR: $\mathcal{N}=1$ SYM contaminated with KK.

$$\begin{aligned} ds^2_{str} &= e^{\phi_D}\Big[dx_\mu dx^\mu + \alpha' g_s N(d\rho^2 + e^{2g(\rho)}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) \\ &+ \frac{1}{4}\sum_a (w^a - A^a)^2)\Big], \quad e^{2\phi_D} = e^{2\phi_{D,0}}\frac{\sinh 2\rho}{2e^{g(\rho)}} \\ H^{RR} &= g_s N\Big[-\frac{1}{4}(w^1 - A^1)\wedge(w^2 - A^2)\wedge(w^3 - A^3) \\ &+ \frac{1}{4}\sum_a F^a \wedge (w^a - A^a)\Big] \end{aligned}$$

$e^{\phi_{D_0}} = \sqrt{g_s N}$

$$\begin{aligned} e^{2g} &= \rho\coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4} \\ A &= \frac{1}{2}\Big[\sigma^1 a(\rho)d\theta_1 + \sigma^2 a(\rho)\sin\theta_1 d\phi_1 + \sigma^3\cos\theta_1 d\phi_1\Big] \\ a(\rho) &= \frac{2\rho}{\sinh 2\rho} \end{aligned}$$

w^a – $SU(2)$ left-invariant one-forms

Scales associated with the $\mathcal{N}=1$ SYM dual of the MN background.

$$M^2_{gb} \sim M^2_{KK} \sim \frac{1}{g_s N\alpha'}, \quad T_s \propto M^2_{gb}\,(g_s N)^{\frac{3}{2}}.$$

The Penrose-Güven limit: Set up

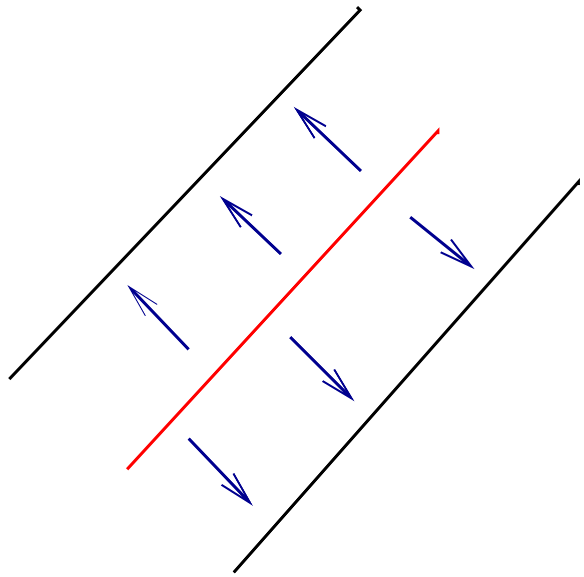
Make the following change of variables

$$dt = dx^0, \qquad x^i \to \frac{1}{L}\, x^i, \qquad \rho = \frac{m_0}{L}\, r,$$
$$\theta_2 = \frac{2\, m_0}{L}\, v, \qquad \phi_+ = \frac{1}{2}(\psi + \phi_2),$$

where $L^2 = \sqrt{g_s N}$ and $m_0 = \frac{1}{\sqrt{g_s N \alpha'}}$ is the glueball mass

$$\hat{\phi}_1 = \phi_1 + \frac{1}{3}\,\phi_+ \qquad \hat{\phi}_2 = \phi_2 - \phi_+.$$

$$x^+ = t, \qquad x^- = \frac{L^2}{2}(t - \frac{1}{m_0}\,\phi_+),$$



The Penrose-Güven Limit

$L \to \infty$; m_0 fixed

$$ds^2 = -2dx^+dx^- - m_0^2\,(\frac{1}{9}z_1^2 + \frac{1}{9}z_2^2 + v_1^2 + v_2^2)(dx^+)^2 + d\vec{x}^{\,2} + d\vec{z}^{\,2} + dv_1^2 + dv_2^2.$$

- 4 massless direction: (three x's from WV and one z).
- 2 directions (v) with mass m_0.
- 2 directions with mass $\frac{1}{3}\,m_0$.

$$H^{RR} = -2\,m_0\,dx^+ \wedge [\,dv_1 \wedge dv_2 + 1/3\,dz_1 \wedge dz_2].$$

- Fermions: 4 with mass $m_0/3$ and 4 with mass $2m_0/3$

The Hamiltonian is(Poincare time/Energy):

$$H = -p_+ = i\partial_+ = E - m_0(-\frac{1}{3}J_1 + J_2 + J_\psi) = E - m_0\,J,$$

$$P^+ = -\frac{1}{2}p_- = \frac{i}{2}\partial_- = \frac{m_0}{\Omega^2}(-\frac{1}{3}J_1 + J_2 + J_\psi) = m_0\frac{J}{\sqrt{g_sN}}.$$

The Annulon Hamiltonian

The light-cone Hamiltonian of the theory has the following simple form:

$$
\begin{aligned}
H &= \frac{P_i^2}{2P^+} + \frac{P_4^2}{2P^+} + \frac{1}{2\alpha' P^+}\sum_{n=1}^{\infty} n(N_n^i + N_4^i) \\
&+ \frac{1}{2\alpha' p^+}\sum_{n=0}^{\infty}\Big(w_n^a(N_n^1 + N_n^2) + w_n^b(N_n^3 + N_n^4)\Big) \\
&+ \frac{1}{2\alpha' p^+}\sum_{n=0}^{\infty}\Big(\omega_n^\alpha \mathcal{S}_n^\alpha + \omega_n^\beta \mathcal{S}_n^\beta\Big)\,. \qquad (2)
\end{aligned}
$$

where $i = 1, 2, 3, \mathbf{4}$, $a = 5, 6$, $b = 7, 8$, $\alpha = 1, 2, 3, 4$ and $\beta = 5, 6, 7, 8$; N and $\mathcal{S}$ are bosonic and fermionic occupation numbers

$$
\begin{aligned}
w_n^a &= \sqrt{n^2 + (m_0 p^+ \alpha')^2}, \qquad w_n^a = \sqrt{n^2 + \frac{1}{9}(m_0 p^+ \alpha')^2}, \\
\omega_n^\alpha &= \sqrt{n^2 + \frac{1}{9}(m_0 p^+ \alpha')^2}, \qquad \omega_n^\beta = \sqrt{n^2 + \frac{4}{9}(m_0 p^+ \alpha')^2} \qquad (3)
\end{aligned}
$$

A string theory of hadrons

The Hamiltonian purely in Field Theory language

$$
H = [\frac{\mathcal{P}_i^2}{2m_0 J} + \frac{T_s}{2m_0 J}(\mathcal{N}_R + \mathcal{N}_L)] + [\frac{T_s}{2m_0 J}(H_0 + H_R + H_L)]\ .
$$

Towards the MN Annulon partition function

- building blocks: Boson off criticality [Itzykson and Saleur].

$$z_{lc}^{(0,0)}(\tau,m)=\int\mathcal{D}X\exp\Big[-\int_T d^2z\bar{X}(-\partial_z\partial_{\bar{z}}+m^2)X\Big], \tag{4}$$

Doubly periodic quantum boson $z=\xi_1+\tau\xi_2$,

$$X(\xi_1,\xi_2)=\sum_{n1,n2\in\mathbf{Z}}X_{n1,n2}\exp[2\pi i(n_1\xi_1+n_2\xi_2)]$$

$$\begin{aligned} d^2z &= d\xi_1 d\xi_2\tau_2, \\ \partial_z\partial_{\bar{z}} &= \frac{1}{4{\tau_2}^2}\left(|\tau|^2{\partial_1}^2-2\tau_1\partial_1\partial_2+{\partial_2}^2\right), \end{aligned}$$

Explicit Gaussian integrals over $X_{n1,n2}$

$$z_{lc}^{(0,0)}(\tau,\mu)=\left[\prod_{n_1,n_2\in\mathbf{Z}}\tau_2\left((\frac{2\pi}{4\tau_2})^2|n_1\tau-n_2|^2+m^2\right)\right]^{-1}.$$

Double Product $\longrightarrow$ Modular properties

$$z_{lc}(-1/\tau,m|\tau|)=z_{lc}(\tau,m)$$

Fermionic Partition function [antiperiodic in ξ_1]

$$z_{lc}^{(1/2,0)}(\tau,\mu)=\prod_{n_1,n_2\in\mathbf{Z}}\tau_2\left((\frac{2\pi}{4\tau_2})^2\left|\frac{2n_1+1}{2}\tau+n_2\right|^2+\frac{\mu^2\,\beta^2}{\tau_2^2}\right)$$

Too Formal!

A Nonholomorphic Generalization of Dedekind $\eta(\tau)$

Performing one of the infinite products

$$
\begin{aligned}
z_{lc}^{(0,0)}(\tau,m) &= \exp\left[2\pi\tau_2\left(m/2+\sum_{n=1}^{\infty}\sqrt{n^2+m^2}\right)\right] \\
&\quad \left[\prod_{n\in\mathbf{Z}}\left(1-\exp[2\pi i(\tau_1 n+i\tau_2\sqrt{n^2+m^2})]\right)\right]^{-1}.
\end{aligned}
$$

Compare with

$$
|\eta(\tau)|^2=\exp\left[-\pi\tau_2/6\right]\left[\prod_{n\in\mathbf{Z}}\left(1-\exp[2\pi in(\tau_1+i\tau_2)]\right)\right]^{-1} \tag{5}
$$

ζ-function regularization of the Casimir Energy

$$
\begin{aligned}
\gamma_0(m)=\frac{m}{2}+\sum_{n=1}^{\infty}\sqrt{n^2+m^2} &= \frac{m}{2}+\left[-\frac{1}{12}+\frac{1}{2}m-\frac{1}{2}m^2\ln(4\pi e^{-\gamma})\right. \\
&+ \left.\sum_{n=2}^{\infty}(-1)^n\frac{\Gamma(n-\frac{1}{2})}{n!\Gamma(-\frac{1}{2})}\zeta(2n-1)m^{2n}\right],
\end{aligned}
$$

γ – Euler constant. Flat space limit $(m\to 0)$

$\gamma_0(m) \longrightarrow \sum_{n=1}^{\infty} n=\zeta(-1)=-1/12.$

The MN Annulon Partition Function

$$
\begin{aligned}
Z(\beta,\mu) \;=\;& \frac{\beta}{4\pi\, l_s} \int_0^\infty \frac{d\tau_2}{{\tau_2}^2} \int_{-1/2}^{1/2} d\tau_1 \sum_{r=1}^{\infty} \left[1-(-1)^r\right] \exp\!\left(-\frac{\beta^2\, r^2}{2\pi\, \alpha'\, \tau_2}\right) \\
\times\;& \left[{\tau_2}^{-1/2} |\eta(\tau)|^{-2}\right]^4 \qquad 4 \quad \text{massless bosons} \\
\times\;& \left[z_{lc}^{(0,0)}(\tau, \frac{m_0\, \beta\, r}{\tau_2})\right]^2 \qquad 2 \quad m_0 \text{ bosons} \\
\times\;& \left[z_{lc}^{(0,0)}(\tau, \frac{m_0/3\, \beta\, r}{\tau_2})\right]^2 \qquad 2 \quad m_0/3 \text{ bosons} \\
\times\;& \left[z_{lc}^{(1/2,0)}(\tau, \frac{m_0/3\, \beta\, r}{\tau_2})\right]^4 \qquad 4 \quad m_0/3 \text{ fermions} \\
\times\;& \left[z_{lc}^{(1/2,0)}(\tau, \frac{2m_0/3\, \beta\, r}{\tau_2})\right]^4 \qquad 4 \quad 2m_0/3 \text{ fermions} \\
+\;& \text{stuff associated with a nonsupersymmetric ground state} \qquad (6)
\end{aligned}
$$

Hagedorn Temperature:

$$-\frac{T_s\beta_H^2}{2\pi}+\frac{2}{3}\pi-4\pi\gamma_0(m_0\beta_H)-4\pi\gamma_0(\frac{m_0}{3}\beta_H)+8\pi\gamma_{1/2}(\frac{m_0}{3}\beta_H)+8\pi\gamma_{1/2}(\frac{2m_0}{3}\beta_H)=0. \tag{7}$$

Limits: $m_0 \to 0$ [IIB Strings in Flat Space]

Large m_0 a lower dimensional theory with $\beta_H = 2\pi/(\sqrt{3}T_s^{1/2})$

Density of States for the MN Annulons

$$S=\frac{2\pi}{\sqrt{3}}\frac{E}{\tilde{T}_s^{1/2}}.$$

$\tilde{T}_s = T_s/J$. In general $S(m_0, T_s/J)$.

Where are the Temporal Windings?

- Using Light-Cone, temporal coordinate gauged away, RR field.
- How can this result be turned into evidence for the proposal.

The above Partition function was written over the strip:

$$E: \quad \tau_2 > 0, \quad -\frac{1}{2} < \tau_1 < \frac{1}{2}.$$

Generalizing a flat space result: Tiling.

$$\begin{aligned} Z(\beta,\mu) \;=\; & \frac{\beta}{4\pi\, l_s} \int_{\mathcal{F}} \frac{d\tau_2}{{\tau_2}^2} \int d\tau_1 \sum_{m,n}{}' \prod_{n_1,n_2 \in \mathbf{Z}} \exp\left(-\frac{\beta^2 |m\tau+n|^2}{2\pi\alpha' \tau_2}\right) \\ \times \left[{\tau_2}^{-1/2} |\eta(\tau)|^{-2}\right]^4 \quad & \left[\frac{\pi^2}{4\tau_2}|n_1\tau - n_2|^2 + \frac{\mu^2\,\beta^2}{\tau_2}|m\tau+n|^2\right]^{-2} \\ & \left[\frac{\pi^2}{4\tau_2}|n_1\tau - n_2|^2 + \frac{\mu^2\,\beta^2}{9\tau_2}|m\tau+n|^2\right]^{-2} \\ & \left[\frac{\pi^2}{4\tau_2}\left|\frac{2n_1+1}{2}\tau + n_2\right|^2 + \frac{\mu^2\,\beta^2}{9\tau_2}|m\tau+n|^2\right]^{4} \\ & \left[\frac{\pi^2}{4\tau_2}\left|\frac{2n_1+1}{2}\tau + n_2\right|^2 + \frac{4\mu^2\,\beta^2}{9\tau_2}|m\tau+n|^2\right]^{4} \\ + \;& \text{stuff associated with a nonsupersymmetric ground state} \end{aligned} \tag{8}$$

$m, n \in \mathbf{Z}^*$

Integration over the fundamental domain:

$$\mathcal{F}: \quad |\tau| > 1, \quad -\frac{1}{2} < \tau_1 < \frac{1}{2}.$$

Mixing $Z_{quantum} Z_{soliton}$: (m,n) dependent masses:

$$\mu \to \mu|m\tau + n|$$

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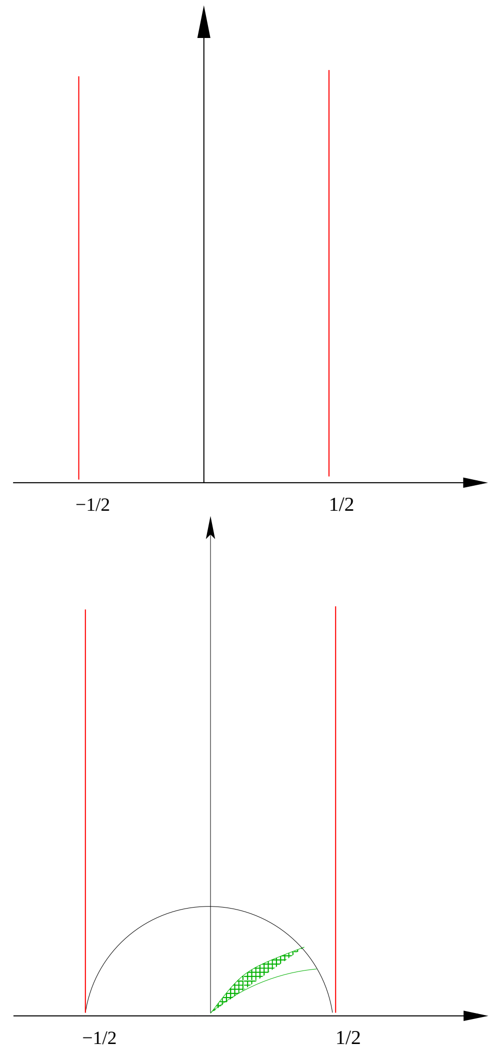


Temporal winding modes as solitons

The World Sheet has Torus Topology:

$$ds^2 = |d\sigma_1 + \tau d\sigma_2|^2 = d\sigma_1^2 + |\tau|^2 d\sigma_2^2 + 2(\mathrm{Re}\,\tau) d\sigma_1 d\sigma_2.$$

String Action [bosonic]:

$$I = \frac{1}{2\pi\alpha'} \int d\sigma_1 d\sigma_2 \sqrt{\gamma}\, g_{\mu\nu} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu.$$

EOM [assumption: $g_{\mu\nu}(r)$]:

$$\partial_\alpha(\sqrt{\gamma}\gamma^{\alpha\beta} g_{00} \partial_\beta X^0) = 0.$$

$$\partial_\alpha \left(\sqrt{\gamma}\gamma^{\alpha\beta} g_{rr} \partial_\beta r\right) - \frac{1}{2}\partial_r g_{00} [\sqrt{\gamma}\gamma^{\alpha\beta} \partial_\alpha X^0 \partial_\beta X^0\Big] = 0.$$

Looking for the winding Soliton:

$$X^0 = m\beta\sigma_1 + n\beta\sigma_2, \qquad r = r(\sigma_1, \sigma_2).$$

Complicated in General

$$\exists\, r_0 : \; g_{00}(r_0) \neq 0, \quad \partial_r g_{00}(r_0) = 0, \quad \partial_r g_{rr}(r_0) = 0$$

$$\begin{aligned} S_\beta(m,n) \;&=\; \frac{1}{2\pi\alpha'}\frac{1}{\tau_2}\int d\sigma_1 d\sigma_2 \Big[\beta^2(n^2 + m^2|\tau|^2 - 2(\mathrm{Re}\,\tau)\, m\, n) g_{00} \\ &+\; g_{rr}\,(|\tau|^2\,\dot{r}^2 + r'^2 - 2(\mathrm{Re})\,\dot{r}r')\Big]. \end{aligned}$$

- For confining backgrounds $r = r_0$ is a solution [String at the AdS Wall] $T_s = g_{00}(r_0)/2\pi\alpha'$

$$S_\beta(m,n) = T_s \frac{1}{\tau_2}\Big[\beta^2(n^2 + m^2|\tau|^2 - 2(\mathrm{Re}\,\tau)\, m\, n)\Big] = T_s \frac{\beta^2|m\tau - n|^2}{\tau_2}.$$

Sketch of Fluctuations I:

- r_0 is now $\tau = 0$.
- $e_3^2 + e_4^2 + e_5^2|_{\tau=0} = \frac{1}{2} d\Omega_3^2$ round S^3 with radius $1/\sqrt{2}$.
- $S^3(\theta, \phi, \psi)$ by fixin $\theta = \pi/2$ becomes $\mathbf{R}^3(y^1, y^2, y^3)$.
- $e^{2g}|_{\tau=0} \approx \tau^2 \longrightarrow \tau$-direction with $S^2(\theta_1, \phi_1)$ into $\mathbf{R}^3(\tau^1, \tau^2, \tau^3)$

$$S_{2b} = S[X^0_{classical}, r = r_0] + \frac{1}{2\pi\alpha'} \int d\sigma_1 d\sigma_2 \sqrt{\gamma}\gamma^{\alpha\beta} \left(\partial_\alpha X^a \partial_\beta X^a g_{00} + \alpha' g_s N g_{00}[\partial_\alpha \tau^i \partial_\beta \tau_i + \frac{1}{4}\partial_\alpha y^i \partial_\beta y_i] + \frac{4\beta^2}{9 Im\tau^2} g_{00} |m\tau - n|^2 \tau^i \tau_i \right) \quad (9)$$

where $a = 1, \ldots, 4$ and $i = 1, 2, 3$.

- The mass $(2/3)\beta\sqrt{\frac{1}{\alpha' g_s N}}|m\tau - n|/Im\tau$.

$$S_{2f} = \frac{i}{2\pi\alpha'} \int \bar{\theta}^I (\sqrt{\gamma}\gamma^{\alpha\beta}\delta^{IJ} - \epsilon^{\alpha\beta}\sigma_3^{IJ}) \partial_\alpha X^0 \Gamma_{\underline{0}} e^{\underline{0}}_0 (\delta^{JK}\partial_\beta + \frac{1}{8 \cdot 3!} e^\phi \sigma_1^{JK} \Gamma^{\mu_1\mu_2\mu_3} F_{\mu_1\mu_2\mu_3} \partial_\beta X^0 \Gamma_{\underline{0}} e^{\underline{0}}_0)\theta^K \quad (10)$$

$F_{(3)} = -\frac{1}{4} g_s N dy_1 \wedge dy_2 \wedge dy_3$
Choose the κ : $\Gamma^+\theta^I = 0$ or $\theta^1 = \theta^2$

Thermalized fermionic degrees of freedom in a given soliton sector, characterized by (m,n) winding numbers

$$
\begin{aligned}
\theta(\sigma_1+1,\sigma_2) &= (-1)^m\theta(\sigma_1,\sigma_1) \\
\theta(\sigma_1,\sigma_2+1) &= (-1)^n\theta(\sigma_1,\sigma_1).
\end{aligned}
\tag{11}
$$

$$
Z_{T^2} = \sum_{m,n\in\mathbf{Z}}{}' \frac{\beta}{2\pi l_s}\int_{\mathcal{F}} d^2\tau \frac{1}{Im\tau^2} e^{-\frac{\beta^2 g_{00}}{4\pi\alpha'}\frac{|m\tau-n|^2}{Im\tau}} z^b_{0,0}(\tau,0)^5 z^b_{0,0}(\tau, M^2 = \frac{4}{9}\beta^2\frac{|m\tau-n|^2}{Im\tau^2}\frac{1}{\alpha' g_s N})^3 z^f_{b_1,b_2}(\tau,0)^8
\tag{12}
$$

$$
z^b_{0,0}(\tau,M) = e^{-\pi Im\tau\sum_{l\in\mathbf{Z}}\sqrt{l^2+M^2}}\prod_{l\in\mathbf{Z}}\left(1-e^{-2\pi Im\tau\sqrt{l^2+M^2}+2\pi i Re\tau l}\right)^{-1}
\tag{13}
$$

$$
z^f_{b_1,b_2}(\tau,M) \;=\; e^{\pi Im\tau\sum_{l\in\mathbf{Z}}\sqrt{(l+b_1)^2+M^2}}\prod_{l\in\mathbf{Z}}(1-e^{-2\pi Im\tau\sqrt{(l+b_1)^2+M^2}+2\pi i Re\tau(l+b_1)-2\pi i b_2}),
\tag{14}
$$

denotes the contribution of a GS fermion, with mass M and in the soliton sector m,n, with twisted boundary conditions $b_1=(1-(-1)^m)/2$ and $b_2=(1-(-1)^n)/2$.

- Partition function:

$$Z_{T^2} \approx \int e^{-\frac{\beta^2 g_{00}}{4\pi\alpha'} Im\tau} e^{-\pi Im\tau \sum_{l\in\mathbf{Z}}(5l+3\sqrt{l^2+\frac{4}{9}\beta^2\frac{1}{\alpha' g_s N}}-8(l+\frac{1}{2}))}, \tag{15}$$

T_H:

$$\frac{1}{4\pi\alpha'}\beta_H^2 g_{00} = -2\pi\Big(5\gamma_0(0) + 3\gamma_0(2\beta_H\sqrt{\frac{1}{\alpha' g_s N}}/3) - 8\gamma_{1/2}(0)\Big). \tag{16}$$

$$d(E) \approx \exp\left(\sqrt{3\pi}\ \frac{E}{T_s^{1/2}}\right). \tag{17}$$

The Density of states depends on the gauge theory quark-antiquark string tension

Outlook:

- Exact Calculation of the Density of States in Hadronic String Theories.

- A proposal for how to compute the Hagedorn Density of States when the full string solution is not available.

- How about transitions: Confinement/Deconfinement?

- What other hadronic properties can one get a handle on?

Regge Trajectories Revisited in teh Gauge/Gravity Correspondence

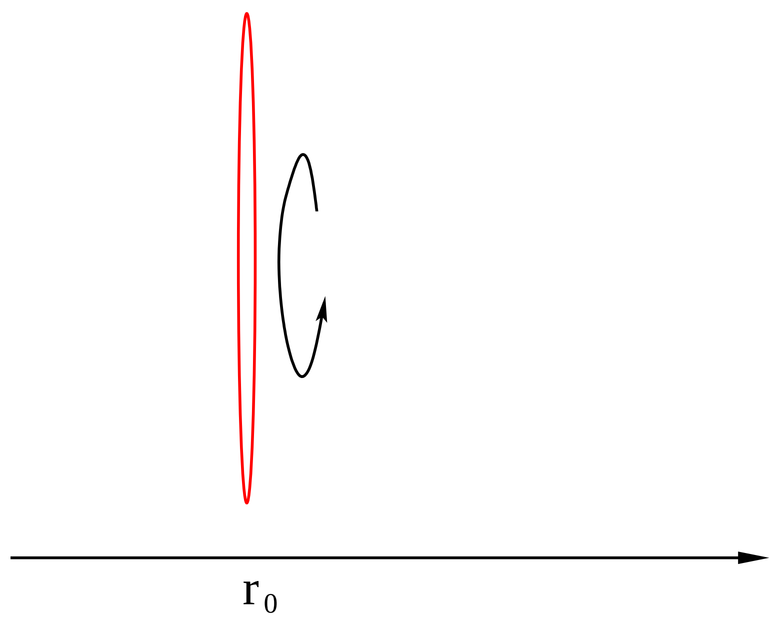
- A Regge trajectory: a line in the Chew-Frautschi plot: $J = \alpha_0 + \alpha' t$

- Well described by simple strings model but *now* we have the *right* string models.

Gauge Theory State	String Theory Configuration
Glueballs	Spinning Folded Closed String
Mesons of heavy quarks	Spinning open strings
Baryons of heavy quarks	Strings attached to a baryonic vertex
Dibaryons	Strings attached to wrapped branes

Table 1: States in gauge theory and their corresponding classical configuration in the string theory.

Closed spinning strings in supergravity backgrounds



$$ds^2 = h(r)^{-1/2}\Big[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\Big] + h(r)^{1/2}dr^2 + \dots \tag{18}$$

The relevant classical equations of motion are

$$\begin{aligned}
\partial_a(h^{-1/2}\eta^{ab}\partial_b t) &= 0, \\
\partial_a(h^{-1/2}\eta^{ab}\partial_b x^i) &= 0, \\
\partial_a(h^{1/2}\eta^{ab}\partial_b r) &= \frac{1}{2}\partial_r(h^{-1/2})\eta^{ab}[-\partial_a t\partial_b t + \partial_a x_i \partial_b x^i].
\end{aligned} \tag{19}$$

They are supplemented by the standard Virasoro constraints.

ansatz

$$\begin{aligned}
t &= e\,\tau, \\
x_1 &= f_1(\tau)\,g_1(\sigma), \qquad x_2 = f_2(\tau)\,g_2(\sigma), \\
x_3 &= \text{constant}, \qquad \mathrm{r} = \mathrm{r}(\sigma).
\end{aligned} \tag{20}$$

The above system can be greatly simplified by further taking

$$f_1 = \cos e\omega\,\tau, \quad f_2 = \sin e\omega\,\tau, \quad \text{and} \quad \mathrm{g}_1 = \mathrm{g}_2. \tag{21}$$

The energy and angular momentum

$$E = \frac{e}{2\pi\alpha'}\int h^{-1/2} d\sigma, \tag{22}$$

$$J = \frac{e\omega}{2\pi\alpha'}\int h^{-1/2} g^2 d\sigma. \tag{23}$$

- A spinning string in the Poincare coordinates is dual to a state of energy E and spin J.

Closed spinning strings in confining theories

- Conditions for confinement in gauge/gravity: g_{00} has a nonzero minimum at some point r_0.

$$\partial_r(g_{00})|_{r=r_0} = 0, \qquad g_{00}|_{r=r_0} \neq 0. \tag{24}$$

$$t = e\,\tau, \qquad x_1 = \frac{1}{\omega}\cos e\omega\,\tau\ \sin e\omega\,\sigma \qquad x_2 = \frac{1}{\omega}\sin e\omega\,\tau\ \sin e\omega\,\sigma \tag{25}$$

$$\begin{aligned} E &= \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega}\int_0^{\bar{g}_0}\frac{d\bar{g}}{\sqrt{1-\bar{g}^2}} = \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega}\arcsin\bar{g}_0 \\ J &= \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega^2}\int_0^{\bar{g}_0}\bar{g}^2\frac{d\bar{g}}{\sqrt{1-\bar{g}^2}} = \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega^2}\frac{1}{2}[\arcsin\bar{g}_0 - \bar{g}_0\sqrt{1-\bar{g}_0^2}], \end{aligned} \tag{26}$$

$\bar{g}_0 \longrightarrow 1$ [Ends spinning at the speed of light]

$$\begin{aligned} E &= \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega}\int_0^{1}\frac{d\bar{g}}{\sqrt{1-\bar{g}^2}} = \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega}\frac{\pi}{2} \\ J &= \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega^2}\int_0^{1}\bar{g}^2\frac{d\bar{g}}{\sqrt{1-\bar{g}^2}} = \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega^2}\frac{\pi}{4}. \end{aligned} \tag{27}$$

Typical Regge trajectories

$$E^2 = 4\,\pi T_s\ S. \tag{28}$$

Semiclassical quantization

- Quadratic fluctuations in flat space.

- New feature of confining backgrounds?

$$\gamma^{\tau\tau}g_{tt}\partial_\tau t\partial_\tau t+\gamma^{\alpha\beta}\partial_\alpha x^i\partial_\beta x^i g_{ii}=\left(\frac{8e^{\phi_0}}{9}\ \kappa^2\cos^2\omega\sigma\right)\ \tau^i\tau_i \tag{29}$$

$$[\partial_\tau^2-\partial_\sigma^2+m_0^2\cos^2(\omega\sigma)]\delta\tau_i=0. \tag{30}$$

where $i=1,2,3$ and $m_0^2=\frac{8e^{\phi_0}}{9}\ \kappa^2$ for the MN solution and $m_0^2=\frac{\varepsilon^{4/3}}{g_sM\alpha'}\frac{a_1}{2^{1/3}a_0^{3/2}}\ \kappa^2$ for the KS solution.

This equation can be put in the standard way of Mathieu differential equation

$$[\frac{\partial^2}{\partial\sigma^2}+\frac{1}{\omega^2}(n^2-\frac{m_0^2}{2})-\frac{m_0^2}{2\omega^2}\ \cos 2\sigma]\delta\tau_i(\sigma)=0. \tag{31}$$

$$\lambda_{r,n}=\frac{n^2}{\omega^2}+\frac{m_0^2}{2\omega^2}+\frac{r^2}{\omega^2}+\frac{1}{2(r^2-1)}\frac{m_0^4}{16\omega^4}+\mathcal{O}(m_0^8), \tag{32}$$

where r and n are integers

$$\Delta E=-\frac{1}{12}+\frac{1}{\sqrt{2}}m_0. \tag{33}$$