On singular effective superpotentials in SUSY gauge theories

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Outline of the talk

- Motivation for studying supersymmetric gauge theories
- Structure of four dimensional $\mathcal{N} = 1$ SUSY gauge theories
- Singular effective superpotentials of $\mathcal{N} = 1$ SU(2) SUSY gauge theories
- Singular effective superpotentials of $\mathcal{N} = 2$ theories in three dimensions
- Singular effective superpotentials of $\mathcal{N} = 1$ SU(N_c) gauge theories
- Conclusion

Motivation for studying supersymmetric gauge theories

- Holomorphicity of the superpotential and gauge couplings, global symmetries and the weak-coupling limit enable one to obtain exact results in supersymmetric gauge theories.

- These theories exhibit a wealth of generic non-perturbative phenomena such as:
  - dynamically generated superpotential
  - chiral symmetry breaking
  - confinement
  - deformed classical moduli space
  - Seiberg duality, etc.

- Since some of these phenomena also arise in non-supersymmetric contexts, supersymmetric gauge theories are usually considered as a window to qualitatively study some non-perturbative and insuperably difficult aspects of ordinary gauge theories in general.

- Therefore having a clear picture of the behavior of supersymetric gauge theories may shed light on a better understanding of the dynamics of strongly-coupled gauge theories with no supersymmetry.

- Four dimensional $\mathcal{N} = 1$ supersymmetric gauge theories, compared to gauge theories with higher supersymmetry, are the closest ones to the real world physics.

Structure of four dimensional $\mathcal{N} = 1$ supersymmetric gauge theories

- The basic field ingredients in the construction of supersymmetric gauge theories are chiral superfields Φ^i, anti-chiral superfields $\overline{\Phi}_i$ and vector superfields V^a.

- The most general gauge-invariant action for the Φ^i, $\overline{\Phi}_i$ and V^a takes the form

$$\begin{aligned} S &= \int d^4x \; d^4\theta \;\; K(\overline{\Phi}, e^V \Phi) \\ &+ \int d^4x \; d^2\theta \left(\frac{\tau}{32\pi i}\right) \mathrm{tr}(\mathcal{W}^2) + h.c. \\ &+ \int d^4x \; d^2\theta \;\; W(\Phi) + h.c., \end{aligned}$$

  where the first term is a kinetic term (non-linear sigma model) for Φ^i and $\overline{\Phi}_i$, the second term is the kinetic term for the gauge fields and the last term is the superpotential.

- $W(\Phi)$ is a holomorphic gauge-invariant function of the chiral fields. It determines many of the coupling constants, interactions and the scalar potential $V(\phi, \overline{\phi})$ in the theory.

- Our problem is to find the low energy behavior of these theories. In the low energy effective theory, the interactions of the light particles are characterized by a low energy effective superpotential W_{eff}.

- The key observation is that the effective superpotential can often be determined exactly by imposing the following constraints (N. Seiberg, hep-th/9309335):

  - symmetry

  - holomorphicity

  - smoothness

- Despite much progress in the effective dynamics of these theories, W_{eff}'s are less understood for a large number of flavors N_f. This is because:

  - For large N_f, there are additional light degrees of freedom at the origin of the moduli space that one needs to include as relevant degrees of freedom.

  - the effective superpotentials are singular when expressed in terms of the local gauge-invariant light degrees of freedom.

  - The dependence of W_{eff}'s on the strong coupling scale of the theory Λ is such that they don't vanish as $\Lambda \to 0$.

- These problems have led some authors to conclude that large N_f effective superpotentials are ill-defined.

The purpose of this talk is to show that $\mathrm{W}_{\mathrm{eff}}$'s should exist and ,despite being singular, are perfectly sensible.

- The basic strategy for finding W_{eff}'s (in SUSY QCD) has been a loose kind of induction in the number of light flavors in which one works one's way up to larger numbers of light flavors by making consistent guesses.

- It is natural to ask whether this procedure can be made more deductive and uniform by turning it on its head, and starting instead with the IR free theories with many massless flavors.

- When there are enough massless flavors so that the theory is IR free, we know what the light degrees of freedom are near the origin (since we have a weakly coupled lagrangian description there).

- One can furthermore argue that a complete set of local gauge-invariant chiral degrees of freedom in these theories are just the usual meson, baryon, and glueball fields. (F. Cachazo, *et al* hep-th/0211170, N. Seiberg hep-th/0212225, E. Witten hep-th/0302194)

- So, when the theory is IR free, W_{eff} should exists as a function of local gauge-invariant chiral fields.

- Once we determine W_{eff} for large numbers of flavors, we can then integrate out flavors to get W_{eff} for fewer flavors.

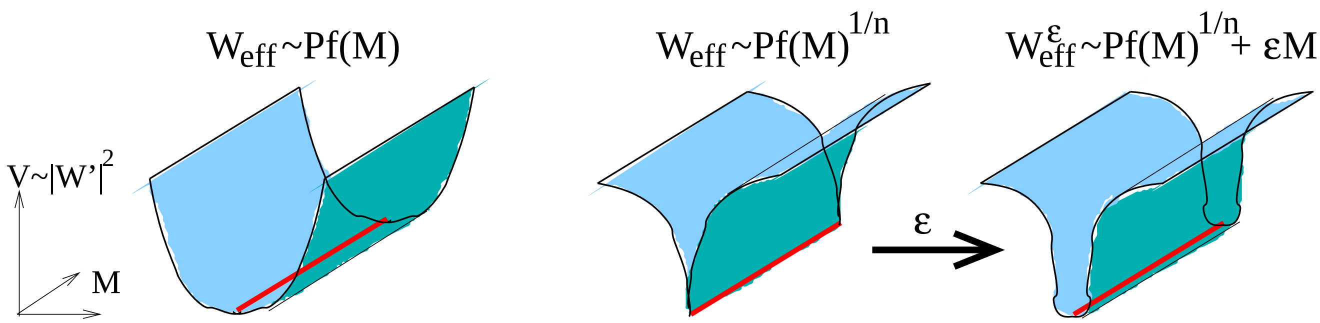
- Therefore effective superpotentials exist for all numbers of flavors in these theories.

- We confirm our observation by doing some consistency checks on W_{eff}.

- For large enough N_f, W_{eff}'s are singular.

- A naive analysis may lead to a wrong conclusion that W_{eff}'s cannot correctly describe the moduli space of vacua and therefore, they are not valid effective superpotentials.

- W_{eff}s' cusp-like singularities can be regularized. We then show that no matter how the regularizing parameters are sent to zero, these superpotentials always give the correct constraint equation(s) describing the moduli space. The basic point is illustrated in figure below.



(a): n=1 (b): n>1

Singular W_{eff}'s of $\mathcal{N}=1$ SU(2) SUSY gauge theories

A) $\mathcal{N}=1$ SU(2) **SUSY gauge theories**

B) Deriving the constraint equation

C) Consistency under RG flow

D) Higer-derivative F-terms

- **A)** $\mathcal{N}=1$ SU(2) **SUSY gauge theories:** Consider an $\mathcal{N}=1$ SU(2) supersymmetric gauge theory with $2N_f$ massless quark chiral fields Q_a^i transforming in the fundamental representation, where $i=1,\ldots,2N_f$ and $a=1,2$ are flavor and color indices, respectively.

- The anomaly-free global symmetry of the theory is SU($2N_f$)×U($1)_R$ under which the quarks transform as $(\mathbf{2N_f}, (N_f - 2)/N_f)$

- The classical moduli space of vacua is conveniently parametrized in terms of

$$M^{[ij]} := Q^i_a \epsilon^{ab} Q^j_b,$$

where ϵ^{ab} is the invariant antisymmetric tensor of SU(2).

- The effective dynamics of the theory varies drastically depending on N_f.

- For $N_f = 1$, the classical moduli space is the space of arbitrary vevs M^{ij}.

- For $N_f \geq 2$, it is all M^{ij} satisfying the constraint

$$\epsilon_{i_1 \cdots i_{2N_f}} M^{i_1 i_2} M^{i_3 i_4} = 0, \tag{1}$$

or, equivalently, $\text{rank}(M) \leq 2$.

- Quantum mechanically, for $N_f = 1$, there is a dynamically generated superpotential (I. Affleck, M. Dine and N. Seiberg, *Nucl. Phys.* **B 241** (1984) 493)

$$W_{\text{eff}} = \frac{\Lambda^5}{\text{Pf}M},$$

where Λ is the strong-coupling scale of the theory and

$$\begin{aligned} \text{Pf}M &:= \epsilon_{i_1 \cdots i_{2N_f}} M^{i_1 i_2} \ldots M^{i_{2N_f - 1} i_{2N_f}} \\ &= \sqrt{\text{det}M}. \end{aligned}$$

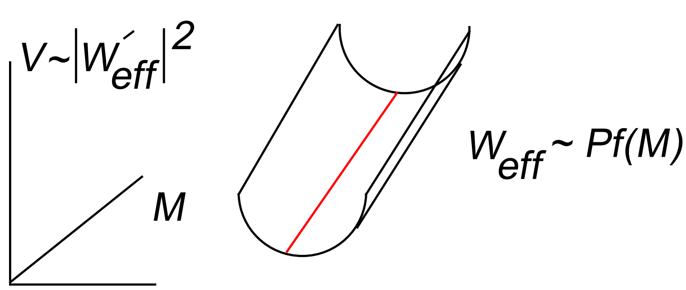
- For $N_f = 2$ the effective superpotential can be written (N. Seiberg, hep-th/9402044)

$$W_{\text{eff}} = \Sigma \left(\text{Pf}M - \Lambda^4 \right),$$

where Σ is a Lagrange multiplier enforcing a quantum-deformed constraint $\text{Pf}M = \Lambda^4$.

- For $N_f = 3$ the effective superpotential is (N. Seiberg, hep-th/9402044)

$$W_{\text{eff}} = -\frac{\text{Pf}M}{\Lambda^3},$$



whose equations of motion reproduce the classical constraint.

- For $N_f > 3$, the classical constraints are not modified (N. Seiberg, hep-th/9402044).

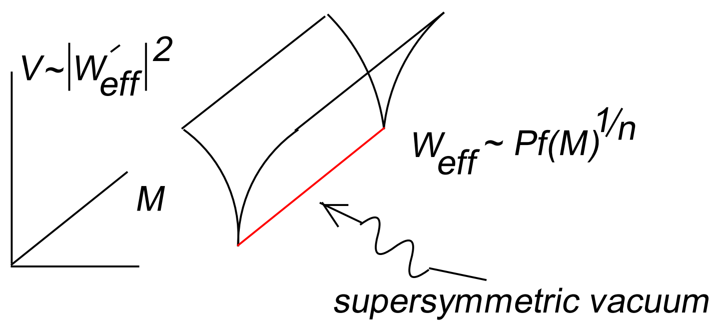
- But there are new light degrees of freedom at the singularity (the origin) when the theory is asymptotically free, $N_f < 6$ (N. Seiberg, hep-th/9411149).

- The only effective superpotential consistent with holomorphicity, weak-coupling limits, and the global symmetries is

$$W_{\text{eff}} = -n \left(\frac{\text{Pf}M}{\Lambda^{b_0}} \right)^{1/n}, \tag{2}$$

where $n := N_f - 2 > 1$, and $b_0 = 6 - N_f$ is the coefficient of the one-loop β-function.

- The fractional power of PfM implies that the potential corresponding to this superpotential has a cusp-like singularity at its extrema.



- But we will show that its cusp-like behavior still unambiguously describes the supersymmetric minima of the theory.

- The first issue is how the classical constraint follows from extremizing the singular W_{eff}.

- **B) Deriving the constraint equation:** We regularize W_{eff} by adding a mass term with an invertible antisymmetric mass matrix ε_{ij} for the meson fields:

$$W_{\text{eff}}^{\varepsilon} := W_{\text{eff}} + \frac{1}{2}\varepsilon_{ij}M^{ij}.$$

Varying $W_{\text{eff}}^{\varepsilon}$ with respect to M^{kl} yields the equation of motion

$$M^{kl} = -\Lambda^{-b_0/n}(\text{Pf}M)^{1/n}(\varepsilon^{-1})^{kl}.$$

Solving for PfM in terms of ε and substituting back gives

$$M^{kl} = -\Lambda^{b_0/2}(\text{Pf}\varepsilon)^{1/2}(\varepsilon^{-1})^{kl},$$

which in turn implies

$$\epsilon_{i_1 \ldots i_{2N_f}} M^{i_1 i_2} M^{i_3 i_4} = \frac{1}{\Lambda^{b_0}} \epsilon_{i_1 \ldots i_{2N_f}} \times (\varepsilon^{-1})^{i_1 i_2} (\varepsilon^{-1})^{i_3 i_4}\ \text{Pf}\varepsilon.$$

- The right hand side of the above expression is a polynomial of order $n > 0$ in the ε_{ij}.

- Therefore, no matter how we send $\varepsilon_{ij} \to 0$, the right hand side will vanish, giving back the classical constraint

$$\epsilon_{i_1 \cdots i_{2N_f}} M^{i_1 i_2} M^{i_3 i_4} = 0.$$

- Besides correctly describing the moduli space, the effective superpotentials should also pass some other tests.

- **C) Consistency under RG flow:** If we add a mass term for one flavor in the superpotential of a theory with N_f flavors and then integrate it out, we should recover the superpotential of the theory with $N_f - 1$ flavors.

- We will now show that our singular W_{eff} will pass this test as well.

- We add a gauge-invariant mass term for one flavor, say $M^{2N_f-1\ 2N_f}$:

$$W_{\text{eff}} = -n\left(\frac{\text{Pf}M}{\Lambda^{b_0}}\right)^{1/n} + mM^{2N_f-1\ 2N_f}.$$

  The equations of motion for $M^{i\ 2N_f-1}$ and $M^{j\ 2N_f}$ ($i \neq 2N_f - 1$ and $j \neq 2N_f$) put the meson matrix into the form

$$M^{ij} = \begin{pmatrix} \widehat{M} & 0 \\ 0 & \widehat{X} \end{pmatrix}$$

  where $\widehat{M}$ is a $2(N_f - 1) \times 2(N_f - 1)$ and $\widehat{X}$ a 2×2 matrix.

- Integrating out $\widehat{X} \sim M^{2N_f-1\ 2N_f} \otimes \sigma_2$ by its equation of motion gives

$$W_{\text{eff}} \ = \ -(n-1)\left(\frac{\text{Pf}\widehat{M}}{\widehat{\Lambda}^{b_0}}\right)^{1/(n-1)}$$

where $\widehat{\Lambda} = m\Lambda^{6-N_f}$ is the strong-coupling scale of the theory with $N_f - 1$ flavors, consistent with matching the RG flow of couplings at the scale m.

- Dropping the hats, we recognize W_{eff} as the effective superpotential of SU(2) SQCD with $N_f - 1$ flavors.

- **D) Higer-derivative F-terms:** We now show that our effective superpotential passes a different, more stringent, test.

- In a paper by C. Beasley and E. Witten (hep-th/0409149) a series of higher-derivative F-terms were calculated by integrating out massive modes at tree level from the non-singular effective superpotentials

$$\begin{aligned} W_{\text{eff}} &= \Sigma\left(\text{Pf}M - \Lambda^4\right) \\ W_{\text{eff}} &= -\frac{\text{Pf}M}{\Lambda^3}. \end{aligned}$$

- Here we show that our singular superpotential for $N_f > 3$ reproduces these F-terms by a tree-level calculation.

- As in our discussion of the classical constraint, the key point in this calculation is to first regularize W_{eff}, and then show that the results are independent of the regularization.

- The higher derivative terms calculated by C. Beasly and E.Witten for SU(2) SQCD with $N_f > 2$, are

$$
\begin{aligned}
\delta S \;=\; & \int d^4x d^2\theta \, \Lambda^{6-N_f} \epsilon^{i_1 j_1 \cdots i_{N_f} j_{N_f}} (M\overline{M})^{-N_f} \\
& \overline{M}_{i_1 j_1} (M^{k_2 \ell_2} \overline{DM}_{i_2 k_2} \cdot \overline{DM}_{j_2 \ell_2}) \times \\
& \cdots (M^{k_{N_f} \ell_{N_f}} \overline{DM}_{i_{N_f} k_{N_f}} \cdot \overline{DM}_{j_{N_f} \ell_{N_f}}),
\end{aligned}
$$

where $(M\overline{M}) := (1/2) \sum_{ij} M^{ij} \overline{M}_{ij}$, and the dot denotes contraction of the spinor indices on the covariant derivatives $\overline{D}_{\dot{\alpha}}$.

- We will now show how δS emerges from the singular effective superpotential W_{eff}.

- To derive on-vacuum effective interactions from an off-vacuum term, we simply have to expand around a given point on the moduli space and integrate out the massive modes at tree level.

- The only technical complication is that, as discussed in this talk, W_{eff} needs to be regularized first, *e.g.* by ε_{ij}, so that it is smooth at its extrema. At the end, we take $\varepsilon_{ij} \to 0$.

- The absence of divergences as $\varepsilon \to 0$ is another check of the consistency of our singular effective superpotential.

- Without loss of generality, we expand M^{ij} around

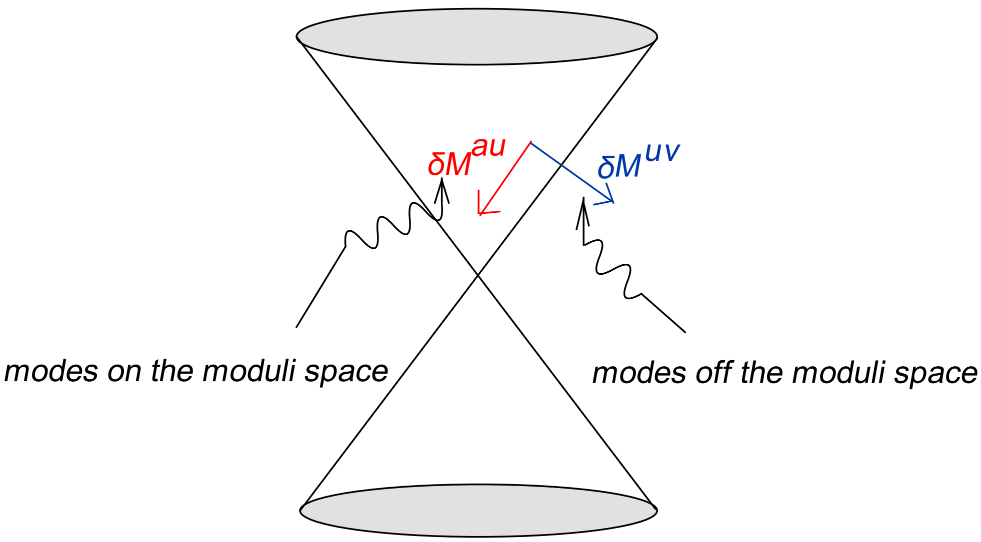
$$M_0{}^{ij} = \begin{pmatrix} \mu & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \otimes i\sigma_2,$$

with μ a non-vanishing constant, by making an appropriate SU($2N_f$) global flavor rotation.

- Note that $M_0{}^{ij}$ breaks the SU($2N_f$) global symmetry to SU(2) $\times$ SU($2N_f - 2$).

- Accordingly we henceforth partition the i, j flavor indices into those transforming under the unbroken SU(2) factor from the front of the alphabet—a, b=$1, 2$—and the remaining SU($2N_f - 2$) indices from the back: $u, v, \ldots = 3, \ldots, 2N_f$.

- Writing $M^{ij} = {M_0}^{ij} + \delta M^{ij}$, implies that the massless modes are δM^{12} and δM^{au}, while the δM^{uv} are all massive. See the figure.



- Expanding δS around ${M_0}^{ij}$ and keeping only the massless modes, we generate an infinite number of terms. The leading term is of order $(\delta\overline{M})^{2N_f-2}$,

$$
\begin{aligned}
\delta S_{l.t.} \;\sim\; & \int d^4x d^2\theta\, \Lambda^{6-N_f}\overline{\mu}^{1-N_f}\mu^{-1} \\
& \epsilon^{u_1 v_1 \cdots u_{N_f-1} v_{N_f-1}} (\overline{D}\delta\overline{M}_{1u_1} \cdot \overline{D}\delta\overline{M}_{2v_1}) \\
& \cdots (\overline{D}\delta\overline{M}_{1u_{N_f-1}} \cdot \overline{D}\delta\overline{M}_{2v_{N_f-1}}).
\end{aligned}
$$

- It suffices to show that this leading term is generated in perturbation theory since δS is the unique non-linear completion of $\delta S_{l.t.}$ (hep-th/0409149).

- In order to demonstrate how $\delta S_{l.t.}$ is generated at tree level from our singular W_{eff}, we first regularize $W_{\text{eff}} \to W^{\varepsilon}_{\text{eff}}$,

$$W^{\varepsilon}_{\text{eff}} := -n\lambda(\text{Pf}M)^{1/n} + \varepsilon_{ij}M^{ij},$$

where we have defined

$$n := N_f - 2 \quad , \quad \lambda := \Lambda^{(N_f-6)/(N_f-2)}.$$

- Without loss of generality, we choose a point on the moduli space of the deformed theory

$$(M^{\varepsilon}_0)^{ij} = \begin{pmatrix} \mu & & & \\ & \varepsilon & & \\ & & \ddots & \\ & & & \varepsilon \end{pmatrix}, \otimes i\sigma_2.$$

and expand M^{ij} around this point.

- Expanding $W^{\varepsilon}_{\mathsf{eff}}$ around this point, we have

$$\begin{aligned} W^{\varepsilon}_{\mathsf{eff}}(M^{\varepsilon}_{0}) &= W^{\varepsilon}_{\mathsf{eff}}(M^{\varepsilon}_{0}) + \lambda t^{ijk\ell}_{i'j'k'\ell'}(\mathsf{Pf}M^{\varepsilon}_{0})^{1/n} \\ & \quad (M^{\varepsilon}_{0})^{-1}_{ij}(M^{\varepsilon}_{0})^{-1}_{k\ell}\delta M^{i'j'}\delta M^{k'\ell'} \\ & \quad + \cdots , \end{aligned}$$

where the numerical tensor $t^{ijk\ell}_{i'j'k'\ell'}$ controls how the $ij\ldots$ indices are contracted with the $i'j'\ldots$ indices.

- We use standard superspace Feynman rules to compute the effective action for the massless δM^{ua} modes by integrating out the massive δM^{uv} modes.

- This means we need to evaluate connected tree diagrams at zero momentum with internal massive propagators and external massless legs.

- The massive modes have standard chiral, anti-chiral, and mixed superspace propagators with masses derived from the quadratic terms in the expansion of $W^{\varepsilon}_{\mathsf{eff}}$:

$$\begin{array}{lcl} \delta M^{uv} - - - - \delta M^{wx} & : & \text{chiral propagator,} \\ \delta\overline{M}_{uv} \text{ ———————— } \delta\overline{M}_{wx} & : & \text{anti-chiral propagator,} \\ \delta\overline{M}_{uv} \text{ ——— } - - \delta M^{wx} & : & \text{mixed propagator.} \end{array}$$

- The higher-order terms in the expansion of $W^{\varepsilon}_{\mathsf{eff}}$

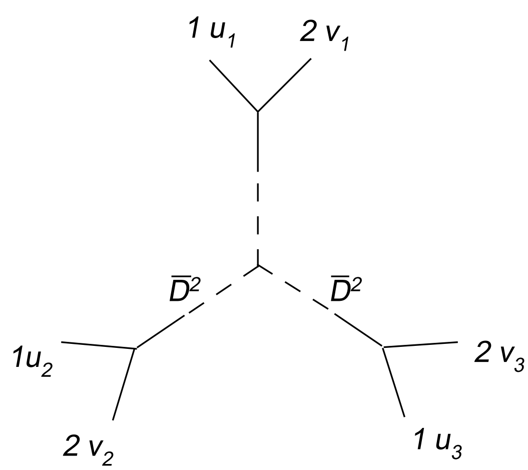
$$\lambda(\mathrm{Pf} M_0^{\varepsilon})^{1/n}(M_0^{\varepsilon})^{-1}_{i_1 j_1}\cdots(M_0^{\varepsilon})^{-1}_{i_\ell j_\ell}\delta M^{i'_1 j'_1}\cdots\delta M^{i'_\ell j'_\ell}$$

give chiral and anti-chiral vertices:

chiral vertices   anti-chiral vertices

- Let's consider $N_f = 4$ case as an example. This is the first case where we have a singular effective superpotential.

- It turns out that the only non-vanishing diagram comes with a total of six external legs and one internal chiral vertex, as follows (P. C. Argyres and M. Edalati hep-th/0510)



- In the limit of zero momentum, the above super Feynman diagram reads

$$\int d^4x\ d^2\theta\ \ \lambda^{-2}\ (\overline{\mu})^{-3}(\mu)^{-1}\ \epsilon^{u_1v_1u_2v_2u_3v_3}$$

$$(\overline{D}\delta\overline{M}_{1u_1}.\ \overline{D}\delta\overline{M}_{2v_1})$$

$$(\overline{D}\delta\overline{M}_{1u_2}.\ \overline{D}\delta\overline{M}_{2v_2})$$

$$(\overline{D}\delta\overline{M}_{1u_3}.\ \overline{D}\delta\overline{M}_{2v_3}).$$

- As we see, there is no ε dependence in the above expression, so, it does not diverge in the limit $\varepsilon \rightarrow 0$.

- The above expression, up to a numerical factor, is the same as $\delta S_{l,t}$ for $N_f = 4$.

- This implies that the effective superpotential of SU(2) superQCD with $N_f = 4$ flavors indeed reproduces the corresponding higher-derivative global F-term.

Singular W_{eff}'s of $\mathcal{N} = 2$ theories in three dimensions

- Singular superpotentials are a generic feature of gauge theories with a large number of flavors, and are not special just to four-dimensional theories.

- For an $\mathcal{N} = 2$ SU(2) supersymmetric gauge theory in three dimensions (hence four supercharges) with $2N_f$ light flavors $\mathcal{Q}^i_{\ a}$, transforming in the fundamental representation (O. Aharony, *et al* hep-th/9703110 and J. de Boer, *et al* hep-th/9703100)

  Classically, the moduli space of the theory has a Coulomb branch as well as a Higgs branch for $N_f \neq 0$.

- The Coulomb branch is paremetrized by the vacuum expectation values of $U = e^{\Phi}$ where Φ is a chiral superfield.

- The scalar component of Φ is $\phi + i\sigma$, where $\phi \in \mathbb{R}/\mathbb{Z}_2$ is the scalar in the vector multiplet of the unbroken U(1) and $\sigma \sim \sigma + 2\pi r$ is the scalar dual to the gauge field.

- The Higgs branch is parametrized by the vacuum expectation values of $V^{ij} = \epsilon_{ab} \mathcal{Q}^i{}_a \mathcal{Q}^j{}_b$.

- For $N_f = 1$, V^{ij} is unconstrained while for $N_f > 1$, V^{ij} is subject to rank $(M) \leq 2$, or equivalently

$$\epsilon_{i_1 \ldots i_{2N_f}} V^{i_1 i_2} V^{i_3 i_4} = 0.$$

- The quantum global symmetry of the theory is SU($2N_f$)$\times$U(1)$_A$$\times$U(1)$_R$ under which the fields parametrizing the Coulomb and the Higgs branch transform as

	SU($2N_f$)	U(1)$_A$	U(1)$_R$
U	$\mathbf{1}$	$-2N_f$	$2(1-N_f)$
V^{ij}	$\wedge^2(\mathbf{2N_f})$	2	0

- For $N_f > 1$, the quantum Higgs branch is the same as the classical Higgs branch, i.e. it is described by

$$\epsilon_{i_1 \ldots i_{2N_f}} V^{i_1 i_2} V^{i_3 i_4} = 0.$$

- We will be interested in the Higgs branch of the moduli space only for $N_f > 2$ where the global symmetry of the theory allows one to write a singular superpotential

$$W = (1-N_f)(U\,\mathrm{Pf}V)^{\frac{1}{(N_f-1)}}. \qquad (3)$$

- In addition to being singular the above superpotential cannot describe the origin of the moduli space where $\mathsf{U} = V^{ij} = 0$.

- But nevertheless for points away from the origin this superpotential perfectly describes the moduli space.

- Analogous to what we did in four dimensions we deform W as follows

$$W \to W^{\zeta,\eta} \ = \ W + \zeta U + \frac{1}{2}\eta_{ij}V^{ij},$$

  where ζ and η_{ij} are some gauge-invariant invertible parameters.

- The equations of motion for U and V^{kl} yields, respectively,

$$\begin{aligned} \left(U^{2-N_f}\mathrm{Pf}V\right)^{\frac{1}{(N_f-1)}} &= \zeta, \\ (U\mathrm{Pf}V)^{\frac{1}{(N_f-1)}}(\eta^{-1})^{kl} &= -V^{kl}. \end{aligned}$$

These equations result in

$$V^{kl} = -(\zeta \mathrm{Pf}\eta)^{\frac{1}{2}}(\eta^{-1})^{kl}.$$

which implies

$$\epsilon_{i_1 \ldots i_{2N_f}} V^{i_1 i_2} V^{i_3 i_4} = \epsilon_{i_1 \ldots i_{2N_f}} \zeta(\mathrm{Pf}\ \eta)(\eta^{-1})^{i_1 i_2}(\eta^{-1})^{i_3 i_4}.$$

- The right hand side of the above expression is a polynomial of order $N_f - 2 > 0$ for η_{ij} and of order one for ζ .

- Therefore independent of how we send ϵ_{ij} and ζ to zero, the right hand side will vanish and we obtain

$$\epsilon_{i_1 \ldots i_{2N_f}} V^{i_1 i_2} V^{i_3 i_4} = 0,$$

which is exactly the constraint equation describing the moduli space.

Conclusion

- We studied $\mathcal{N} = 1$ supersymmetric SU(2) gauge theory in four dimensions with a large number of massless quarks.

- We argued that effective superpotentials as a function of gauge-invariant local chiral fields should exist for these theories.

- Using a series of consistency checks, we showed that large N_f effective superpotentials, albeit singular, are perfectly sensible.

- We also gave some evidence that singular superpotentials can perfectly-well describe the moduli space in supersymmetric gauge theories in three dimensions with four supercharges.