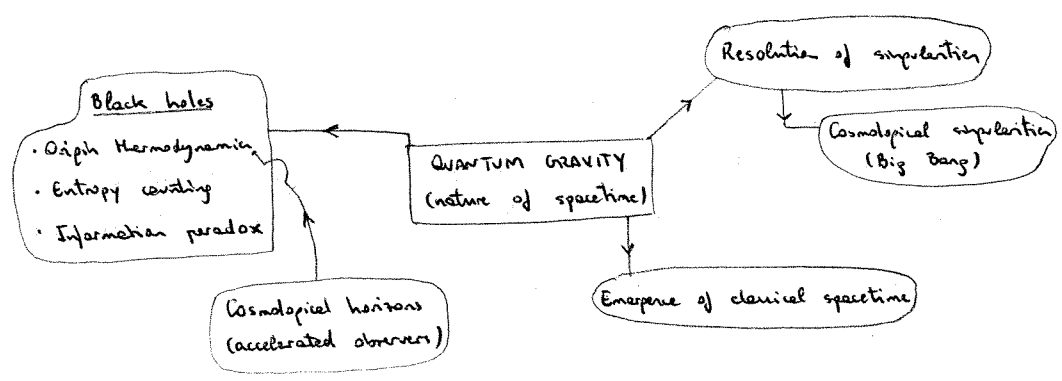


COARSE-GRAINING GRAVITY: BLACK HOLES AND SINGULARITIES

INTRODUCTION / MOTIVATION / PROPOSAL



All notions listed above exist or have been derived (extrapolated) by using scales $L \gg l_p$.
 Equivalently, we use an effective theory (GR or generalization of it) in which there is a natural scale l_p .
 Strictly speaking we refer some of the issues listed above to regimes in which effective theory breaks down
 (this is NOT necessarily true for black holes, since horizon physics can be classical if charges are large enough)

Black holes.

- Question: Where are states of a black hole? What properties should they satisfy?
 - ↳ NO HORIZON! (simple microstate $\Rightarrow S=0$) + global charges of the hole
 - Can it be realized geometrically? \rightarrow depends on curvature! (charge)
 - Small curvature \Rightarrow foamish picture (quantum mechanically)
 - Macroscopic horizons \Rightarrow details matter at horizon scale (Mathur proposal)
- \exists scale (Planck scale or larger): if you coarse-grain above it \Rightarrow no distinction over microstates

PROPOSAL (V. Balasubramanian, V. Jejjala, J. de Boer, J.S.)

- i) Origin of gravitational thermodynamics \leftrightarrow difficulty/inability to tell apart different microstates (analogy with a gas)
 - ↳ gauge theory intuition (loop operators/polynomial of large invariant operator \sim identification of state?)
 - * Large charges are assumed (\sim large degeneracy) \Rightarrow statistical mechanics/information theory to describe "typical state"
(TOOL; generic at this point)
- ii) Each microstate \rightarrow "foamish" description at a given scale (l_p or another \sim dependence of charges)
 - ↳ by coarse-graining system ($L \gg l_p$), we lose the ability to identify uniquely the state \rightarrow emergence of semiclassical spacetime
 - ↳ we have access to "typical" properties (charges, multipole moments) \rightarrow "typical" state \rightarrow singular spacetime
 - * Singular classical spacetimes emerge as "geometrical averages" after coarse-graining details over l_p scale.

What shall we do?

- Embed our ideas into string theory: AdS/CFT ~~etc~~
- Find a subsector of the theory with enough symmetry so that BQFT, field theory and gravity are under control
- Put such systems into test
- Check our philosophy.

(*) Emphasize: dual field theory provides the tools/variables to identify "the atoms of spacetime" in this particular construction. In general, we do lack such description [in particular, for Big Bang physics]

- Focus on extremal Reissner-Nordström black holes in $AdS_2 \times S^2$ (simple Reissner-Nordström):

$$\Delta = M \cdot L = \frac{1}{2} N^2 w \quad w = g_s / L^2$$

$$N_c = N \cdot w \quad (\neq \text{giant gravitons})$$

SET-UP

• We shall consider $\frac{1}{2}$ BPS vector ($\Delta=R$) of $\mathcal{N}=4$ SYM

Field Theory Considerations (RS^3)

Absence of angular momentum ($J=\vec{J}=0$) \Rightarrow zero modes on S^3 } $\Pi(\text{Tr } X^k)^j \leftrightarrow \mathcal{B}_{[0, k, 0]}(0,0)$ representation
 Simple R-charge $\Rightarrow X, \bar{X}$ (no Y, Z excitations)
 \Downarrow reduction of $\mathcal{N}=4$ SYM on S^3 \Downarrow microscopic counting: partition problem of Δ boxes
 matrix quantum mechanics: one single complex matrix + $\Delta=R$ constraint \Rightarrow real matrix quantum mechanics

Degrees of freedom: eigenvalues of the matrix $\{ \lambda_i \}$ $i=1, \dots, N$

\hookrightarrow Van der Monde determinant: N fermions in an harmonic potential \Rightarrow spectrum, wavefunctions, ... fully known

Characterization of states: $E_i = \epsilon_i \hbar + \frac{\hbar}{2}$ ($w=1$) (single particle spectrum)

$E_i^0 = (i-1)\hbar + \frac{\hbar}{2}$ (ground state energies; Pauli exclusion principle; $i=1, \dots, N$)

\Downarrow

$$\boxed{r_i = \frac{1}{\hbar}(E_i - E_i^0) = \epsilon_i - i + 1} \quad (\text{excitation of particle } i \text{ over ground state})$$

$\{r_i\}$ non-decreasing int of integers \iff Young diagram



Useful variables.

$$c_N = r_1 ; \quad c_{N-i} = r_{i+1} - r_i \quad i=1, \dots, (N-1) \quad c_j \equiv \# \text{ cols. of length } j \quad (\text{maximum length is } N!!)$$

$$r_{i+1} = \epsilon_{i+1} - i = c_{N-i} + \dots + c_N$$

Physical interpretation

i) Columns \iff giant gravitons (gravitons with large momentum, due to Myers effect, blow up into spherical D3-branes spinning on $S^2 \iff$ R-charge)

\hookrightarrow They carry a maximum angular momentum of N , which matches the maximum length (# boxes) of a given column

(*) # columns = # giant gravitons (important later)

ii) Rows \iff dual giant gravitons

Gravity considerations

We are looking for asymptotically $AdS_5 \times S^5$ with $S^3 \times S^3 \times \mathbb{R}$ global symmetry and 16 supercharges
 $\begin{matrix} S^3 & \times & S^3 & \times & \mathbb{R} \\ \downarrow & & \downarrow & & \downarrow \\ AdS_5 & & S^5 & & \Rightarrow \{y, x^i, x^t\} \text{ dimensions} \end{matrix}$

- Assume $\bar{\Phi} = \bar{\Phi}_0$ (constant dilaton), no 2-form flux and "standard" RR 5-form flux (to stabilize $AdS_5 \times S^5$ itself)
 ↳ This matches with previous for non-extremal R-charge black holes

This problem was fully solved by LM:

$$g = -h^2 (dt + v_i dx^i)^2 + h^2 (dy^2 + dx^i dx^i) + \eta e^{\phi} g_{S^3} + \eta e^{-\phi} g_{S^3} ; \quad \eta^2 \equiv \text{product of radii of time sphere}$$

$$z = \frac{1}{2} \tanh \phi$$

Solution (full) is characterized by a single scalar function $z = z(y, x^i, x^t)$ satisfying a linear diff. equation

$$\partial_i \partial^i z + \eta^2 \partial_y \left(\frac{\partial_y z}{\eta} \right) = 0 \quad i=1,2$$

(*) $\bar{\Phi}(y; x^i, x^t) = z \cdot \eta^{-2}$ satisfies a Laplace eq. for an electrostatic potential in 6 dimensions being spherically symmetric in 4 directions: $\mathbb{R}^6 = \mathbb{R}^4 \times \mathbb{R}^2 \mapsto \bar{\Phi}(y; x^i, x^t)$ and is characterized by a boundary condition at $y=0$

$$z(y; x^i, x^t) = \frac{\eta^2}{\pi} \int \frac{dx^i dx^t}{[(x-x^i)^2 + \eta^2]^2} z(0; x^i, x^t)$$

Comments:

1. A given supergravity configuration $\iff z(0; x^i, x^t) \equiv$ charge distribution in (x^i, x^t) plane

2. As $\eta \rightarrow 0$, S^3 or S^5 (or both) shrink to zero size: singularities?

↳ LM concluded that only regular boundary conditions correspond to $\begin{cases} +1/2 & S^3 \hookrightarrow S^5 \text{ zero size} \\ -1/2 & S^3 \hookrightarrow AdS^5 \text{ zero size} \end{cases}$

3. Charges:

$$\Delta = R = \int_D \frac{d^2x}{2\pi k} \frac{1}{2} \frac{x^t + x^t}{k} = \frac{1}{2} \left(\int_D \frac{d^2x}{2\pi k} \right)^2$$

$D \equiv$ droplet (region) in x^i-x^t plane where $z = -1/2$.

$k = 2\pi R^4$ [it has L^4 units because both x^i, x^t have L^4 units]

Quantization of flux: $N = \int_D \frac{d^2x}{2\pi k}$

Introduce: $u(0; x^i, x^t) = \frac{1}{2} - z(0; x^i, x^t) \implies$

$$\Delta = \int_{\mathbb{R}^2} \frac{dx^i dx^t}{2\pi k} \frac{(x^t)^2 + (x^i)^2}{2k} u(0; x^i, x^t) - \frac{1}{2} N^2$$

i) Integral over (x^i, x^t) of Hamiltonian ($\frac{1}{2} \dot{q}^2 + \frac{1}{2} p^2 \sim$ harmonic oscillator) minus vacuum energy $\frac{1}{2} N^2$.

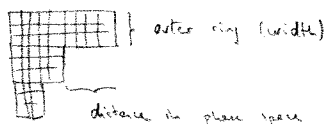
ii) Suggest to interpret $u(0; x^i, x^t)$ as a "density" (Peter phase space density function)

4. Examples:

Vacuum: disk \iff absence of diagram

$U(1)$ invariant states: bands of chips \iff

Non- $U(1)$ invariant states



Comment for talk:

i) Keep metric LM, Δ formula on the blackboard [leave space to write separator and compare]

TYPICAL STATES

General Philosophy.

Large $N \sim$ large degeneracy \Rightarrow canonical ensemble [fix β so that $\langle E \rangle = \Delta$] (statistical mechanics in general)

$\langle c_i \rangle, \langle c_j \rangle \rightarrow$ averaged tableau/diagram (state) \rightarrow average state / typical state (overwhelming many of them will lie close to that state)
 fluctuations \rightarrow deviation from typical state

Comment: large degeneracy (scaling with N) does NOT guarantee a MACROSCOPIC gravitational horizon.

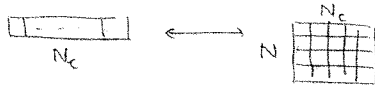
Different ensembles: consider system with fixed # fermion (rows)

i) Unfixed energies: # columns is not constrained

Averaged energy grows without bound as the temperature grows [$S > 0, E > 0, T > 0$]

ii) Fixing # columns = fixing # giant positions [relevant to discuss the physics of extremal R-charge black holes]

$$\Delta \in [N_c, N \cdot N_c]$$



$$S(\Delta = N_c) = S(\Delta = N \cdot N_c) = 0 \quad (1 \text{ microstate!})$$

iii) As Δ increases from N_c , entropy increases, heating up system: $T > 0$ branch

But as $\Delta \rightarrow N_c(N+1)/2$ [half filled diagram], entropy should reach a maximum ($T \rightarrow \infty$)

iii) As Δ continues to grow, S decreases: $T < 0$ branch, and decreases in absolute value.

This physical picture is confirmed by the 1-1 map between Young diagrams with Δ and $N_c(N+1) - \Delta$

$$S(\Delta) = S(N_c(N+1) - \Delta)$$

Details

$$Z = \sum_{c_1, \dots, c_N} e^{-\beta \sum_j c_j - \lambda (\sum_j c_j - N_c)}$$

where λ Lagrange multiplier:

$$\sum_j c_j = N_c$$

$$Z = \zeta^{-N_c} \prod_{j=1}^N \frac{1}{1 - \zeta q^j} \quad q = e^{-\beta}; \quad \zeta = e^{-\lambda}$$

$$\langle E \rangle = \Delta = q \partial_q \log Z = \sum_{j=1}^N \frac{j \zeta q^j}{1 - \zeta q^j}$$

$$N_c = \sum_{j=1}^N \langle c_j \rangle = \sum_{j=1}^N \frac{\zeta q^j}{1 - \zeta q^j}$$

These are the equations fixing β, λ as a function of Δ, N_c (data from physics) !!

Checks

Low temperature: large β

$$1. T > 0 \Rightarrow q = e^{-\beta} \ll 1 \Rightarrow \Delta \sim \frac{\zeta q}{1 - \zeta q}; \quad N_c \sim \frac{\zeta q}{1 - \zeta q} \Rightarrow \text{single row } (\Delta = N_c)$$

$$2. T < 0 \Rightarrow q = e^{-\beta} \gg 1 \Rightarrow N_c \sim \langle c_N \rangle = \frac{\zeta q^N}{1 - \zeta q^N} \quad (\text{rectangular diagram})$$

$$\Delta = \sum_{s=1}^{\infty} \zeta^s \sum_{j=1}^N j q^{sj} = \sum_{s=1}^{\infty} \zeta^s \left\{ q^s \frac{1 - q^{s(N+1)}}{1 - q^s} + q^{2s} \frac{1 - Nq^{(N-1)s} + (N-1)q^{Ns}}{(1 - q^s)^2} \right\} \sim N \sum_{s=1}^{\infty} (\zeta q^N)^s = N \cdot N_c$$

High temperature.

$$S = \beta \langle E \rangle + \log Z$$

Solving N_c constraint: $\zeta(N, N_c) = \frac{w}{1+w} \left(1 + \beta \frac{A_1}{N} + \beta^2 \frac{N+1}{2} [(w+2) - w(N-1)] \right) + O(\beta^3)$; $w = \frac{N_c}{N}$; $A_1 = \sum_{i=1}^N i = \frac{N(N+1)}{2}$

Solving $\langle E \rangle$ constraint: $\Delta = w A_1 - \beta \frac{A_1}{6} w(1+w)(N-1) + O(\beta^2)$

Comment: $\Delta = \frac{1}{2} N \cdot N_c + O(\beta) + O(N_c) \Rightarrow$ dominant contribution at large N and large temperature \Leftrightarrow superstar energy

$$S = -\log \left(\frac{w^{N_c}}{(1+w)^{N+N_c}} \right) - \beta^2 \frac{w(1+w)}{24} N(N^2-1) + O(\beta^4)$$

Thus, $\beta=0$ ($T \rightarrow \infty$) is indeed a maximum for the entropy.

Conjecture: $\beta \rightarrow 0$, large N limit describes typical states for the superstar (critical $\frac{1}{2}$ BTZ R-charged black hole)

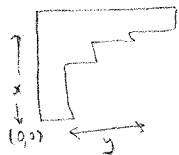
Check.

$$S \sim (\log \text{cont.}) \cdot \sqrt{\Delta} \stackrel{N_c=wN}{\sim} N \sim \frac{L^5 \rho_h^3}{G_N} \sim \frac{L^8}{L^2} \left(\frac{\rho_h}{L} \right)^3 \sim N^2 \left(\frac{\rho_h}{L} \right)^3 \Rightarrow \rho_h \sim \frac{L}{N^{1/3}} \Leftrightarrow \frac{\rho_h}{L} \sim N^{-1/3} = \frac{1}{N^{1/2}} \rightarrow 0$$

Even though entropy is maximum, degeneracy (wN) is NOT enough to generate a macroscopic horizon \sim superstar etc.

Limit Curve.

- Introduce two coordinates (x, y) to describe the diagram: x along the rows and y along the columns.



Consider: $N \rightarrow \infty$, N th fixed, receding diagram \Rightarrow continuum limit

- Notice that: $\rho_{ih} = E_{ih} - i = c_{N-i} + \dots + c_N \Leftrightarrow y(x) = \sum_{i=N-x}^N c_i$ (excitation of particle "x")

$$\langle y(x) \rangle = y(x) = \sum_{i=N-x}^N \langle c_i \rangle \sim \int_{N-x}^N di \langle c_i \rangle$$

$$\alpha q^{N-x} + \beta q^y = 1$$

$$\alpha = \frac{1-q^{N_c}}{1-q^{N+N_c}}; \quad \beta = \frac{1-q^N}{1-q^{N+N_c}}$$

- In particular: as $\beta \rightarrow 0$ (to check our conjecture) [Comment on triangular diagram]

$$y = \frac{N_c}{N} x = wx$$

MATCHING GRAVITY: SEMICLASSICAL CONSIDERATIONS

- LHM supported $(x', x'') \leftrightarrow (q, p)$ phase space of a single particle in the eigenvalue picture
- We shall give one step further:

$$\boxed{U(x', x'') = U_{\text{Wigner}}} \quad (\text{equality meaning up to normalisation and taking semiclassical limit appropriately})$$

WIGNER

$$W(\bar{q}; \bar{p}) = \frac{1}{(2\pi\hbar)^n} \int_{-\infty}^{\infty} d\bar{y} \langle \bar{q}-\bar{y} | \hat{\rho}_n | \bar{q}+\bar{y} \rangle e^{2i\bar{p}\bar{y}/\hbar} \quad (\text{definition, given a density matrix } \hat{\rho}_n)$$

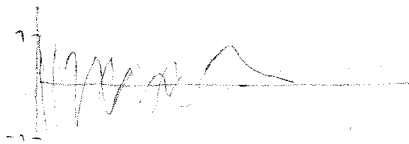
$$\langle \hat{A}_w \rangle = \text{tr}(\hat{\rho}_n \hat{A}_w(\bar{q}, \bar{p})) = \int d\bar{q} d\bar{p} A(\bar{q}, \bar{p}) W(\bar{q}, \bar{p}) \quad [\text{it computes expectation value of Weyl ordered operator}]$$

Properties:

- Though bounded, it oscillates rapidly in the classically allowed phase space, and decays exponentially outside.

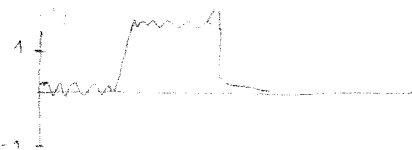
In our setup: (after integrating out $N-1$ particles !!)

$$W(q, p) = \frac{1}{\pi\hbar} e^{-i(p+q^2)/\hbar} \sum_{j \in \mathcal{J}} (-1)^j L_j\left(\frac{2}{\hbar}(p+q^2)\right) \quad \mathcal{J} \text{ set of excitation levels}$$



Main observation: oscillation have $\sim \hbar$ scale \Rightarrow very quantum mechanical sensitive quality
 & maximum at classical energy

- Even if we add the effect of many fermions, \exists quantum mechanical oscillation

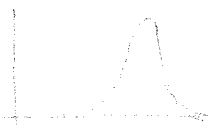


Suppression: coarse-grain Wigner at \hbar scale \Rightarrow Husimi distribution.

HUSIMI

$$H_{\text{us}}(q, p) = \frac{1}{2\pi\hbar} \int dq' dp' Z(q, p; q', p') W(q', p') \quad \text{where } Z = \exp\left[-\frac{1}{2\hbar} [(q-q')^2 + (p-p')^2]\right] \quad (\text{Gaussian kernel of variance } \hbar)$$

- It contains the same quantum mechanical information, but it computes normal ordered observables.



$$H_{\text{us}}(\zeta) \sim \frac{1}{2\pi\hbar} \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{(\zeta-n)^2}{2\hbar}} \quad \text{where } \zeta = (p^2 + q^2)/2\hbar$$

It is peaked at $\zeta = n$ (classical energy), with variance $\sim n \Rightarrow \sigma(\zeta) \sim \sqrt{n}$ (variance in radial variable)

\hookrightarrow it is a well-localised lump of energy as $\hbar \rightarrow 0$, BUT ---

$\sigma(\zeta) \sim \sqrt{n} \hbar \Rightarrow \Delta E = \hbar \Rightarrow$ localized ring does not capture physics. (classical limit)

GRAYSCALE DISTRIBUTION

- Assume \hbar semiclassical limit ($\hbar \rightarrow 0$, h fixed)
- In phase space, we want to coarse-grain at scales $\Delta E/\hbar \rightarrow \infty$

A natural coarse-grained observable would depend on the coarse-grained Husimi distribution:

$$R(\bar{E}, \Delta E) = 2\pi\hbar \int_E^{E+\Delta E} dg dp H(g,p) \sim \# \text{ fermions between } E \text{ and } E+\Delta E$$

Effective density:

$$\overline{g(E)} = 2\pi\hbar \left(\frac{R(\bar{E}, \Delta E)}{2\hbar \Delta E} \right) = \hbar \frac{\Delta x}{\Delta E} = \frac{\hbar}{\partial E/\partial x}$$

- If we assume \hbar semiclassical limit: $E = \hbar(x+y(x)) \Rightarrow \overline{g(E)} = \frac{1}{1+y'}$

phase space distribution in the semiclassical limit (derived from limit wave) equals the notion of integrating out smaller scales (grayscale)

Equivalently: $\frac{u_0(r^2)}{2\hbar} dr^2 = dx$ (# particle in dx , $r, r+dr$)
 $r^2 \frac{u_0(r^2)}{4\hbar^2} dr^2 = (y+x)dx$ (energy in next band)

$\Rightarrow y(x) + x = \frac{r^2}{2\hbar} \checkmark, \quad \left| u_0(r^2) = \frac{1}{1+y'} \right|$

MATCHING SUPERSTAR

- Gravity data: $g = -\frac{\delta}{H} \{ dt^2 + \frac{\sqrt{\delta}}{f} dr^2 + \sqrt{\delta} r^2 g_{S^2} + \sqrt{\delta} L^2 d\theta^2 + \frac{L^2}{\sqrt{\delta}} \sin^2 \theta g_{S^1} + \frac{H}{\sqrt{\delta}} \omega^2 [Ld\phi + (H^{-1}-1)dt]^2$

$H = 1 + \delta/r^2; \quad f = 1 + 2H/L^2; \quad \delta = 1 + \delta \sin^2 \theta / r^2$

Comparing sizes of S^1 's:

$$y e^b = \sqrt{\delta} r^2; \quad y e^{-b} = \frac{L^2 \sin^2 \theta}{\sqrt{\delta}} \Leftrightarrow y^2 = r^2 L^2 \sin^2 \theta; \quad e^b = \frac{r\sqrt{\delta}}{L \sin \theta} \Rightarrow z = \frac{1}{2} \frac{r^2 \delta - L^2 \sin^2 \theta}{r^2 \delta + L^2 \sin^2 \theta}$$

We want to compute $z(0)$:

$$y \rightarrow 0 \Rightarrow \begin{cases} \sin \theta = 0 \Rightarrow z = 1/2 \text{ (no } P\text{-brane excited!)} \\ r=0 \text{ (density of point does not vanish)} \Rightarrow \left| z(r=0) = \frac{1}{2} \frac{w-1}{w+1} \right| \text{ where } w = \delta/L^2 = N_c/N \end{cases}$$

MAGIC EXISTS... !!

Our field theory analysis + typical state analysis + proposal to match:

$$z = \frac{1}{2} - u = \frac{1}{2} - \frac{1}{1+w} = \frac{1}{2} - \frac{1}{1+w} = \frac{1}{2} \frac{(w-1)}{w+1}$$

MATCH!

LESSONS (interpretation)

- i) Simple microstates \rightarrow not classical geometry (smooth; topologically complex)
- ii) Typical microstates can not be told apart \rightarrow limit wave (difference integrated out) [origin of prethermal thermodynamics]
- iii) Emergent spacetime is singular

(*) Quantum mechanical correlation function: are they universal? (indistinguishability of parton microstates) [field theory side]

(**) Including parton effects in gravity (knowing higher scale details) [gravity side]

SCHWARZSCHILD ADS BLACK HOLES

- One may wonder whether the previous lesson could shed some light on the misconception of "real" black holes in AdS.

Gravity results.

$$r_h \sim l \Rightarrow \frac{r_h}{l_p} \sim N^{1/4} \text{ (macroscopic horizon)} ; \quad \Delta = M \cdot L \sim N^2$$
$$S \sim N^2$$

Field Theory expectations.

3 degeneracy of microstates: $|\Psi\rangle = |O\rangle$; $\Delta(O) = N^2$ (at large N)

↳ LONG OPERATORS \sim built of words (traces) made of strings of letters (fields) that belong to an alphabet ($U(N)$ SYM)

Again, the crucial notion is: typical operator \sim typical word

Large $N \Rightarrow$ use information theory (statistical mechanics) to determine/characterize "typical word"

Information Theory (Sanov's Theorem)

typical word \sim statistical randomness of the presence of fields in the long polynomial

$$P(x) = \frac{1}{k} \quad k = \# \text{ alphabet} \quad \text{and} \quad x = \text{field (letter)}$$

The probability that a given distribution differs from $P(x)$ is exponentially suppressed in the unfermed dimension.

Caveats

- i) Non-BPS operators ($X\bar{X}$ are allowed): Δ renormalizes
 - ii) Mixing of operators
- vs. $\left. \begin{array}{l} \text{Coupling (difference between a pair of particles and a} \\ \text{black hole)} \\ \text{(strong-weak AdS/CFT duality)} \end{array} \right\}$
- Correspondence principle (classical results)

Our proposal.

Correlation functions of probe operators O_p computed in very heavy states $|\Psi\rangle$ depend only on Δ and global charges up to corrections of $O(e^{-\Delta})$, for almost all states $|\Psi\rangle$ and probe O_p .

- In free field theory, the contribution from planar diagrams is controlled by the number of times the probe operator appears within the state (pattern matching) \Rightarrow this number is mostly determined by the statistics of long random polynomials.

FUTURE DIRECTIONS

1. Loss of information (N particle \rightarrow 1 particle phase space); gravitational counterpart (α' effects??)
 - universality of correlation functions (proton version for origin of thermodynamics)
 - non-UM invariant states; non-pure states \rightarrow identification of quantum microstates \sim lens
2. Non-extremal supersets: perturbative expansion (MCCG) \rightarrow connection to Schw. black hole physics
Toy model: throw q and perturbations $(\hbar \bar{d}\bar{d}), (\hbar \bar{X}\bar{X})$ in large BPS operator (pants)
3. Cosmological singularity: is coarse-graining over Planck ^{time} scales responsible for the existence (classical) of the Big Bang singularity?
 - \hookrightarrow relevance of coupling effects, addition of degrees of freedom (*)

(*) In our talk, there was no need for adding those since we knew we had all of them included!