Exploring Type II Flux Vacua:
SUSY, Non-SUSY, and Non-geometric

University of Wisconsin, Madison
September 2005
W. Taylor (MIT)

hep-th/0505160 (w/ O. De Wolfe, A. Giryavets, S. Kachru)

hep-th/0508133 (w/ J. Shelton, B. Wecht)

Outline

1. Introduction/Motivation
2. IIB vacua
3. IIA vacua
4. Synthesis: non-geometric compactifications
5. Summary + open questions

1. Introduction/Motivation

Type IIA/IIB string compactification:

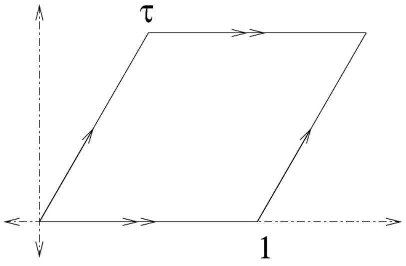
$$X_6 \longrightarrow M_{10}$$

$$M_4$$

SUSY, no fluxes: $X_6 =$ Calabi-Yau, $M_4 = \mathbb{R}^4$

Generic Calabi-Yau: Moduli

Example: $(T^2)^3$ in type IIB



Complex structure: τ

Kähler modulus: $U = B_{xy} + i \times \text{volume}$

Axiodilaton: $S = \chi + ie^{-\phi}$

Moduli space: manifold of SUSY vacua

Moduli stabilization

Turn on integrally quantized (topological) fluxes

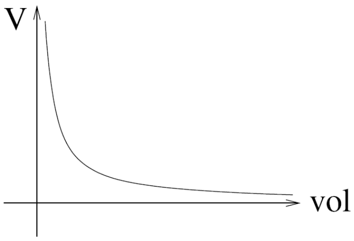
$$H_{abc}, \qquad F^{(p)}_{a_1\cdots a_p}$$

$\Rightarrow$ 4D potential

$$V \sim \int_{M_6} \sqrt{g}\,\left(e^{-2\phi}|H|^2 + |F|^2 + \cdots\right)$$

is Moduli dependent

Problems:



A) Runaway moduli ($V \sim H^2/\text{volume}^2$)

B) Tadpoles

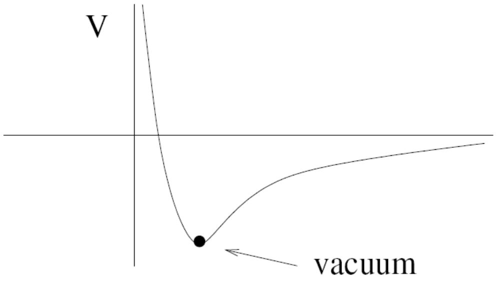
e.g., $\int A_4 \wedge F_3 \wedge H_3$ in IIB Chern-Simons term

$\Rightarrow \nabla_6^2 A_4 \sim F_3 \wedge H_3$

One solution: Orientifold planes

$$T_{O_p} < 0, \qquad D{-}\text{charge}(O_p) < 0$$

Fluxes + O-planes → moduli stabilization



Goal: Study “landscape” of string vacua

Motivations:

- May connect to phenomenology
- May connect to cosmology
- May shed light on foundational aspects of string theory

“Anthropic”/environmental selection issues of limited practical consequence without a better global picture of range of possible vacua, some dynamical principle/definition of string theory

Summary of talk:

- We know of many flux vacua
- There probably exist many many more

2. Type IIB flux vacua

Consider integral (topological) IIB fluxes: H_{abc}, F_{abc}

Two ways to study: (Giddings-Kachru-Polchinski, . . .)

A) 10D SUGRA $S \to$ 4D potential $V(\mathrm{moduli})$

B) Superpotential W for 4D SUGRA (Gukov-Vafa-Witten)

Begin with A:

$$S = \int \sqrt{g} \left(e^{-2\phi}(R + (\partial\phi)^2 - |H|^2) - \sum_p |F^{(p)}|^2 \right)$$

$$-A_4 \wedge H_3 \wedge F_3 + \delta^{(6)}_{\mathrm{D3,O3}}(T_{\mathrm{D3,O3}} - A_4)$$

A_4 tadpole cancellation:

$$N_{D3} + \int F_3 \wedge H_3 = N_{O3}$$

Varying zero-modes (moduli) gives

$$S \to V(\mathrm{moduli})$$

where zero-modes of ϕ, B, g, A^{p-1} are moduli

B) Analysis using 4D superpotential

Potential can be written

$$V = e^K (DW\overline{DW} - 3|W|^2)$$

where

$$DW = \partial W + (\partial K)W$$

and

$$W = \int G \wedge \Omega \quad \text{(GVW)}$$

(depends only on CS moduli, axiodilaton S)

"no-scale" dependence on Kähler moduli:

$$D_K W \overline{D_K W} = 3|W|^2$$

gives

$$V = e^K (D_{\rm CS} W \overline{D_{\rm CS} W})$$

SUSY solutions: $DW = W = 0$

Summary of IIB vacuum analysis to date

- Equations of motion $\partial_{\mathrm{moduli}} V = 0, DW = 0$

  $\Rightarrow$ some moduli stabilized

- Potential can be written

$$V \sim \frac{|iG^{(3)} - {}^*G^{(3)}|^2}{\mathrm{vol}^2} + \cdots$$

where $G^{(3)} = F^{(3)} - SH^{(3)}$. $iG^{(3)} = {}^*G^{(3)} \Leftrightarrow$ ISD.

- Generically stabilizes complex structure moduli, S

- SUSY $DW = 0$ solutions ISD, $V = 0$, $M_4 = \mathbb{R}^4$

- Kähler moduli only stabilized nonperturbatively

  (Denef/Douglas/Florea/Grassi/Kachru)

- Tadpole constraint + ISD

  $\Rightarrow$ finite # of inequivalent solutions

- Statistical analysis of IIB vacua begun

  (Douglas, Ashok/Douglas, Denef/Douglas, DGKT, . . .)

3. Type IIA flux vacua

Can have fluxes H_3, F_6, F_4, F_2, F_0 (massive IIA)

Use Orientifold 6-plane to cancel A_7 tadpole

$$F_0 H_3 + N_{D6} = N_{O6}$$

Both analysis methods again possible.

A) Explicit computation of 4D potential V

$$V \sim e^{2\phi} \frac{H^2}{\mathrm{vol}^2} + e^{4\phi} \frac{F_4^2}{\mathrm{vol}^{7/3}} + e^{4\phi} \frac{F_0^2}{\mathrm{vol}} - e^{3\phi} \frac{O_6}{\mathrm{vol}^{3/2}} + \cdots$$

Note: volume dependence allows Kähler stabilization

B) Superpotential formalism (Grimm/Louis)

$$W^Q = \int \Omega_c \wedge H_3$$

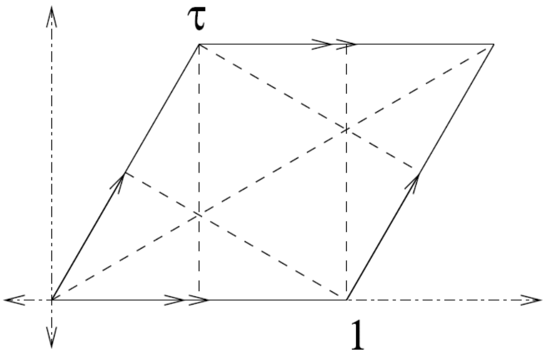
$$W^K = \int J_c \wedge F_4 - \frac{F_0}{6} \int J_c \wedge J_c \wedge J_c + \cdots$$

Summary of IIA vacuum analysis

- Kähler moduli generically stabilized
- Some models: all moduli stabilized
  (DGKT example: $T^6/\mathbb{Z}_3^2$)
- Other models: unstabilized axions
  — needed to cancel anomaly on branes
  (Camara/Font/Ibañez)
- F_4 unconstrained by tadpole $\Rightarrow$ ∞ # of vacua
- No no-scale structure: for SUSY $DW = 0$ vacua
  $W = 0 \Rightarrow$ Minkowski, $W \neq 0 \Rightarrow$ AdS$_4$
- Exist controlled families of vacua,
  $g \to 0, \text{volume} \to \infty$
- non-SUSY vacua exist in controlled regime
  SUSY breaking from flux sign change

Simple example of IIA vacua: $T^6/\mathbb{Z}_3^2$

Consider $(T^2)^3$ with $\tau = e^{2\pi i/3}$



Mod out by

$$T : (z_1, z_2, z_3) \rightarrow (\alpha^2 z_1, \alpha^2 z_2, \alpha^2 z_3)$$

$$Q : (z_1, z_2, z_3) \rightarrow (\alpha^2 z_1 + \frac{1+\alpha}{3},\ \alpha^4 z_2 + \frac{1+\alpha}{3},\ z_3 + \frac{1+\alpha}{3})$$

Singular limit of CY, $\chi = 24$, 9 $\mathbb{Z}_3$ singularities

$$h^{2,1} = 0, \qquad h^{1,1} = 12$$

3 Kähler moduli from tori, 9 from singularities

Orientifold: fixed plane of

$$\sigma :\ z_i \rightarrow -\bar{z}_i$$

Holomorphic 3-form

$$\Omega = \frac{i}{3^{1/4}}\, dz_1 \wedge dz_2 \wedge dz_3 = \frac{1}{\sqrt{2}}\, (\alpha_0 + i\, \beta_0)$$

Moduli of $T^6/\mathbb{Z}_3^2$ model:

$$A_{(3)} = \xi\,\alpha_0, \qquad \phi \qquad (\text{axion}, \text{dilaton})$$

$$ds^2 = \sum_{i=1}^{3} \gamma_i\, dz^i d\bar{z}^i$$

$$B_2 = \sum_{i=1}^{3} \beta_i\; dz^i \wedge d\bar{z}^i$$

Metric, B-field components γ_i $\beta_i \Rightarrow$ 3 Kähler moduli

Remaining 9 Kähler moduli from blow-up modes.

Fluxes (quantized):

$$H_3^{\text{bg}} = -p\,\beta_0$$

$$F_4^{\text{bg}} = \text{constant}\left(e_1\; dz^2 \wedge d\bar{z}^2 \wedge dz^3 \wedge d\bar{z}^3 + \text{cyclic}\right)$$

Tadpole condition $m_0 p = -2(2\pi\sqrt{\alpha'})$

No tadpole constraint on e_i.

Can explicitly solve EOM to get

$$B = A_{(3)} = 0$$

Potential ($v_i = \text{constant} \times \gamma_i, \phi$)

$$V = \frac{1}{2} p^2 \frac{e^{2\phi}}{\text{vol}^2} + (\sum_{i=1}^{3} e_i^2 v_i^2) \frac{e^{4\phi}}{\text{vol}^3} + m_0^2 \frac{e^{4\phi}}{\text{vol}} + 2\sqrt{2} m_0 p \frac{e^{3\phi}}{\text{vol}^{3/2}}$$

(vol $=$ constant $\times \gamma_1 \gamma_2 \gamma_3$)

Solving

$$ds^2 = \left(\frac{1}{9\kappa}\right)^{1/6} \sqrt{5 \left|\frac{e_1 e_2 e_3}{m_0}\right|} \sum_{i=1}^{3} \frac{1}{|e_i|} dz^i d\bar{z}^i ,$$

$$e^{\phi} = \frac{3}{4} |p| \left(\frac{5}{12} \frac{\kappa}{|m_0 e_1 e_2 e_3|}\right)^{1/4} .$$

Scaling of solutions for large $e_i \sim E$:

$$\text{vol} \sim E^{3/2}$$

$$e^{\phi} \sim E^{-3/4}$$

$$\Lambda \sim E^{-9/2}$$

$$HR \sim E^{-1/2}$$

So solutions are parametrically under control

Further comments on solutions

- SUSY solutions: all e_i have same sign
  - other signs: non-SUSY controlled solutions
  - $\sim$ skew-whiffing (Duff/Nillson/Pope)

- Can check B-mode stability
  - SUSY solutions: all modes stable
  - non-SUSY solutions: BF-allowed tachyons

- Can stabilize blow-up modes with additional F_4 fluxes
  - can choose in regime where blow-up modes $\ll R$

- Number of vacua with $R \leq R^*$ goes as $(R^*)^4$
  - cutoff dominated

- Expect similar results for other models
  - some axions not stabilized, fix anomalies (CFI)

4. Synthesis: non-geometric vacua

Upshot so far: IIA, IIB vacua seem very different

But mirror symmetry: IIA $\leftrightarrow$ IIB??

Reconciliation: non-geometric fluxes

Example: Consider T^3 with $B_{xy} = Nz \Rightarrow H_{xyz} = N$ flux

T-duality T_x: "geometric flux" f^x_{yz}

$$ds^2 = (dx + f^x_{yz} z dy)^2 + dy^2 + dz^2$$

(twisted tori: Scherk/Schwarz, Kaloper/Myers, . . . ;
SU(3) structure: Hitchin, Gurrieri/Louis/Micu/Waldram, . . .)

T_y: "non-geometric flux" Q^{xy}_z
Locally geometric T^2 bundle over T^1, duality twist in BC's

$$\begin{aligned} ds^2 &= \frac{1}{1+N^2z^2}\left(dx^2 + dy^2\right) + dz^2 \\ B_{xy} &= \frac{Nz}{1+N^2z^2}. \end{aligned}$$

(Dabholkar/Hull, Hellerman/McGreevy/Williams,
Flourney/Wecht/Williams, . . .)

T_z: more non-geometric flux R^{xyz}; not yet understood

T-duality rules for NS-NS fluxes

$$H_{abc} \xleftrightarrow{T_a} f^a_{bc} \xleftrightarrow{T_b} Q^{ab}_c \xleftrightarrow{T_c} R^{abc}$$

Like T-duality rules for R-R fluxes

$$F_{x\alpha_1\cdots\alpha_p} \xleftrightarrow{T_x} F_{\alpha_1\cdots\alpha_p}$$

Generalize Buscher rules to include 0-forms

Example: $T^6 = (T^2)^3$ in IIA, IIB

- Duality $\Rightarrow$ superpotential, constraints
- Demonstrates consistency of NG fluxes

moduli	IIB	IIA
τ	CS	Kähler
S	axiodilaton	axiodilaton
U	Kähler	CS

Previously known flux superpotentials

IIB: $W = P_1^{(3)}(\tau) + SP_2^{(3)}(\tau)$
(geometric, coefficients F, H)

IIA: $W = P_1^{(3)}(\tau) + SP_2^{(1)}(\tau) + UP_3^{(1)}(\tau)$
(w/ geometric flux; Villadoro/Zwirner, Camara/Font/Ibanez)

Claim: full IIA/IIB superpotential is

$$W = P_1^{(3)}(\tau) + SP_2^{(3)}(\tau) + UP_3^{(3)}(\tau)$$

coefficients: NS-NS fluxes $H_{abc}, f^a_{bc}, Q^{ab}_c, R^{abc}$

Explicit construction (O6 on α, β, γ)

Term	IIA flux	IIB flux
1	$\bar{F}_{\alpha i \beta j \gamma k}$	$\bar{F}_{ijk}$
τ	$\bar{F}_{\alpha i \beta j}$	$\bar{F}_{ij\gamma}$
τ^2	$\bar{F}_{\alpha i}$	$\bar{F}_{i\beta\gamma}$
τ^3	$F^{(0)}$	$\bar{F}_{\alpha\beta\gamma}$
S	$\bar{H}_{ijk}$	$\bar{H}_{ijk}$
U	$\bar{H}_{\alpha\beta k}$	$Q^{\alpha\beta}_k$
$S\tau$	f^{α}_{jk}	$\bar{H}_{\alpha jk}$
$U\tau$	$f^j_{k\alpha}, f^i_{\beta k}, f^{\alpha}_{\beta\gamma}$	$Q^{\alpha j}_k, Q^{i\beta}_k, Q^{\beta\gamma}_{\alpha}$
$S\tau^2$	$Q^{\alpha\beta}_k$	$\bar{H}_{i\beta\gamma}$
$U\tau^2$	$Q^{\gamma i}_{\beta}, Q^{i\beta}_{\gamma}, Q^{ij}_k$	$Q^{i\beta}_{\gamma}, Q^{\gamma i}_{\beta}, Q^{ij}_k$
$S\tau^3$	$R^{\alpha\beta\gamma}$	$\bar{H}_{\alpha\beta\gamma}$
$U\tau^3$	$R^{ij\gamma}$	Q^{ij}_{γ}

Black: already known; Blue: T-dual of black

Green: rotation of blue; Purple: T-dual of Green

Use T-duality to find (Bianchi/tadpole) constraints

NS-NS constraints ($\sim \int dH = 0$)

$$
\begin{aligned}
\bar{H}_{x[ab} f^{x}_{cd]} &= 0 \\
f^{a}_{x[b} f^{x}_{cd]} + \bar{H}_{x[bc} Q^{ax}_{d]} &= 0 \\
Q^{[ab]}_{x} f^{x}_{[cd]} - 4 f^{[a}_{x[c} Q^{b]x}_{d]} + \bar{H}_{x[cd]} R^{[ab]x} &= 0 \\
Q^{[ab}_{x} Q^{c]x}_{d} + f^{[a}_{xd} R^{bc]x} &= 0 \\
Q^{[ab}_{x} R^{cd]x} &= 0.
\end{aligned}
$$

R-R constraints ($\sim \int (d+H)F = 0$)

$$
\begin{aligned}
\bar{F}_{[abc} \bar{H}_{def]} &= 0 \\
\bar{F}_{x[abc} f^{x}_{de]} - \bar{F}_{[ab} \bar{H}_{cde]} &= 0 \\
\bar{F}_{xy[abc} Q^{xy}_{d]} - 3 \bar{F}_{x[ab} f^{x}_{cd]} - 2 \bar{F}_{[a} \bar{H}_{bcd]} &= 0 \\
\bar{F}_{xyz[abc]} R^{xyz} - 9 \bar{F}_{xy[ab} Q^{xy}_{c]} & \\
-18 \bar{F}_{x[a} f^{x}_{bc]} + 6 F^{(0)} \bar{H}_{[abc]} &= 0 \\
\bar{F}_{xyz[ab]} R^{xyz} + 6 \bar{F}_{xy[a} Q^{xy}_{b]} - 6 \bar{F}_{x} f^{x}_{[ab]} &= 0 \\
\bar{F}_{xyza} R^{xyz} - 3 \bar{F}_{xy} Q^{xy}_{a} &= 0 \\
\bar{F}_{xyz} R^{xyz} &= 0.
\end{aligned}
$$

Comments on NG flux compactification

- Constructed explicit superpotential, constraints for T^6
- Can solve $DW = 0$ to find SUSY vacua
- Explicitly T-duality invariant, IIA vacua $=$ IIB vacua
- Non-geometric Q^{ab}_c explicit through T-duality
- NS-NS NG 0-form fluxes R^{abc} needed for completeness
- Generic vacua may be string scale
- May need new methods (beyond SUGRA) for analyzing
- Generic vacua may be non-geometric in any duality frame

Crude estimate for vacua satisfying physical constraints

$$N_{\rm vac} \sim e^{N_F + N_H + N_f + N_Q + N_R - {\rm constraints}}$$

may take $\sim 10^{500}$ geometric vacua $\rightarrow \sim 10^{2000}$ NG vacua

- Need to generalize to mirror symmetry on general CY using (Strominger-Yau-Zaslow) T-duality on T^3 fiber (generalization of GLMW to nongeometric spaces)

5. Summary + open questions

Summary of results

- IIB vacua: tadpole + SUSY EOM $\Rightarrow$ finite # solutions
- IIA vacua: unconstrained F_4 flux $\Rightarrow \infty$ solutions
- IIA vacua: all moduli can be stabilized classically
- IIA vacua: vacua with parametric control
- IIA vacua: SUSY breaking from flux choice
- nongeometric fluxes: unify IIA and IIB pictures
- nongeometric fluxes: new compactification structures Q, R
- nongeometric fluxes: may be generic

Open questions

- Generalize NG fluxes to general Calabi-Yau
- Understand generic type II flux compactifications
- Understand perturbative + nonperturbative corrections
- Develop string description of NG fluxes, particularly R^{abc}
- Understand SUSY breaking in IIA vacua
- Understand S-duality of NG fluxes