Introduction / Motivation / Proposal

All notions listed above exist or have been derived (interpreted) by very scale $L \gg \ell_P$.

Equivalently, we use an effective theory (e.g. or generalization of it) in which $L$ is a natural scale of $\ell_P$.

Strictly speaking, we refer some of the issues listed above to regions in which effective theory breaks down (then is NOT generally true for black holes, since known physics can be classical if chosen on large enough).

Black holes:
- Question: When we refer to a black hole? What property should we study?
- By NO horizons! (singularity means $s = 0$) + global charge of the hole
  Can it be realistic practically? $\rightarrow$ depends on curvature! (charge)
  Small curvature $\rightarrow$ fermion picture (position mechanically)
  Macromagic horizon $\rightarrow$ details matter at horizon scale (Machian penchant)

- $L$ scale (Planck scale or larger): if we coarse-grain above it $\rightarrow$ no distinction and microstate.

Proposal (V. Balasubramanian, V. F. J. Boer, T. S.)

1) Origin of gravitational thermodynamics $\otimes$ difficulty/uncertainty to tell apart different microstates (analogy with a gas)
   $\otimes$ gauge theory matter (large operation/pseudorotation or gauge matrix operator $\rightarrow$ identification of states)
   * large charge are assumed ($\otimes$ large degeneracy) $\Rightarrow$ statistical mechanics (thermodynamics) to decide "typical state"
   (too: gauge at this point)

2) Each microstate $\rightarrow$ "random" description at a given scale ($L$ or another $\rightarrow$ dependence of charge)
   $L$ by coarse-graining system ($L \gg \ell_P$), we lose the ability to identify uniquely the state $\rightarrow$ ensemble of microstate
   $\otimes$ we have access to "typical" properties (charge, multiple moments) $\rightarrow$ "typical" state (singular operation)
   Singlet classical equation emerge on "average" after coarse-graining details over $L$ scale.
What shall we do?

- Embed our ideas into string theory: ADS/CFT
- Find a covariant of the theory with enough symmetry to hat both, field theory and gravity are under control
- Put such system into test
- Check our philosophy

(a) Emphasis: dual global theory provides the tools/variables to identify "the axioms of spacetime" in this particular construct. In general, we do lack such description. [In particular, for Big Bang physics]

- Focus on external Robbins which acts in $AdS_5 \times S^5$ (simple Robbins):
  \[ \Delta = M L = \frac{1}{2} N \omega \quad \omega = 9/15 \]
  \[ N = N \omega \quad (\text{a fixed point}) \]
SET-UP

- We shall consider $\mathbb{C}^3$ vector ($A=\mathbb{R}$) of $\mathfrak{sl}_3$ sym

Field Theory considerations ($\mathbb{R}^3$)

- Absence of angular momentum ($\mathbb{C}^3$) $\Rightarrow$ zero modes on $S^3$
- Single R-charge $\Rightarrow X, i\bar{X}$ ($\propto Y, \bar{Y}$ cannot be both)
  \[ \mathfrak{sl}_3 \] reduction of $\mathfrak{so}_4$ from an $S^3$
- Metric quantum mechanics: one single complex metric + $A=\mathbb{R}$ constraint $\Rightarrow$ real metric quantum mechanics

Degree of freedom: eigenvalue of the metric $\frac{1}{2}d_i^2$ $i=1,...,N$

- Van der Monde determinant: $N$ fermions in an harmonic potential $\Rightarrow$ spectrum, wavefunctions, ... fully known

Characterisation of states:

- $E_i = E_k + \frac{1}{2}$ ($w=1$) (single particle system)

- $E_i = (i-1)\kappa + \frac{1}{2}$ (fused state energy; quasi classical principle; $i=1,...,N$)

- \[ E_i = \frac{1}{\kappa}(E_k - E_1) = E_i - 1 \]

(spin labels, $i$ particle is $i-th$ fused state)

\( \{E_i\} \) non-decreasing as $i$ increases $\Leftrightarrow$ Young diagram

Useful variables:

- $C_i = E_i$; $C_{i-k} = E_{i-k}$ $i=1,...,N+1$ $\quad C_j$ are columns of length $j$ (maximum length is $N+1$)
- $C_0 = C_{N+1} = C_{N+2} = \ldots = C_N$

Physical Interpretation:

- i) Columns $\Rightarrow$ giant graviton (particles with large momentum, due to Myers effect, blow up into spherical D-brane spinning on $S^3 \equiv R$-charge).

- They carry a maximum angular momentum of $N$, which matches the maximum length (branons) of a given column

- ii) Columns $\Rightarrow$ dual giant graviton

- iii) Rows $\Leftrightarrow$ dual giant graviton
Gravity considerations

- We are looking for asymptotically AdS$_5 \times S^5$ with $SO(1,5|N) \times U(N)$ global symmetry and its supergravity
- $\mathcal{N} = 2$ $\Rightarrow$ $\mathcal{N}, s, x^i$ $\Rightarrow$ $\mathcal{N}, s, x^i$ $\Rightarrow$ $\mathcal{N}, s, x^i$ $\Rightarrow$ $\mathcal{N}, s, x^i$

- Assume $\mathcal{N} = 2$, (Lorentz dilation), no $2$-form (trace) and "standard" $R \neq 5$-form flux (to oblate AdS$_5 \times S^5$ shell).
- This renders, with (9) spin $\mathcal{N}$-stabilized $R \neq 5$-charge black hole.

This point was fully solved by LLM:

$$\gamma = -\frac{1}{2} \left(d\theta + \frac{\alpha}{2} d\phi + \theta^{(d \phi + d\theta)} + \frac{1}{2} \theta^{(d \phi + d\theta)} + \frac{1}{2} \theta^{(d \phi + d\theta)}\right) + \frac{1}{2} \gamma - \text{a product of radius of two spaces}$$

Solution (111) is characterized by a single scalar function $z = z(\gamma, x^i)$ satisfying a linear differential equation:

$$\frac{\partial}{\partial x^i} + \frac{\partial}{\partial y}(\frac{\partial}{\partial \gamma}) = 0$$

(12)

$$f(\gamma, x^i, z) \rightarrow z$$

for a stable solution in $6$-dimension being specifically invariant in $4$-dimension $U(1)$ $\rightarrow f(\gamma, x^i)$ and is characterized by a boundary condition at $\gamma = 0$

$$f(\gamma, x^i, z) \rightarrow \frac{\partial}{\partial \gamma} \quad \text{at} \quad \gamma = 0$$

Comments:

1. A given supergravity configuration $\leftrightarrow z(\gamma; x^i)$ is charge distribution in $(\gamma, x^i)$ plane.
2. As $\gamma \rightarrow 0$, $\gamma^3$ or $\gamma^4$ (or both) should be zero size: simplicity?
   - LLM concluded that only unique boundary condition correspond to $z = \frac{\delta}{\delta z} f(\gamma, x^i)$ $\rightarrow$ zero size
3. Charges:
   $$\Delta \equiv R \int_{0}^{1} \frac{d\gamma}{\delta z} \frac{\partial}{\partial \gamma} \quad \text{at} \quad \gamma = 0$$

Quantization of gauge:

$$N \equiv \int_{0}^{1} \frac{d\gamma}{\delta z}$$

Introduction:

$$\{U(1), x^i \rightarrow \frac{1}{2} - z(\gamma; x^i)\} \Rightarrow \Delta = \int_{U(1)} \frac{d\gamma d\phi^{(1)}}{2\pi} \frac{u(\gamma; x^i) \bar{u}(\gamma; x^i)}{\phi^{(1)}}$$

is integral over $(\gamma, x^i)$ of Hamiltonian ($\frac{1}{2} \phi^{(1)} \frac{\partial}{\partial \gamma} \phi^{(1)}$ harmonic oscillator)

with zero norm every $\gamma$.

ii) Suppose to interpret $u(\gamma; x^i)$ as a "density" (also plane wave delta function)

vacuum: disk $\leftrightarrow$ absence of diagram

initial state: bond of spins $\leftrightarrow$

final state: boundary spins
Connect for help:

1) Keep near USN, A formless in the shudder [Please space to write reporter and compare]
Typical States

General Philosophy.

Large $N \sim$ large degeneracy $\Rightarrow$ canonical ensemble (for $\beta \to 0$ but $\langle \varepsilon \rangle = \Delta$) (statistical mechanics in general)

$\langle \varepsilon \rangle, \langle c \rangle \Rightarrow$ average hardcore/singlet state $\Rightarrow$ average state/typical state (converging many of them will be close to that state).

Fluctuation $\Rightarrow$ deviation from typical state.

Connect: large degeneracy (equality with $N$) does not guarantee a macroscopic quasibound boson.

Different ensemble: consider system with fixed a few terms (own).

i) Typical ensemble: $N$ columns is not constrained.

Average energy grows without bound at the temperature $\rho_o$.

$S(N = \infty) = S(N = N_c) = 0$ (as expected!)

ii) Fixing $2$ columns is fixing a good procedure (relevant to discuss the physics of external & deep well hosts).

$\Delta \in [N_c, N-N_c]$

$N_c \in [N_c, N-N_c]$

$S(N = \infty) = S(N = N_c) = 0$ (as expected!)

This physical picture is confirmed by the fact that a two-term map between Yang diagram with $\Delta$ and $N_c$ is fixed.

$S(N = \infty) = S(N = N_c) = 0$ (as expected!)

Details

$Z = \prod_{\langle \varepsilon \rangle = \Lambda} c_\varepsilon^\varepsilon e^{-\varepsilon} \varepsilon^{-\lambda} (\varepsilon - \varepsilon_c) = \prod_{\langle \varepsilon \rangle = \Lambda} c_\varepsilon^\varepsilon e^{-\varepsilon}

Z = \prod_{\langle \varepsilon \rangle = \Lambda} c_\varepsilon^\varepsilon e^{-\varepsilon} \varepsilon^{-\lambda}

\langle \varepsilon \rangle = \Delta = \frac{2}{1-\lambda} \lambda = \frac{1-\lambda}{1-\lambda}

\langle c \rangle = \frac{N_c}{1-\lambda} \lambda = \frac{1-\lambda}{1-\lambda}

N_c = \frac{\lambda}{1-\lambda} \lambda = \frac{1-\lambda}{1-\lambda}

Connect: low temperature $\Rightarrow$ large $\beta$

1. $T < \rho_o \Rightarrow q = e^{\Delta} \Rightarrow \Delta = \frac{\lambda}{1-\lambda} \lambda = \frac{1-\lambda}{1-\lambda}$

2. $T > \rho_o \Rightarrow q = e^{\Delta} \Rightarrow N_c \sim \langle c \rangle = \frac{\lambda}{1-\lambda} \lambda = \frac{1-\lambda}{1-\lambda}$ (ground state diagram).

$\Delta = \frac{2}{1-\lambda} \lambda = \frac{1-\lambda}{1-\lambda}$

$N \sim \frac{\lambda}{1-\lambda} \lambda = \frac{1-\lambda}{1-\lambda}$
High Temperature.

\[ S = \beta (\varepsilon) + \beta \ln 2 \]

Solving the constraint:

\[ \varepsilon(n, N) = \frac{N}{n} \left( \frac{N}{A_n} + \beta \ln \frac{N}{A_n} \right) + \alpha (n) ; \quad \omega = \frac{N}{A_n} ; \quad A_n = \frac{N}{n} \]

Solving (C) constraint:

\[ \Delta = \frac{N}{A_n} - \beta \ln \alpha (n) + \frac{\alpha (n)}{\beta} \]

Comment:

\[ \Delta = \frac{1}{2} N N \ln N + O(1) \Rightarrow \text{dominant contribution at large } N \text{ and large temperature} \Rightarrow \text{uniformity} \]

\[ S = -\beta \ln \left( \frac{\omega (\varepsilon (N, N))}{(4 \pi n)^{3/2}} \right) - \frac{\beta^2}{2} \frac{\omega (\varepsilon (N, N)) N (N^2 - 1) + O(1)}{\epsilon_n} \]

Thus, \( \beta = 0 \) (T = 0) is added a maximum for the entropy.

Conjecture: \( \beta \rightarrow 0 \), large \( N \) limit describes typical state for the system (extremal \( \frac{1}{2} \) Boltzmann black hole).

Show:

\[ S \sim \Omega_T \sim N/N^{3/2} \sim (2 \pi)^{3/2} / \ln \frac{2 \pi n}{1/2} \sim N^{1/10} \Rightarrow S / N^{1/10} \sim N^{1/10} \sim N N^{1/10} \rightarrow 0 \]

Even though entropy is maximum, degeneracy \( \omega (n) \) is NOT enough to generate a macroscopic horizon = evaporate.

Limit case.

- Introduce two coordinates \( (x, y) \) to describe the diagram: \( x \) along the row and \( y \) along the column.

- Notice that:

\[ \langle x \rangle = \frac{x + 1}{2} \Rightarrow \frac{1}{2} \frac{N}{x} \]

\[ \langle y(x) \rangle = \frac{y}{2} \sum_{x=1}^{N} \frac{1}{x} \sim \int_{N-x}^{N} dx \Rightarrow \]

\[ \left[ \langle x \rangle \right]^{N-\Delta} = \left[ \frac{N-x}{N} \right]^{N-\Delta} \Rightarrow \lambda \sim \frac{N-x}{N} \Rightarrow \beta = 1 - \frac{N-x}{N} \]

- In particular, as \( \beta \rightarrow 0 \) (to check our conjecture) \[ \text{[Drawn on triangular diagram]}. \]
Matching Gravity: Semiclassical Considerations

LM suggested $\{\ell, m\} \rightarrow (l, s)$ plan wave of a single particle in the eigenvalue picture.

We shall give one step further:

\[
\langle \Psi(\vec{r}); \Psi(\vec{r}') \rangle = W_{\text{Wigner}}(\vec{r}, \vec{r}')
\]

(roughly meaning up to normalization and taking semiclassical limit appropriately)

**Wigner**

\[
W_l(\hat{\mathbf{r}}; \hat{\mathbf{r}'}) = \frac{1}{\pi}\int_{-\infty}^{\infty} dx \left< \frac{x}{\sqrt{x^2 + y^2}} \right> e^{i\hat{\mathbf{r}} \cdot \hat{\mathbf{r}'}}
\]

[definition, given a density matrix \( \hat{\rho} \)]

\[
\langle \hat{A}_\mathbf{r} \rangle = \text{tr}(\hat{\rho} \hat{A}_\mathbf{r}) = \int d\mathbf{r} d\mathbf{r}' \ A(\mathbf{r}, \mathbf{r'}) W_l(\mathbf{r}, \mathbf{r'})
\]

[\( A \) complete vectorial \( \rightarrow \) well defined operator]

Properties:

- Trivial bounded, it oscillates rapidly as the classically allowed phase space, and decay exponentially outside.

In our setup: (after averaging on \( \nu \)-particle !)

\[
W_l(\mathbf{r}, \mathbf{r'}) = \frac{1}{\sqrt{\pi}} e^{-\frac{(x-x')^2}{2\sigma^2}} \text{, \( \sigma \) is of oscillation length}
\]

Main Obs: oscillation length \( \sigma \) scale \( \rightarrow \) very position/energy sensitive widely.

- Even if we add the effect of many bosons, it's quantum mechanical oscillation

Support: even power Wigner \( \circ \) c scale \( \Rightarrow \) Wigner distribution.

**Husimi**

\[
H_l(\mathbf{r}, \mathbf{r'}) = \frac{1}{\pi \sigma \sqrt{\pi}} \int d\mathbf{r} d\mathbf{r}' \ 2(\mathbf{r}, \mathbf{r'}, \mathbf{r}, \mathbf{r'}) W_l(\mathbf{r}, \mathbf{r'}) \text{ where } 2 = \exp \left[ -\frac{1}{2\sigma^2} (\mathbf{r}-\mathbf{r'})^2 \right]
\]

(Bernstein kernel of variance \( \sigma^2 \))

- It contains the same quantum mechanical information, but it computes normal ordered observable.

\[
H_l(\mathbf{r}, \mathbf{r'}) \sim \frac{1}{\pi \sigma \sqrt{2\pi}} e^{-\frac{(r-r')^2}{2\sigma^2}} \text{ where } \sigma = \sqrt{(\mathbf{r}-\mathbf{r'})^2 / \sigma}
\]

It is peaked at \( \mathbf{r} = \mathbf{r'} \) (classical energy), with variance \( \sim \sigma \) \( \Rightarrow \) \( \sigma \mathbf{r} \sim \sigma \mathbf{r'} \) (variance as radial variable)

\( \mathbf{r} \) it is a well-behaved loop of energy on \( \mathbf{r} \), but...

\( \sigma \mathbf{r} \sim \sigma \mathbf{r'} \Rightarrow \) localized ring does not capture physics.

(classical limit)
Grayscale Distribution

- Assume a nondimensional limit \((k=0, \text{ not fixed})\)
- In plane space, we want to approach a limit \(\Delta E/k \to \infty\)

A natural choice of a product variable would depend on the nondimensional choice:

\[
R(E, \Delta E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{2\Delta x^2}} dx 
\]

**Effective density:**

\[
\langle \Delta E \rangle E = \frac{\text{d} \langle \Delta E \rangle}{\text{d} E} = \frac{E}{\text{d} x / \Delta x}
\]

- If we assume a nondimensional limit: \(E = h(x + y_G) \Rightarrow \langle \Delta E \rangle \approx \frac{1}{h(y_G)}

Equivalent:

\[
\frac{\text{d} \langle \Delta E \rangle}{\text{d} x} = \frac{1}{h(y_G)} (\text{a posteriori, \text{ e.g., \text{ riddle}}})
\]

\[
\frac{\text{d}}{\text{d} t} \frac{\text{d} \langle \Delta E \rangle}{\text{d} x} = \frac{1}{h(y_G)} (\text{every even odd bond})
\]

\[
\frac{\text{d}}{\text{d} t} \frac{\text{d} \langle \Delta E \rangle}{\text{d} x} = \frac{1}{h(y_G)} (\text{every odd even bond})
\]

\[
\text{MATHEMATICAL SUPERSTAR!}
\]

- Gravity data:

\[
g = -\sqrt{\frac{k}{h}} \left[ \left( \frac{\text{d} k^2}{\text{d} x} \right)^2 + \left( \frac{\text{d} k^2}{\text{d} y} \right)^2 + \left( \frac{\text{d} k^2}{\text{d} z} \right)^2 \right]^{1/2}
\]

\[
h = \frac{1}{2} k \Delta t^2 / \Delta x^2
\]

- Compress size of \(S^3\)s:

\[
\eta \Delta k = \frac{\Delta k}{k} \text{ for } \eta \Delta k \ll 1
\]

\[
\eta \Delta k < \frac{1}{2} \Delta k < \frac{1}{2} \Delta k \text{ for } \eta \Delta k \ll 1
\]

We want to compute \(z(0)\):

\[
\eta \approx 0 \Rightarrow \text{no \text{ zero \text{ excited!}}}
\]

\[
0 = 0 \text{ density of \text{ first \text{ non \text{ excited}}} \Rightarrow \langle \Delta E/\Delta x \rangle = \frac{1}{2} \Delta t^2
\]

MAGIC EXISTS... !

Our code 

- high mass analysis + typical state analysis + proposal to match:

\[
2 = \frac{1}{2} - \frac{1}{2} \frac{x^2}{\Delta x^2} = \frac{1}{2} - \frac{1}{2} \frac{\Delta x^2}{\Delta t^2} \Rightarrow \frac{(x_0-x)^2}{2\Delta x^2}
\]

\[
\text{MATCH!}
\]

LESSON (interpretation):

\[
\text{a) simple \text{ micrometeor} \Rightarrow \text{ not \text{ classical \text{ activity; typ. \text{ complex)}}}
\]

\[
\text{b) typical \text{ micrometeor} \text{ cannot be \text{ bad \text{ apart \text{ limit \text{ some \text{ phenomenon}}}}}}
\]

\[
\text{c) \text{ neutron \text{ reaction \text{ is \text{ regular}}} \Rightarrow}
\]

\[
\text{d) Quantum mechanical correlation \text{ function: one \text{ key \text{ moment!}} \text{ \text{ distinguish \text{ ability of \text{ dark \text{ micrometeor}}}}}
\]

\[
\text{e) Including quantum \text{ effects \text{ for \text{ parity \text{ breaking \text{ scale \text{ details}}}}} \text{ \text{ parity \text{ side!}}}
\]
Once any notion whether the persons known could shed some light on the microwaves of "real" black holes, or Ads.

\[ \text{Gravity results:} \]
\[ S \sim L \Rightarrow S_\text{AdS} \sim N^4, \quad \Delta = M \cdot L \sim N^2 \]
\[ S \sim N^2 \]

Field theory expectations:

1. 
2. 
3. 

\[ \text{Large} \text{ operators} \sim \text{built of words (loops)} \text{ made of things } q \text{ letters } (fields) \text{ that belong to an alphabet } (N \text{ sym}) \]

Again, the crucial notion is: 

\[ \text{Typical operator} \sim \text{Typical word} \]

\[ \text{Large } N \Rightarrow \text{an approximate heavy boson (Statistical mechanics) to determine characteristic "typical words"} \]

Information Heavy (Sena's theorem)

\[ \text{Typical word} \sim \text{statistical random of the sequence of fields in the long polynomial } \]
\[ \text{P}(x) = \prod_{k} \text{ks} = \text{alphabet and } x = \text{field (fields)} \]

The probability that a given distribution differs from \( \text{P}(x) \) is exponentially suppressed in the infrared dimension.

**Conjectures**

1. Non-fermionic operators (R^4 are allowed): A normalization
2. Mixing of operation

\[ \text{Coupling (difference between a pair of particle and a hole) } \]
\[ \text{vs} \]
\[ \text{(strong-wk AdS/CFT duality)} \]
\[ \text{correspondence principle (classical limit)} \]

Our proposal:

**Correlation functions of probe operator \( \mathcal{O}_p \) computed in very heavy states \( \mathcal{O} \) depend only on \( \Delta = \text{planck length} \)

up to correction of \( \mathcal{O}(e^{-\Lambda}) \), for almost all states \( \mathcal{O} \) and probe \( \mathcal{O}_p \).

In the field theory, the contribution from other diagrams is controlled by the number of times the probe operator appears within the state (pattern matching) \( \Rightarrow \) this number is mostly determined by the statistics of long random polynomials.
Future Directions

1. Loss of information (N particle $\rightarrow$ (9 particle plus vacuum)); gravitational counterpart (r' Hitch 91)
   - Unpredictability of correlation function (point vs. six) or (aspects of renormalization)
   - Non-via incarnate state; non-free state $\rightarrow$ identification of point microstates $\rightarrow$ learn

2. Non-extremal epsilon: perturbative expansion (pace $q$) $\rightarrow$ connection to Schwabl's; see physics
   Toy model: How to perturbations (1) $\rightarrow$ in any big question (piets)

3. Cosmological fluctuation: is zero-pruning one Planck scales responsible for the existence (denial) of
   the Big Bang cosmology?
   In reference of coping effects, additive of degree of freedom ($k$)

(*) In our talk, there was no need for adding these since we knew we had all the main included!