On two dimensional black holes and matrix models

Based on:

—*On Black Hole Thermodynamics of 2-D Type 0A*,
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Motivation:

- Black Holes are the Hydrogen Atoms of Quantum Gravity

- Black Hole Thermodynamics:

  Effective, macroscopic $\leftrightarrow$ Fundamental, microscopic

- 2-D Black Holes are the "simplest" gravity wise

- The dual theory is presumably manageable [Matrix Model]

Outline

- Why take vacation in Flatland?
- Black Holes in 2-d Type OA
- Thermodynamics of 2-d Black Holes
- Matrix Models "dual" of Black Holes
- Where is the Black Hole/Matrix Duality?

The Bosonic 2-d Black Hole [Witten]

Background

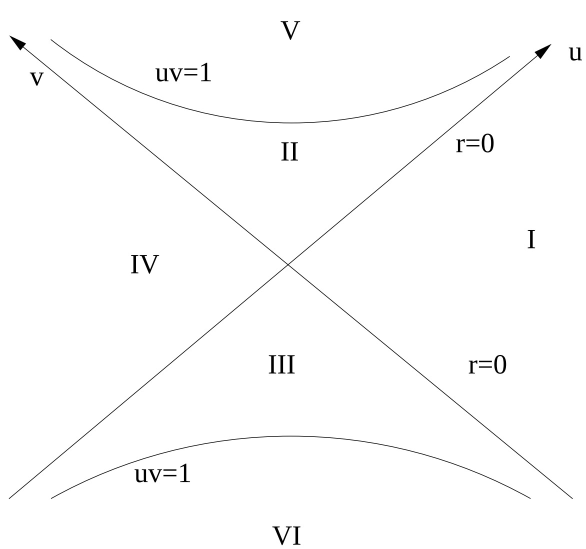
$$ds^2 = -\tanh^2 r dt^2 + dr^2, \qquad \Phi = 2 \ln \cosh r + \Phi_0. \tag{1}$$

$$R = 4/\cosh^2 r \tag{2}$$

- Regular

Null Coordinates

$$ds^2 = -\frac{dudv}{1-uv} \tag{3}$$



2-D Type 0A Low energy "Effective" Action

`Takayanagi, Toumbas` and
`Douglas,Klebanov,Kutasov,Maldacena,Martinec,Seiberg`

$$S = \int d^2x\sqrt{-g}\Big[\frac{e^{-2\Phi}}{2\kappa^2}\left(\frac{8}{\alpha'} + R + 4(\nabla\Phi)^2 - f_1(T)(\nabla T)^2 + f_2(T) + \dots\right) - \frac{2\pi\alpha'}{4}f_3(T)(F^{(+)})^2 - \frac{2\pi\alpha'}{4}f_3(-T)(F^{(-)})^2 + \dots\Big], \tag{4}$$

$f_i(T)$ – functions of the tachyon field T.

- Dualize RR field Strength, q electric and magnetic D0 branes:

$$S = \int d^2x\sqrt{-g}\Big[e^{-2\Phi}\left(c + R + 4(\nabla\Phi)^2 - (\nabla T)^2 + \frac{2}{\alpha'}T^2\right) + \Lambda(1+2T^2)\dots\Big], \tag{5}$$

$c = 8/\alpha'$, $\Lambda = -q^2/(2\pi\alpha')$ units $2\kappa^2 = 1$.

The Black Hole Solution

$$ds^2 = l(\phi)\, dt^2 + \frac{d\phi^2}{l(\phi)}, \tag{6}$$

$$l(\phi) = 1 - \frac{4}{c} e^{\sqrt{c}\phi} \left(\frac{1}{4} \Lambda \sqrt{c}\, \phi + m \right), \tag{7}$$

$$\Phi = \sqrt{c}\, \phi / 2 \tag{8}$$

$\Lambda < 0$ the Reissner-Nordstrom [Two different roots for $l(\phi)$]

$$R = -\partial_\phi^2 l(\phi), \tag{9}$$

- Singular at $\phi \rightarrow \infty$

This is Really a Brane-like Solution

- Flat space (with linear dilaton) $\phi \to -\infty$

$$ds^2 = -dt^2 + d\phi^2, \qquad \Phi = \sqrt{\frac{c}{4}}\,\phi. \tag{10}$$

- The near horizon region of the extremal black hole is AdS_2

$$ds^2 = -\phi^2\, dt^2 + \frac{d\phi^2}{\phi^2}, \qquad \Phi = \text{Constant} \tag{11}$$

- The extremal black hole interpolates between flat space and AdS_2

ADM Mass

General Case [Mann]

$$S[g,\Phi] = \int d^2x\sqrt{-g}\Big[D(\Phi)\,R + H(\Phi)g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + V(\Phi,\Phi_M)\Big], \quad (12)$$

$$S = \int d^2x\sqrt{-g}\Big[e^{-2\Phi}\left(R + 4(\nabla\Phi)^2 + c\right) + \Lambda\Big] \quad (13)$$

In our case

$$D(\Phi) = e^{-2\Phi},\ H(\Phi) = 4e^{-2\Phi},\ V = \Lambda + c\,e^{-2\Phi}. \quad (14)$$

a topologically conserved current, need no timelike Killing vector

$$S_\mu = T_{\mu\nu}\epsilon^{\nu\rho}\partial_\rho F \quad (15)$$

$T_{\mu\nu}$ the stress-energy tensor

$$F = F_0\int^{\Phi} ds\ D'\ \exp\left(-\int^{s} dt\,\frac{H(t)}{D'(t)}\right). \quad (16)$$

Mass

$\mathcal{M}$: $S^{\mu} = \epsilon^{\mu\nu}\partial_{\nu}\mathcal{M}$

$$\mathcal{M} = F_0\Big[\int^{\Phi} ds D'(s)\, V(s)\, \exp\left(-\int^{s} dt\, \frac{H(t)}{D'(t)}\right) - (\nabla D)^2 \exp\left(-\int^{\Phi} dt\, \frac{H(t)}{D'(t)}\right)\Big]. \tag{17}$$

$$\mathcal{M} = 4F_0 e^{-2\Phi}\Big[(\nabla\Phi)^2 - \frac{c}{4} + \frac{1}{2}\Lambda\ \Phi e^{2\Phi}\Big] \tag{18}$$

For the solution

$$\mathcal{M} = \frac{4}{\sqrt{c}}\, m. \tag{19}$$

Temperature

Absence of conical singularity

$$ds^2 = \frac{4}{l'^2}\left(dr^2 + \frac{1}{4}l'^2 r^2 dt^2\right). \tag{20}$$

$$T = \frac{1}{4\pi}|l'(\phi)|_{\phi=\phi_h}. \tag{21}$$

$$\phi_h = -\frac{4m}{\Lambda\sqrt{c}} + \frac{1}{\sqrt{c}}W\left(\frac{c}{\Lambda}\,e^{4m/\Lambda}\right). \tag{22}$$

Lambert function: $W(z)\exp(W(z)) = z$.

$$T = \frac{\sqrt{c}}{4\pi}|1 + \frac{\Lambda}{c}\exp\left(-4\frac{m}{\Lambda} + W(\frac{c}{\Lambda}e^{4m/\Lambda})\right)|. \tag{23}$$

Extremal and Near-extremal case

$$\phi_0 = -\frac{1}{\sqrt{c}} - 4\,\frac{m_0}{\Lambda\sqrt{c}} = \frac{1}{\sqrt{c}}\ln\left(-\frac{c}{\Lambda}\right), \qquad m_0 = -\frac{1}{4}\,\Lambda\Big[1 + \ln\left(-\frac{c}{\Lambda}\right)\Big]. \tag{24}$$

Positivity of ADM mass [Park–Strominger] $\longrightarrow$ Bound on the flux

$$q^2 < 16\pi e. \tag{25}$$

$m = m_0 + \delta m$ with $|\delta m/m_0| \ll 1$

$$\phi_h = \frac{1}{\sqrt{c}}\ln\left(-\frac{c}{\Lambda}\right) + \frac{2\sqrt{2}}{\sqrt{c}}\left(-\frac{\delta m}{\Lambda}\right)^{1/2} - \frac{4}{3\sqrt{c}}\frac{\delta m}{\Lambda} + \mathcal{O}\left(\left(-\frac{\delta m}{\Lambda}\right)^{3/2}\right). \tag{26}$$

- The horizon is pushed outward by adding a small amount of matter

$$T = \frac{\sqrt{c}}{\sqrt{2}\pi}\left(-\frac{\delta m}{\Lambda}\right)^{1/2}. \tag{27}$$

Dilaton Charge

Conserved without equations of motion: topological

$$\nabla_\mu j^\mu = -\epsilon^{\mu\nu}\left[f''(\Phi)\nabla_\mu\Phi\nabla_\nu\Phi + f'(\Phi)\nabla_\mu\nabla_\nu\Phi\right]. \tag{28}$$

$$\begin{aligned} D &= \int_\Sigma d\Sigma n^\mu j_\mu \\ &= -\int_{\phi_W}^{\phi_0} d\phi\,\sqrt{g_{\phi\phi}} n^t g_{tt}\epsilon^{t\nu}\nabla_\nu f(\Phi) \\ &= -\int_{\phi_W}^{\phi_0} d\phi\, f'(\Phi). \end{aligned} \tag{29}$$

Canonical choice for $f(\Phi)$: $D = e^{-2\Phi}$

- D is the the function multiplying the Ricci scalar in the action.

Thermodynamical action

- Everything in term of observables at a Wall: T_W and D_W

$$I_{onshell} = \int_{\mathcal{M}} \sqrt{g}\Lambda + 2\int_{\partial\mathcal{M}} \sqrt{h}e^{-2\Phi}\left(K - 2n^a\nabla_a\Phi\right). \tag{30}$$

$$F = T_W I, \qquad S = -\frac{\partial F}{\partial T_W}, \qquad E = F + T_W S, \qquad \psi = -\frac{\partial F}{\partial D_W}. \tag{31}$$

$$\begin{aligned} I &= \beta\,\Lambda(\phi_W - \phi_h) + \beta e^{-\sqrt{c}\,\phi_W}\left(-2l(\phi_W)\sqrt{c} + l'(\phi_W)\right) \\ &= \beta\,\Lambda(\phi_W - \phi_h) - \beta\sqrt{c}\,e^{-\sqrt{c}\,\phi_W}\left(l(\phi_W) + 1 + \frac{\Lambda}{c}\,e^{\sqrt{c}\,\phi_W}\right), \end{aligned} \tag{32}$$

Tolmann Relation

$$T_W = T\frac{1}{\sqrt{l(\phi_W)}}, \tag{33}$$

$$T = \frac{\sqrt{c}}{4\pi}\left|1 + \frac{\Lambda}{c}D_h^{-1}\right|, \tag{34}$$

m as a function of the position of the horizon $m(\phi_h)$

$$m(\phi_h) = \frac{c}{4}\,D_h + \frac{\Lambda}{4}\,\ln D_h, \tag{35}$$

The On-Shell Action

$$I_W = \frac{1}{T}\Lambda(\phi_W - \phi_h) - \frac{D_W}{T}\left(1 + \frac{T^2}{T_W^2} + \frac{\Lambda}{c\,D_W}\right). \tag{36}$$

$$T_W = T\frac{1}{\sqrt{l(\phi_W)}}, \tag{37}$$

where

$$T = \frac{\sqrt{c}}{4\pi}\Big|1 + \frac{\Lambda}{c}D_h^{-1}\Big|, \tag{38}$$

The free energy:

$$F = -\frac{\Lambda}{\sqrt{c}}\,\frac{T_W}{T}\ln\frac{D_W}{D_h} - \sqrt{c}\,D_W\frac{T_W}{T}\left(1 + \frac{T^2}{T_W^2} + \frac{\Lambda}{c\,D_W}\right), \tag{39}$$

T and D_h should be understood as functions of (T_W, D_W)

The zero RR flux limit[Sanity Check]

$$D_h = D_W \left(1 - \frac{c}{16\pi^2 T_W^2}\right). \tag{40}$$

Allows us to identify $m(\phi_h)$ as:

$$m(D_W, T_W) = \frac{c\, D_W}{4} \left(1 - \frac{c}{16\pi^2 T_W^2}\right). \tag{41}$$

Thermodynamic properties: free energy, entropy and energy of the zero RR flux 2-d black hole [`Gibbons, Perry`]:

$$\begin{aligned} F &= -4\pi\, D_W \left(T_W + \frac{c}{16\pi^2 T_W}\right), \\ S &= 4\pi\, D_W \left(1 - \frac{c}{16\pi^2 T_W^2}\right), \\ E &= -8\pi\, D_W \frac{c}{16\pi^2 T_W}. \end{aligned} \tag{42}$$

The Extremal Black Hole

$$\phi_h = \frac{1}{\sqrt{c}} \ln\left(-\frac{c}{\Lambda}\right) + \delta\phi_h. \tag{43}$$

$$\delta\phi_h = -4\pi \frac{T_W}{c}\left(1 + \frac{\Lambda}{c\, D_W}\left(1 + \ln(-\frac{cD_W}{\Lambda})\right)\right)^{1/2}. \tag{44}$$

Evaluate the free energy

$$F = \frac{T_W}{T}\left(-\frac{\Lambda}{\sqrt{c}}\ln(-\frac{c\, D_W}{\Lambda}) - \sqrt{c}\, D_W - \frac{\Lambda}{\sqrt{c}} - \Lambda\delta\phi_h\right) - \sqrt{c}\, D_W \frac{T}{T_W}, \tag{45}$$

Independent of the temperature → Robust entropy

$$S = -\frac{4\pi\,\Lambda}{c} = \frac{1}{4} q^2. \tag{46}$$

The entropy of the extremal black hole with q units of electric and magnetic RR fluxes.

Energy and Mass

$$E = -2\sqrt{c}\, D_W - \frac{\Lambda}{\sqrt{c}} \ln D_W + \frac{\Lambda}{\sqrt{c}} \left(\ln(-\frac{\Lambda}{c}) - 1 \right) . \qquad (47)$$

Background Subtraction

$$M = \frac{\Lambda}{\sqrt{c}} \left(\ln(-\frac{\Lambda}{c}) - 1 \right) = -\frac{q^2}{4\pi\sqrt{2\alpha'}} \left(\ln \frac{q^2}{16\pi} - 1 \right) . \qquad (48)$$

- Coincides with ADM mass.
- Coincides with Matrix Model result

Arbitrary Non-Extremal 0A Black Hole

The general case:

$$F = -\frac{\Lambda}{\sqrt{c}}\frac{T_W}{T}\ln\frac{D_W}{D_h} - \sqrt{c}\,D_W\frac{T_W}{T}\left(1 + \frac{T^2}{T_W^2} + \frac{\Lambda}{c\,D_W}\right), \qquad (49)$$

$D_W \to \infty, T_W \to T$, while keeping the temperature of the black hole T (and thus D_h)

Free Energy

$$F = -2\sqrt{c}D_W - \frac{\alpha\Lambda}{\sqrt{c}}\ln(\frac{D_W}{D_h}) - \frac{\Lambda}{\sqrt{c}} + \mathcal{O}(D_W^{-1}). \qquad (50)$$

$$\begin{aligned} E &= \frac{\sqrt{D_W(c(D_W - D_h) + \Lambda\ln(D_W/D_h))}}{2\Lambda c D_W - 4c\Lambda D_h + 2\Lambda^2\ln(D_W/D_h) - c^2D_h^2 - \Lambda^2}\Big(- 2\Lambda c D_W + 3\Lambda c D_h \\ &+ \Lambda^2(1+\alpha)\ln(D_h/D_W) - \alpha\Lambda c D_h - \alpha\Lambda^2\Big). \end{aligned} \qquad (51)$$

The limits

$D_W \to \infty$

$$E = -2\sqrt{c}D_W + \frac{\alpha\Lambda}{\sqrt{c}}\ln(\frac{D_W}{D_h}) - \alpha\sqrt{c}D_h - (1+\alpha)\frac{\Lambda}{\sqrt{c}} + \mathcal{O}(D_W^{-1}) \quad (52)$$

$$M = \sqrt{c}D_h + \frac{\Lambda}{\sqrt{c}}\ln D_h = \frac{4}{\sqrt{c}}m \quad (53)$$

(T_W, D_W)

$$S = 4\pi D_h \quad (54)$$

Thermodynamic Properties of 2-d Black Holes in Type 0A

	M	T	S	F	ψ
$\Lambda = 0$	$\sqrt{c}D_h$	$\frac{\sqrt{c}}{4\pi}$	$4\pi D_h$	0	$2\sqrt{c}$
$\Lambda \neq 0$	$\sqrt{c}D_h + \frac{\Lambda}{\sqrt{c}}\ln D_h$	$\frac{1}{4\pi\sqrt{c}D_h}\lvert cD_h + \Lambda\rvert$	$4\pi D_h$	$\frac{\Lambda}{\sqrt{c}}(\ln D_h - 1)$	$2\sqrt{c}$

Matrix Models

- The matrix model for type 0A [`DKKMMS`]. The Jevicki-Yoneya model.

$$V(\lambda) = -\lambda^2 + \frac{q^2 - 1/4}{\lambda^2}. \tag{55}$$

  - The excitement

$$E = \int^{\mu} \epsilon\rho(\epsilon)d\epsilon \sim -\frac{1}{8\pi}q^2 \ln q^2 + \dots \tag{56}$$

  - Other quantities

$$\mathcal{F} = -\frac{1}{8\pi}q^2 \log\frac{q^2}{L^4} + \frac{1}{48\pi}\left[1 + (2\pi T)^2\right] \log\frac{q^2}{L^4} \dots \tag{57}$$

T – temperature, L – IR cut-off.

- The first non-vanishing contribution to the entropy is one-loop

The Disagreement

$$S = -\frac{\pi}{12}T \log\frac{q^2}{L^4}. \tag{58}$$

Vortex Condensation

- Another Candidate [Kazakov, Kostov, Kutasov]
  - Summing over all possible $SU(N)$ twists around the Euclidean time circle.

$$Z_N(\beta,\lambda) = \sum_r \int [D\Omega]\chi_r(\Omega^\dagger) \exp\Big(\sum_{n\in\mathbf{Z}} \lambda_n tr(\Omega^n)\Big) Tr_r e^{-\beta H_r} \tag{59}$$

$\chi_r(\Omega^\dagger)$ – Weyl character

H_r is the Hamiltonian in the representation r

$$H_r = P_r \sum_{k=1}^{N} \Big(-\frac{1}{2}\partial^2_{x_k} - \frac{1}{2}x_k^2\Big) + \frac{1}{2}\sum_{i\neq j} \frac{\tau^r_{ij}\tau^r_{ij}}{(x_i - x_j)^2} \tag{60}$$

x_i – eigenvalues of the matrix with the inverted harmonic oscillator potential
τ^r_{ij} – $SU(N)$ generators

- Free Energy

$$F = \frac{1}{2\pi R}\left(\frac{1}{4}(2-R)^2\lambda^{4/(2-R)} - \frac{R+R^{-1}}{48}\ln(\lambda^{4/(2-R)}) + \sum_{h=2}^{\infty} f_h(R)\lambda^{4(1-h)/(2-R)}\right) \tag{61}$$

R – radius of the compactified time circle, temperature $2\pi R = 1/T$

- The genus zero contribution to the free energy is of the order

$$l^{\frac{2}{(R-2)}} \sim M \sim \frac{1}{g_s^2} \tag{62}$$

- Nonsinglet sector gives a large entropy:

$$S = \beta_{Hagedorn} M + \dots \tag{63}$$

- $M \sim \frac{1}{g_s^2}$

- Our calculation $S = 4\pi D_h$ with $D_h = 1/g_s^2$

- Since the string coupling is related to the

Order of magnitude agreement

AdS_2 in 2-d Type 0A

- AdS_2 Exist not only as a limit but also as a bona fide solution

$$ds^2 = \frac{1}{\phi^2}\left(-dt^2 + d\phi^2\right) \tag{64}$$

$$\Phi = \Phi_0 \tag{65}$$

$$R = -8/\alpha' \tag{66}$$

Apologies for “effective” action

A Matrix Model for AdS_2 `Strominger`

- Type 0A potential

$$V(\lambda) = -\lambda^2 + \frac{q^2 - 1/4}{\lambda^2}. \tag{67}$$

- Flat Space
- AdS_2

Conjecture: Decoupling

The AdS_2 solution is dual to the $1/\lambda^2$ potential

The gravity side

- The GR calculation gives a vanishing result

  - A candidate for black hole in global AdS_2

$$ds^2 = -l^2(1 - br - r^2)dt^2 + l^2(1 - br + r^2)^{-1}dr^2 \qquad (68)$$

  - Related to some identification of AdS_2 for some values of b [BTZ]

Outlook:

- The statistical entropy of 2-d black holes from matrix model.
- Implications for 4-d black holes
- How about the dynamics? Phase transitions?[Gross-Witten]