# The generalized geometry of Calabi-Yau orientifolds with fluxes

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Nucl. Phys. B718, 2005 [hep-th/0412277] with J. Louis
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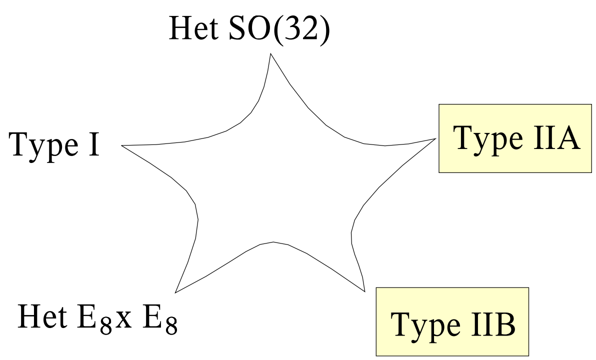
## Introduction and Motivation

➪ From String theory to $D = 10$ supergravity

String Theory $\xrightarrow[\text{energies}]{\text{low}}$ $D = 10$ Supergravity

effective theory for massless string modes: "weak coupling"

- concentrate on the two maximal supersymmetric theories in $D = 10$




Type IIA and IIB String Theory

$\downarrow$

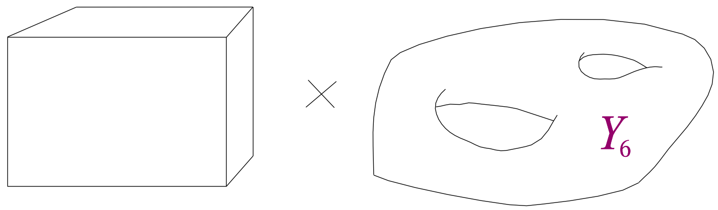
$D = 10$ Type IIA and IIB Supergravity with $N = 2$

phenomenology:

Four-dimensional setups with gauge theory and N=1 supersymmetry

➪ Four-dimensional setups: Compactification

space-time background: $\mathcal{M}_{1,3} \times Y_6$



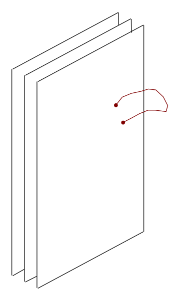

Non-compact visible space

Internal compact six-manifold

- minimal supersymmetry in $D = 4$: $Y_6$ is special manifold – Calabi-Yau

➪ Gauge theory: Type II string theories allow for D-branes

- extended objects with gauge-theory on their world-volume
- boundaries for open strings



stack of N D-branes:
U(N) gauge theory

- supersymmetric D-branes break half of SUSY on their world-volume

➪ Brane-world setups: necessity of orientifolds

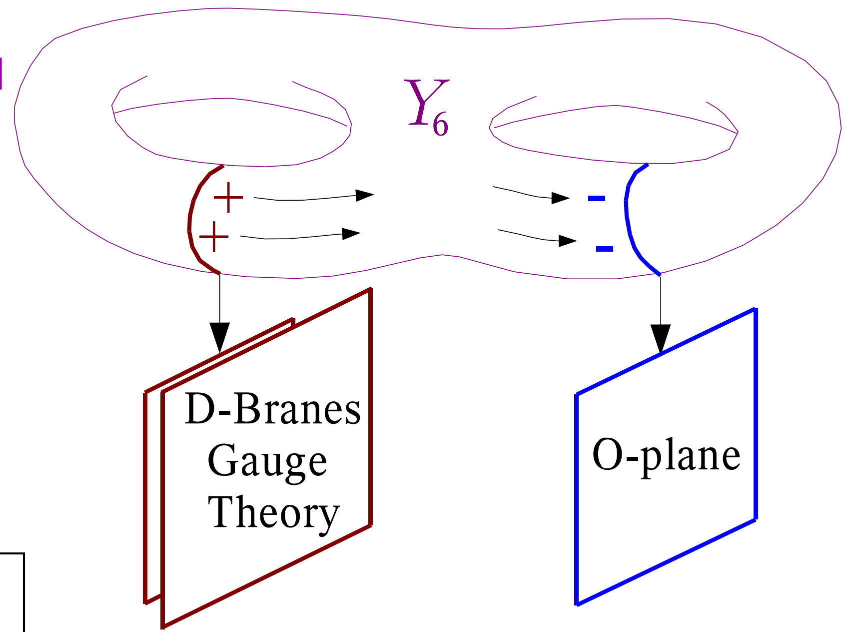
- minimal supersymmetry:
  $Y_6$ – compact Calabi-Yau manifold

- non-Abelian gauge groups:
  space-time filling D-branes

$\Rightarrow$ consistency: orientifold planes

$\downarrow$ Kaluza-Klein reduction

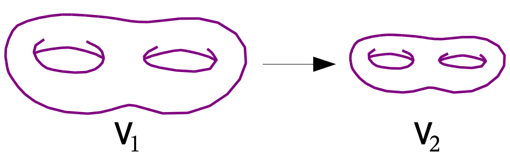
| Effectiv four-dimensional $N=1$ Supergravity Theory |
|---|




Problem:

| many moduli fields – flat directions of the potential |
|---|

example: size of $Y_6$



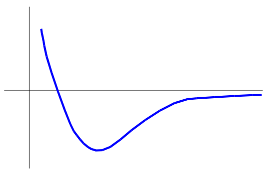

v(x) corresponds to four-dimensional field

➪ Our goal:

1) Determine effective $N = 1$ supergravity theory for these moduli fields in general Calabi-Yau orientifolds of type IIA and IIB string theory

2) Discuss geometry of $N = 1$ moduli space

3) Include mechanism to generate a potential: Background fluxes $\Rightarrow$ moduli stabilization



## Outline of the talk

1) Effective action of Type II Calabi-Yau orientifolds

   – Type IIB Calabi-Yau orientifolds – O3/O7 example

   – Type IIB Calabi-Yau orientifolds with several linear multiplets

2) Type IIA Calabi-Yau orientifolds

   – Kähler potential and generalized geometry of moduli space

3) Fluxes in Type II orientifolds

# 1. Effective action of type II Calabi-Yau orientifolds

➪ $d = 10$  $N = 2$ massless (bosonic) spectrum:

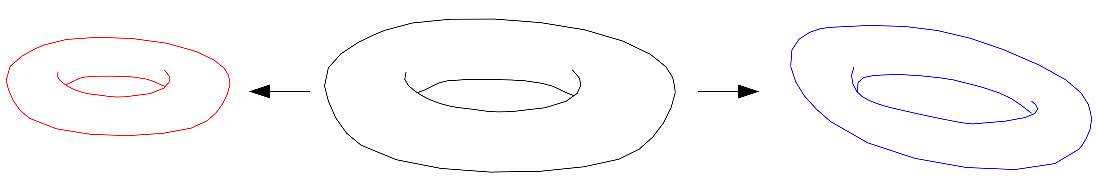
| | Type IIA | Type IIB |
|---|---|---|
| NS-NS: | $\hat{\phi},\ \hat{G}_{MN},\ \hat{B}_2$ | $\hat{\phi},\ \hat{G}_{MN},\ \hat{B}_2$ |
| R-R: | $\hat{C}_1,\ \hat{C}_3$ | $\hat{C}_0,\ \hat{C}_2,\ \hat{C}_4$ |

➪ compactification on compact Calabi-Yau $Y_6$:

Calabi-Yau manifold $\equiv$ exists globally defined two-form $J$ and $(3,0)$-form $\Omega$ s.t. $dJ = 0, \quad d\Omega = 0$

$J$ – Kähler form:
$\delta J$ Kähler structure deformations
$\equiv$ size moduli $v^A$

$\Omega$ – holomorphic three-form:
$\delta\Omega$ complex structure deformations
$\equiv$ shape moduli $z^K$



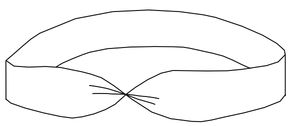
## ➪ Defining the orientifold

Acharya, Aganagic, Brunner, Hori, Vafa

- mod out (gauge-fix) discreate symmetries of the string theory
- focus on Type IIB

1) world sheet parity $\Omega_p$

'orienti-' – allow for non-orientable world-sheets: e.g. Klein bottle, Möbius strip



2) geometric symmetry $\sigma$ of $\mathcal{M}_{10} = \mathcal{M}_4 \times Y_6$, involution $\sigma$ ($\sigma^2 = 1$)
'-fold' – like in orbifold $\Rightarrow$ orientifold planes – fix-points of $\sigma$

- demand $N = 1$ supersymmetry

$\sigma$ is holomorphic and isometric involution: $\sigma^* J = J$

| $\sigma^* \Omega = -\Omega$ | $\sigma^* \Omega = +\Omega$ |
|---|---|
| orientifolds with O3/O7 planes | orientifolds with O5/O9 planes |
| $\mathcal{O} = (-)^{F_L} \Omega_p \sigma^*$ | $\mathcal{O} = \Omega_p \sigma^*$ |

- supergravity: truncate spectrum such that: $\mathcal{O}(\text{Field}) = \text{Field}$

## ➪ Four-dimensional $N = 1$ Spectrum    Type IIB $O3/O7$ orientifolds

- Kaluza-Klein reduction:

| expand fields in zero modes of $Y_6$ consistent with orientifold projection |
|---|

involution splits cohomologies    $H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}$

$$\hat{\phi} = \phi\,, \qquad \hat{B}_2 = b^a\,\omega_a\,, \qquad \hat{C}_0 = C_0 \qquad\qquad \omega_a \in H_-^{(1,1)}$$

$$\hat{C}_2 = c^a\,\omega_a\,, \qquad \hat{C}_4 = \rho_\alpha\,\tilde{\omega}^\alpha + V^\lambda\,\alpha_\lambda\,, \qquad\qquad \tilde{\omega}^\alpha \in H_+^{(2,2)}\,,\ \alpha_\lambda \in H_+^{(3)}$$

| | | | | |
|---|---|---|---|---|
| chiral multiplets | $h_-^{(2,1)}$ | $z^k$ | $z^k$ | shape moduli |
| | $h_+^{(1,1)}$ | $(v^\alpha, \rho_\alpha)$ | $T_\alpha$ | size moduli |
| | $h_-^{(1,1)}$ | $(b^a, c^a)$ | $G^a$ | |
| | 1 | $(\phi, C_0)$ | $\tau$ | |
| vector multiplets | $h_+^{(2,1)}$ | $V^\lambda$ | | |
| gravity multiplet | 1 | $g_{\mu\nu}$ | | |

## ➪ Four-dimensional $N = 1$ effective action

- Kaluza-Klein reduction:

| determine effective action consistent with orientifold projection |
|---|

- $N = 1, D = 4$ effective action in standard form: Wess,Bagger

$$\begin{aligned} L &= -\tfrac{1}{2}R - K_{I\bar{J}} DM^I D\bar{M}^{\bar{J}} - V \\ &\quad - \tfrac{1}{2}\mathrm{Re} f_{\lambda\kappa}\, (F^\lambda)_{\mu\nu}(F^\kappa)^{\mu\nu} - \tfrac{1}{2}\mathrm{Im} f_{\lambda\kappa}\, (F^\lambda)_{\mu\nu}(\tilde{F}^\kappa)^{\mu\nu} \ , \\ V &= e^K \big(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2\big) + \tfrac{1}{2}\, (\mathrm{Re}\ f)^{-1\ \lambda\kappa} D_\lambda D_\kappa \ . \end{aligned}$$

$M^I \equiv (z^k, T_\alpha, G^a, \tau)$: all scalar fields, $\quad F^\lambda = dV^\lambda$

Kähler metric: $K_{I\bar{J}} = \partial_I \bar{\partial}_J K(M, \bar{M})$

holomorphic superpotential: $W(M), \quad D_I W = \partial_I W + (\partial_I K) W$

holomorphic gauge kinetic function: $f(M)$

$\Rightarrow$ determine $K$ (and $f$) from orientifold effective action

later: determine $W$, $D_\alpha$ due to background flux

## ➪ The Kähler potential — Type IIB orientifolds with $O3/O7$ planes — TWG,Louis

chiral moduli fields

$$\Omega(z)\ , \qquad \mathrm{Re}\Big(e^{-\phi}e^{-\hat{B}_2+iJ}\Big) - i\, e^{-\hat{B}_2}\wedge \hat{C} = \tau + G^a\,\omega_a + T_\alpha\,\tilde{\omega}^\alpha$$

- $\tau, G^a, T_\alpha$ – complicated def of complex coordinates on $N=1$ moduli space

Kähler potential

$$K \;=\; -\ln\int_{Y_6}\Omega(z)\wedge\bar{\Omega}(\bar{z}) - \ln(\tau-\bar{\tau}) - 2\ln e^{-\frac{3}{2}\phi}\int_{Y_6} J\wedge J\wedge J\ ,$$

- general form of Kähler potential in terms of topological data of $Y_6$
- of no-scale type: positive potential $V\geq 0$ (if no superpotential for $T_\alpha$)
- last term in K:

  implicit function of real parts of $\tau, G^a, T_\alpha$

  $\Rightarrow$ $\mathrm{Im}\, T_\alpha$ admits shift symmetry: $\mathrm{Im}\, T_\alpha \to \mathrm{Im}\, T_\alpha + c$

  $\Rightarrow$ $K$ becomes explicit in the linear multiplet picture

## ➪ Type II orientifolds with several linear multiplets – O3/O7 example

idea: replace chrial multiplets $T_\alpha$ with linear multiplets $(L^\alpha, D_2^\alpha)$
(Im $T_\alpha$ possess shift symmetries)

Dual picture

linear multiplets $(L^\alpha, D_2^\alpha)$ coupled to chiral multiplets $N^I = z^k, \tau, G^a$

$\Rightarrow$ standard effective action for chiral/linear multiplet system Binetruy, Girardi, Grimm

- kinetic terms and couplings encoded by kinetic potential

$$\tilde{K}(N, L) = K(N, L) - 3F(N, L)$$

- Kähler potential $K(N, T)$ and chiral coordinates $T_\alpha + \bar{T}_\alpha$ are Legendre transform of $\tilde{K}(N, L)$ and $L^\alpha$:

$$\boxed{T_\alpha + \bar{T}_\alpha = \tilde{K}_{L^\alpha}\ , \qquad K(N, T) = \tilde{K} - \tilde{K}_{L^\alpha} L^\alpha}$$

## $O3/O7$ orientifold example

$$K(z,\tau,G,L) \;=\; -\ln\int\Omega(z)\wedge\bar{\Omega}(\bar{z}) - \ln(\tau-\bar{\tau}) + \ln(\mathcal{K}_{\alpha\beta\gamma}L^\alpha L^\beta L^\gamma)$$

$$F(\tau,G,L) \;=\; -i(\tau-\bar{\tau})^{-1}\mathcal{K}_{\alpha ab}L^\alpha(G-\bar{G})^a(G-\bar{G})^b$$

## The scalar potential

potential in the presence of linear multiplets

$$V = e^K\Big(\tilde{K}^{N^A\bar{N}^B}D_{N^A}W\,\overline{D_{N^B}W} - (3 - L^\alpha K_{L^\alpha})|W|^2\Big)$$

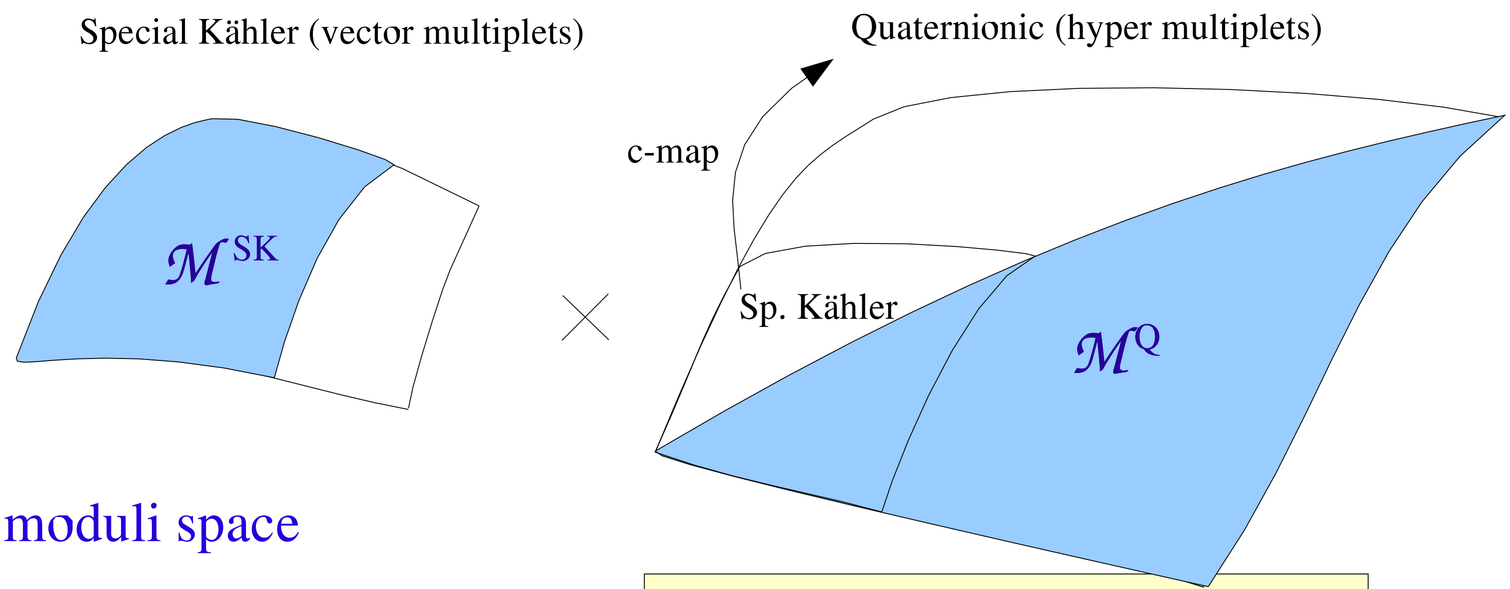
- since $L^\alpha K_{L^\alpha} = 3$ in $O3/O7$ orientifolds $\Rightarrow$ trivially $V \;\geq\; 0$

➪ similar analyis for $O5/O9$ orientifolds possible

# What about type IIA orientifolds?

Just the mirror of both O3/O7 and O5/O9 setups?

# N=2 moduli space




# N=1 moduli space

$\mathcal{M}^{SK}$ remains special Kähler
(same complex structure)

$\mathcal{M}^{Q}$ Kähler – half of Quaternionic

complicated in Type IIA, since
Sp. Kähler is obtained from a general
Pre-potential $\mathcal{F}(z)$

## 2. Type IIA Calabi-Yau orientifolds

TWG,Louis

➪ orientifold projection

$\mathcal{O} = (-1)^{F_L}\Omega_p\sigma$ $\sigma$ is anti-holomorphic, isometric involution of $Y_6$

$$\sigma^* J = -J \qquad \sigma^*\Omega = e^{2i\theta}\bar{\Omega}$$

$\Rightarrow$ $O6$ planes wrap special Lagrangian cycles in $Y_6$ calibrated with $\mathrm{Re}(e^{-i\theta}\Omega)$

➪ chiral moduli fields

$$J_c = B_2 + iJ = t^a\omega_a$$

$h_-^{(1,1)}$ chiral multiplets

- coupling to the string world-sheet

$$\Omega_c = C_3 + i\mathrm{Re}(C\Omega) = N^k\alpha_k + T_\kappa\beta^\kappa$$

$h^{(2,1)} + 1$ chiral multiplets

- $C \propto e^{-\phi - i\theta}$ $\Rightarrow$ $\Omega_c$ coupling to wrapping supersymmetric $D2$ branes
- gauge-couplings for space-time filling $D6$ branes Blumenhagen, Braun, Körs, Lüst

## ➪ Kähler potential

$$K^{\mathrm{SK}}(t) + K^{\mathrm{Q}}(N,T) = -\ln \int_{Y_6} J \wedge J \wedge J - 2\ln \int_{Y_6} C\Omega \wedge \overline{C\Omega}$$

- $K^{\mathrm{Q}}$ calculated by using Legendre transformation (linear multiplet formalism)

define: $V = \int_{Y_6} C\Omega \wedge \overline{C\Omega}$ , $C\Omega = CZ^K \alpha_K - C\mathcal{F}_K \beta^K$

chiral picture: orientifolded Hitchin's generalized geometry

$K^{\mathrm{Q}}(q^k, q_\lambda) = -2\ln V[q]$ , $q^k = \mathrm{Re}(CZ^k)$ $q_\lambda = \mathrm{Re}(C\mathcal{F}_\lambda)$

↓ Legendre transformation ↓

dual picture: orientifolded $N=2$ special geometry

$\tilde{K}^{\mathrm{Q}}(q^k, \pi^\lambda) = -2\ln V[q,\pi] + \frac{1}{V}\mathcal{F}[q,\pi]$ , $q^k = \mathrm{Re}(CZ^k)$ $\pi^\lambda = \frac{1}{V}\mathrm{Im}(CZ^\lambda)$

➪ Generalized complex geometry (very brief) Hitchin,Gualtieri

- differential geometry on $\mathcal{T} \equiv TY_6 \oplus T^*Y_6$ instead on $TY_6$ alone:

  $\Rightarrow$ generalized metric on $\mathcal{T}$ $\cong$ metric, B-field and dilaton on $Y$

  $\Rightarrow$ generalized complex structure on $\mathcal{T}$

  $\cong$ complex/symplectic structure, B-field and dilaton on $Y$

- $TY_6 \oplus T^*Y_6$ has natural $SO(6,6)$ structure $\Rightarrow$ spinors of $Spin(6,6)$?
  two Weyl representations

$$S^{\mathrm{ev}} = \Lambda^{\mathrm{even}} T^*Y \qquad\qquad S^{\mathrm{odd}} = \Lambda^{\mathrm{odd}} T^*Y$$

  special complex spinors define generalized complex structure on $\mathcal{T}$
  examples:

$$e^{-\phi} e^{B_2 + iJ} \in S^{\mathrm{ev}}\,, \qquad\qquad C\Omega \in S^{\mathrm{odd}}$$

  We used the real parts of these forms in defining the $N = 1$ Kähler coordinates on the truncated quaternionic space $\mathcal{M}^{\mathrm{Q}}$!

- Hitchin defines a functional $V(\rho)$ on the real parts of these forms
  $\Rightarrow$ evaluated on light modes of the theory: Kähler potentials on $\mathcal{M}^{\mathrm{Q}}$

The spinors $e^{-\phi}e^{-B_2+iJ}$ and $C\Omega$ are the special cases corresponding to Calabi-Yau orientifolds.

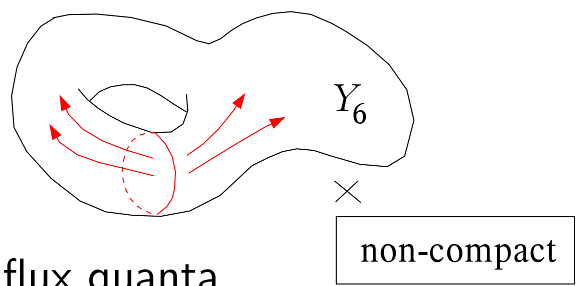
The mathematical framework is much more powerful
It can incorporate orientifolds of non-Calabi Yau spaces.
'Generalized complex orientifolds'

## 3. Fluxes in type II Calabi-Yau orientifolds

➪ Background flux

- Problem: no potential for moduli fields
- background value for field strengths of form fields

| NS-NS flux | R-R fluxes | |
|---|---|---|
| $\langle dB_2 \rangle = H_3$ | $\langle dC_q \rangle = F_{q+1}$ | |
| $\int_{\mathcal{C}^a} H_3 = e_a$ | $\int_{\mathcal{D}^k} F_{q+1} = p^k$ | $\longrightarrow$ flux quanta |




$\Rightarrow$ Background flux induces a potential for the N=1 chiral scalars

$$V \;=\; e^{K}\left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2\right) + \tfrac{1}{2}\,(\mathrm{Re}\; f)^{-1\;ab} D_a D_b \;.$$

➪ Type IIB orientifolds with $O3/O7$ planes: superpotential $W$, no D-term

$$W(\tau, z) = \int \Omega(z) \wedge (F_3 - \tau H_3)$$ Gukov,Vafa,Witten

➪ $O5/O9$ orientifolds: superpotential + D-term (electric NS-NS fluxes)

$$W(z) = \int \Omega(z) \wedge F_3 \qquad D_a = e_a \, e^{\phi} \, \mathrm{Vol}^{-1}$$

⇒ magnetic NS-NS fluxes: massive linear multiplet $(\phi, C_2)$

➪ Type IIA orientifolds with $O6$ planes: superpotential, no D-term

$$W(t, N, T) = \int e^{J_c} \wedge F_{RR} + \int \Omega_c \wedge H_{3\,NS}$$

- compactify massive type IIA theory Romans
- flux-superpotential depends on **all** moduli fields ⇒ suggests that all geometric moduli are stabilized in supersymmetric vacuum: recently confirmed Villadoro, Zwirner; DeWolfe, Giryavets, Kachru, Taylor

## 5. Conclusions

- calculated $N = 1$ effective action for IIA and IIB Calabi-Yau orientifolds
  - complex coordinates and Kähler potentials
  - orientifolds with several linear multiplets
  - geometry of the moduli space
    $\Rightarrow$ Hitchin functionals and generalized complex geometry
- allowed for all possible NS-NS and R-R fluxes
  - IIB with $O3/O7$: Gukov-Vafa-Witten superpotential
  - IIB with $O5/O9$: superpotential, D-term, massive linear multiplet
  - IIA with $O6$: superpotential for all moduli in the theory

## Outlook

- generalized complex orientifolds ($Y_6$ is not necessarily Calabi-Yau)
- vacuum statistics for type IIA and generalized complex orientifolds
- wave-function for $N = 1$ flux compactification

## 4. M- and F-theory embedding

➪ The idea: orientifolds are special limits of higher-dimensional theories
orientifold planes and D-branes can admit a geometrical interpretation

➪ Type IIA: M-theory (11d supergravity) on special $G_2$ manifold

$$X = (CY \times S^1)/\hat{\sigma} \qquad \hat{\sigma} = (\sigma, -1)$$ Joyce,Harvey,Moore,Kachru,McGreevy

$\Rightarrow$ KK-reduction: $N = 1,\ D = 4$ supergravity

- chiral moduli fields $\Phi + iC_3$: $\Phi$ deformations of $G_2$ metric, $C_3$ three-form
- decompose on $X$: $\quad \Phi + iC_3 = J_c \wedge dy^7 + \Omega_c$
- matches $N = 1$ data of the theories: also (part of) background fluxes

➪ Type IIB with $O3/O7$ planes: F-theory on special elliptic Calabi-Yau fourfold

$$Y_4 = (CY \times T^2)/\hat{\sigma} \qquad \hat{\sigma} = (\sigma, -1, -1)$$

- use duality to M-theory on $Y_4$:

$$\text{M-theory}/Y_4 \ \equiv \text{Type IIB}/(CY/\mathcal{O}_{(2)} \times S^1)$$

- compare three-dimensional theories $\Rightarrow$ F-theory lift