Generalized $N = 1$ orientifold compactifications

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based on: [hep-th/0602241] Iman Benmachiche, TWG
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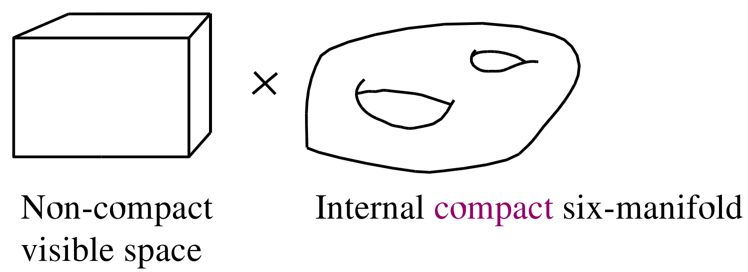
Introduction and Motivation

➪ Phenomenology

String Theory $\longrightarrow$ Four-dimensional $N = 1$ Supergravity



- Address moduli problem: generate potential for massless scalar fields due to background fluxes and non-Calabi-Yau geometries

- Explore generic features of $N = 1$ compactifications – Landscape of String vacua?

- Complete dualities in the presence of background flux

- Four-dimensional gauge theory and specific models

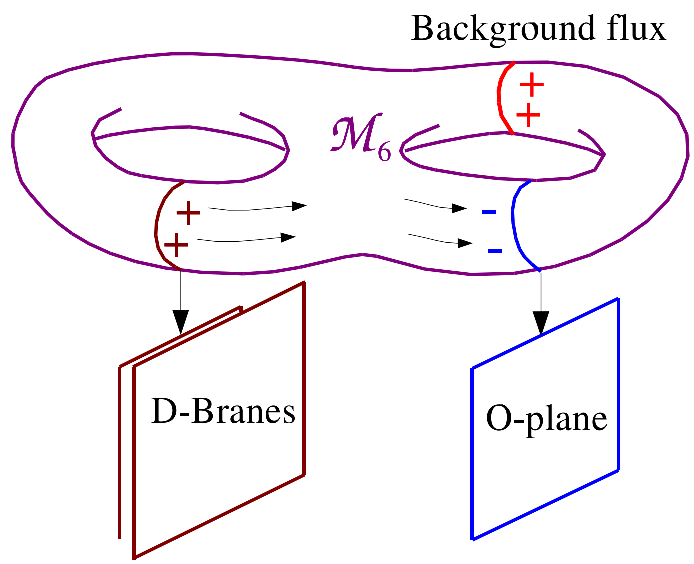
➪ A realization in Type II String theory:

- minimal supersymmetry: background $M_{1,3} \times \mathcal{M}_6$
  $\mathcal{M}_6$ – special manifold

- moduli stabilization:
  background fluxes
  non-Calabi-Yau geometry

- non-Abelian gauge groups:
  space-time filling D-branes

$\Rightarrow$ consistency: orientifold planes



Outline of the Talk

- Conditions on supersymmetric theories and vacua
- Some basics on generalized complex geometry
- Orientifold projection and $N = 1$ spectrum
- $N = 1$ chiral field space: Kähler potential and superpotential
- Mirror symmetry / T-duality with fluxes: a conjecture

Conditions on supersymmetric theories and vacua

➪ Condition for supersymmetric $D = 4$ theory: (Type IIB example)

- metric ansatz: $$ds^2_{10} = e^{A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + g_{ij}(y)\, dy^i dy^j$$

- Two ten-dimensional gravitinos $\Psi^1_{\mathcal{M}}$, $\Psi^2_{\mathcal{M}}$ decompose as:

  $\Psi^i_\mu = \psi^i_{+\mu} \otimes \eta^i_+ + \psi^i_{-\mu} \otimes \eta^i_-$ ψ^1_μ, ψ^2_μ become four-dimensional gravitinos

  $\Psi^i_m =$ four-dimensional fermionic modes for other multiplets

Effective four-dimensional theory possesses $N = 2$ susy $\Rightarrow$ two globally defined spinors η^1 and η^2 on $\mathcal{M}_6$ exist, they locally or globally coincide

Graña,Louis,Waldram

- η^1, η^2 reduce structure group $SO(6)$ of $T\mathcal{M}_6$: $\mathbf{4} \rightarrow \mathbf{1} + \mathbf{3}$

$$\eta^1 : \quad SO(6) \hookleftarrow SU(3)_1$$
$$\eta^2 : \quad SO(6) \hookleftarrow SU(3)_2$$

➪ Supersymmetry conditions on $D = 10$ flux-background

$\Rightarrow$ differential conditions relating η^1 and η^2 to the NS-NS and R-R background flux

Behrndt,Cvetic,Liu; Lüst,Tsimpis; Witt; Graña,Minasian,Petrini,Tomasiello

- vanishing of the gravitino and dilatino variations:

$$\delta\psi^i_M = \nabla_M \epsilon^i + (\text{Flux})_M \epsilon^i = 0 \qquad \delta\lambda^i = (\partial\phi)\ \epsilon^i + (\text{Flux})\epsilon^i = 0$$

- Susy spinors ϵ^i decompose as: $\epsilon^i = \zeta^i_+ \otimes \eta^i_+ + \zeta^i_- \otimes \eta^i_-$ (Type IIB example)

- Susy conditions imply: $\nabla_m \eta^1 = (\text{Flux})^1_m, \quad \nabla_m \eta^2 = (\text{Flux})^2_m$

Examples for $\mathcal{M}_6$:

– Calabi-Yau manifolds: $\eta = \eta^1 = \eta^2$ and $\nabla_m \eta = 0$

– Manifolds with $SU(3)$ structure: $\eta = \eta^1 = \eta^2$, but $\nabla_m \eta \neq 0$

– Manifolds with $\eta^1 = \eta^2$ only locally: manifolds with $SU(3) \times SU(3)$ structure

new framework: **Generalized complex geometry** Hitchin,Gualtieri,Witt

Some basics on generalized complex geometry

➪ Central object: $T \oplus T^* \equiv T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$ generalized tangent bundle

- $T \oplus T^*$ takes the role of tangent bundle T in standard geometry:

recall: metric g_{ij} on $\mathcal{M}_6$ is a map $T \times T \to \mathbb{R}$
almost complex structure I_i^j on $\mathcal{M}_6$ defines a split $T_\mathbb{C} = T^{(1,0)} \oplus T^{(0,1)}$

$\Rightarrow$ generalized metric G_{IJ} as a map $(T \oplus T^*) \times (T \oplus T^*) \to \mathbb{R}$

$$G_{IJ} = \begin{pmatrix} -g^{-1}B & g^{-1} \\ g - Bg^{-1}B & Bg^{-1} \end{pmatrix}$$

$\Rightarrow$ generalized almost complex structure $\mathcal{J}_I^J$

defines split $(T \oplus T^*)_\mathbb{C} = L^{(1,0)} \oplus L^{(0,1)}$

$$\mathcal{J}L^{(1,0)} = iL^{(1,0)} \qquad \mathcal{J}L^{(0,1)} = -iL^{(0,1)} \qquad \mathcal{J}^2 = -\mathbf{1}$$

➪ $T \oplus T^*$ has more structure:

- natural inner product on $T \oplus T^*$: $\qquad V, W \in T \qquad \eta, \zeta \in T^*$

$$(V+\eta, W+\zeta) = (V\ \eta) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} W \\ \zeta \end{pmatrix} = \eta(W) + \zeta(V)$$

$\Rightarrow$ generalized structure group of $T \oplus T^*$ is $SO(6,6)$

- $SO(6,6)$ induces a Clifford action on differential forms Φ:

$$(V+\eta) \cdot \Phi = V \lrcorner \Phi + \eta \wedge \Phi$$

Indeed we have $\{V+\eta, W+\zeta\} \cdot \Phi = (V+\eta, W+\zeta)\Phi$.

Differential forms Φ on $\mathcal{M}_6$ are spinors of $SO(6,6)$:

$$S^{\mathrm{ev}} = \Lambda^{\mathrm{ev}} T^* \otimes \sqrt{\Lambda^6 T} \qquad\qquad S^{\mathrm{odd}} = \Lambda^{\mathrm{odd}} T^* \otimes \sqrt{\Lambda^6 T}$$

➪ $SO(6)$ spinors η^1, η^2 vs. $SO(6,6)$ spinors Φ

Gualtieri; Witt; Graña,Minasian,Petrini,Tomasiello

- Globally defined even and odd forms naturally associated to η^1 and η^2:

$$\Phi^{\mathrm{ev}} = \sum_{i=1}^{6} \eta_+^{\dagger\,2}\, \gamma_{m_1\ldots m_i} \eta_+^1 \; dx^{m_1} \wedge \ldots \wedge dx^{m_i}$$

$$\Phi^{\mathrm{odd}} = \sum_{i=1}^{6} \eta_-^{\dagger\,2}\, \gamma_{m_1\ldots m_i} \eta_+^1 \; dx^{m_1} \wedge \ldots \wedge dx^{m_i}$$

- Conditions on η^1, η^2 translate into conditions on forms $\Phi^{\mathrm{ev/odd}}$

- pure spinors: $(V+\eta)\cdot\Phi^{\rm ev/odd}=0 \qquad V+\eta\in L^{\rm ev/odd}\subset T+T^*$

  $L^{\rm ev/odd}$ are of dimension six and isotropic

  $L^{(1,0)}=L \quad\Rightarrow\quad \Phi^{\rm ev/odd}$ define two generalized almost complex structures

- non-degeneracy: $\langle\Phi^{\rm ev},\bar\Phi^{\rm ev}\rangle\neq 0 \qquad \langle\Phi^{\rm odd},\bar\Phi^{\rm odd}\rangle\neq 0$

  $\langle\cdot,\cdot\rangle$ are the Mukai pairings: e.g. for $\Phi^{\rm ev}=\Phi_0+\Phi_2+\Phi_4+\Phi_6$

  $\langle\Phi^{\rm ev},\bar\Phi^{\rm ev}\rangle=\Phi_0\wedge\bar\Phi_6-\Phi_2\wedge\bar\Phi_4+\Phi_4\wedge\bar\Phi_2-\Phi_6\wedge\bar\Phi_0$

- compatibility: $\langle\Phi^{\rm ev},\bar\Phi^{\rm ev}\rangle=\frac{3}{4}\langle\Phi^{\rm odd},\bar\Phi^{\rm odd}\rangle$ and $\langle\Phi^{\rm odd},(V+\eta)\cdot\Phi^{\rm ev}\rangle=0$

$\Rightarrow$

$\Phi^{\rm ev/odd}$ reduce structure group of $T\oplus T^*$ to $SU(3)\times SU(3)$
$\mathcal{M}_6$ is generalized almost complex manifold with $SU(3)\times SU(3)$ structure
flux background: $d\Phi^{\rm ev}\neq 0 \qquad d\Phi^{\rm odd}\neq 0$

➪ Important example: $SU(3)$ structure manifolds

$$\Phi^{\rm ev} = e^{iJ} \qquad\qquad \Phi^{\rm odd} = \Omega$$

where:

- J is a globally defined $(1,1)$ form
- Ω is a globally defined $(3,0)$ form

$\Rightarrow$ J, Ω define the $SU(3)$ structure:

$$J \wedge J \wedge J = \tfrac{3i}{4}\Omega \wedge \bar{\Omega}$$

$$J \wedge \Omega = 0$$

➪ Remark on NS-NS B-field: note that world-sheet coupling is $B + iJ$

include by a B-transform:

$$\boxed{\Phi \quad \mapsto \quad \Phi_B \equiv e^B \wedge \Phi}$$

- B-transform is a symmetry of Mukai pairings: $\langle \Phi_B, \Psi_B \rangle = \langle \Phi, \Psi \rangle$
- B-transform is $SO(6,6)$ rotation on $T \oplus T^*$: maps a pure spinor Φ into new pure spinor Φ_B
- B has a non-trivial background flux $H = dB$ $\Rightarrow$ Φ_B obtained by 'twisting with a gerbe'

Orientifold projection and $N = 1$ spectrum

➪ The orientifold projection (Type IIA example): Acharya,AganagicBrunner,Hori,Vafa

Bosonic Type IIA spectrum

NS-NS: $\phi,\ g_{MN},\ B_2$	R-R: $C^{\text{odd}} = C_1 + C_3 + C_5 + C_7 + C_9$

- mod out (gauge-fix) discrete symmetries of the string theory:

$\mathcal{O} = (-)^{F_L}\,\Omega_p\,\sigma^*$

1) world sheet parity Ω_p

2) geometric symmetry σ of $\mathcal{M}_6$: $\sigma^2 = 1$ (identity on $M_{3,1}$)

- demand $N = 1$ supersymmetry $\quad \lambda(\omega_{2n}) = (-1)^n \omega_{2n} \qquad \lambda(\omega_{2n-1}) = (-1)^n \omega_{2n-1}$

$$\sigma^* \Phi^{\text{odd}} = \lambda(\bar{\Phi}^{\text{odd}}) \qquad \sigma^* \Phi^{\text{ev}} = \lambda(\Phi^{\text{ev}})$$

Benmachiche,TWG

Calabi-Yau case: σ is anti-holomorphic and isometric involution – $O6$ planes.

- truncate spectrum such that: $\mathcal{O}(\text{Field}) = \text{Field}$

$$\sigma^*\phi = \phi \qquad \sigma^* B_2 = -B_2 \qquad \sigma^* C^{\text{odd}} = \lambda(C^{\text{odd}})$$

➪ Four-dimensional spectrum:

- $\mathcal{M}_6$ generalized complex manifold: two-grading of forms

$$\Lambda^{\rm ev}T^* \;\cong\; S^{\rm ev} \qquad\qquad \Lambda^{\rm odd}T^* \;\cong\; S^{\rm odd}$$

- $\mathcal{M}_6$ possesses orientifold symmetry $\lambda\sigma^*$: four-grading of forms

$$\Lambda^{\rm ev}T^* \;=\; \Lambda^{\rm ev}_+ \,\oplus\, \Lambda^{\rm ev}_- \qquad\qquad \Lambda^{\rm odd}T^* \;=\; \Lambda^{\rm odd}_+ \,\oplus\, \Lambda^{\rm odd}_-$$

Orientifold: restrict ten-dimensional fields to appropriate eigenspaces of σ^*

- R-R sector: four eigenspaces correspond to four fields in $D = 4$:

scalars $C^{\rm odd}_{(0)} = C^{\rm odd}|_{\Lambda^{\rm odd}_+}$ two-forms $C^{\rm odd}_{(2)} = C^{\rm odd}|_{\Lambda^{\rm odd}_-}$

vectors $C^{\rm odd}_{(1)} = C^{\rm odd}|_{\Lambda^{\rm ev}_-}$ three-forms $C^{\rm odd}_{(3)} = C^{\rm odd}|_{\Lambda^{\rm ev}_+}$

So far: Infinite set of scalars, two-forms as well as vectors, three-forms related by duality condition on the ten-dimensional field strengths:

$$*\,F^{\rm ev} = \lambda(F^{\rm ev}) \qquad\qquad F^{\rm ev} = dC^{\rm odd} + H\wedge C^{\rm odd}$$

- NS-NS sector

We define

$$\varphi^{\rm odd} = e^{-\phi} e^{B_2} \wedge \Phi^{\rm odd} \qquad \qquad \varphi^{\rm ev} = e^{B_2} \wedge \Phi^{\rm ev}$$

Four-dimensional graviton and $\varphi^{\rm ev/odd}$ encode all degrees of freedom in the NS-NS sector.

Not all degrees of freedom in $\varphi^{\rm ev/odd}$ are independent:

– $\varphi^{\rm odd}$ pure spinor and non-degeneracy $\Rightarrow$ Im$(\varphi^{\rm odd})$ is function of Re$(\varphi^{\rm odd})$ Hitchin

– $\varphi^{\rm ev}$ complicated function $\varphi^{\rm ev}(t)$ of the true scalar deformations t of the generalized complex manifold (e.g. complex rescalings of $\varphi^{\rm ev}$ are unphysical)

Gualtieri; Graña,Louis,Waldram

(compare with Calabi-Yau case $\varphi^{\rm ev} = e^{B+iJ} = e^{t^a \omega_a}$)

➪ Performing the Kaluza-Klein reduction

- What are the light modes on $SU(3)$ or $SU(3) \times SU(3)$ structure manifold?

  $\Rightarrow$ determine a finite set of forms on $\mathcal{M}_6$ used in the Kaluza-Klein expansion:

$$\Delta^{\text{finite}} = \Delta^{\text{ev}} \oplus \Delta^{\text{odd}}$$

  Δ^{finite} often constructed to match mirrors of Calabi-Yau compacitifications with fluxes

  Gurrieri,Louis,Micu,Waldram; Tomasiello; Graña,Louis,Waldram; Benmachiche,TWG

- How to perform a Kaluza-Klein reduction with a warp factor?

  $\Rightarrow$ restrict to regime of constant warping, large volume limit

  Giddings,Kachru,Polchinski

$N = 1$ chiral field space: Kähler potential and superpotential

1. How do the the scalars combine into $N = 1$ chiral multiplets?

- correct D-brane couplings: combine

$$\varphi_c^{\rm odd} \equiv e^{B_2} \wedge C^{\rm odd} + i\,\mathrm{Re}(\varphi^{\rm odd})\,|_{\Delta_+^{\rm odd}}$$

linear in the $N = 1$ complex scaler fields

- $\varphi^{\rm ev}(t)$ is holomorphic function of complex scalars t^a

2. What is the metric on the scalar field space?

- $N = 1$ susy $\longrightarrow$ Kähler metric, i.e. $G_{AB} = \partial_A \bar\partial_B K$
- Kähler potential: Benmachiche, TWG

$$K(t, \varphi_c^{\rm odd}) = -\ln \int_{\mathcal{M}_6} \langle \varphi^{\rm ev}, \bar\varphi^{\rm ev} \rangle - 2 \ln \int_{\mathcal{M}_6} \langle \varphi^{\rm odd}, \bar\varphi^{\rm odd} \rangle$$

- first term: as in $N = 2$ scale invariant functional on even forms Hitchin; Graña, Louis, Waldram
- second term: Kähler space inside the $N = 2$ quaternionic manifold,
  metric encoded by generalized Hitchin functional of Re$(\varphi^{\rm ev})$

➪ The $N = 1$ superpotential:

The background fluxes and non-Calabi-Yau geometry induce a potential for the scalar fields.

- We allow for non-trivial NS-NS background flux H_3 and R-R fluxes $F^{\rm ev}$ on $\mathcal{M}_6$

$$H_3 = \langle dB_2 \rangle_{\mathcal{M}_6} \, , \qquad\qquad F^{\rm ev} = \langle dC^{\rm odd} \rangle_{\mathcal{M}_6}$$

- The manifold with $SU(3) \times SU(3)$ structure generically has $d\varphi_c^{\rm odd} \neq 0$ and $d\varphi^{\rm ev} \neq 0$. This deviation from a Calabi-Yau manifold contributes to the potential.

- The induced superpotential can be derived by a fermionic reduction. Benmachiche,TWG

$$W(t, \varphi_c^{\rm odd}) = \int_{\mathcal{M}_6} \langle F^{\rm ev} + d_H \varphi_c^{\rm odd}, \varphi^{\rm ev} \rangle$$

  Here $d_H = d + H_3 \wedge$ denotes the H-twisted differential. This superpotential reduces to the known cases on general Calabi-Yau and $SU(3)$ structure orientifolds.

  TWG,Louis; Villadoro,Zwirner; Graña,Louis,Waldram

- A similar analysis can be performed for type IIB set-ups: essentially exchanges the role of even and odd forms

➪ Comments on the Hitchin functionals

Hitchin

- let ρ be a real even or odd form on $\mathcal{M}_6$ $\quad(\rho \in S^{\mathrm{ev/odd}})$
- define invariant quartic form of ρ

$$q(\rho) = \mathrm{Tr}\mu(\rho)^2 = \langle \Gamma_{MN}\rho, \rho\rangle\langle\Gamma^{MN}\rho, \rho\rangle$$

– $\mu(\rho)(a) = \langle\sigma(a)\rho, \rho\rangle$ is moment map of $SO(6,6)$, $\sigma(a)$ representation of $so(6,6)$
– Γ^M are Gamma matrices of $SO(6,6)$

Proposition: $\quad q(\rho) < 0 \quad \Leftrightarrow \quad \rho = \mathrm{Re}\Phi$ for pure spinor Φ and $\langle\Phi, \bar{\Phi}\rangle \neq 0$
$\hat{\rho} = \mathrm{Im}\Phi$ is unique up to ordering

$\Rightarrow$ on the open set of non-degenerate pure spinors define Hitchin functional

$$H[\rho] = \int_{\mathcal{M}_6} \sqrt{-q(\rho)} = i \int_{\mathcal{M}_6} \langle\Phi, \bar{\Phi}\rangle$$

- can express $\mathrm{Im}\Phi(\mathrm{Re}\Phi)$

$$\partial_\rho H[\rho] = \hat{\rho}$$

➪ Some other applications of the Hitchin functionals

- Variational problem: Extrema of $H[\rho]$ for variations in a cohomology class correspond to generalized Calabi-Yau manifolds with $d\Phi = 0$ Hitchin

- $H[\rho]$ is entropy functional for an $N = 2$ black hole: on harmonic forms it is the Legendre transform of the pre-potential $\mathcal{F}$ OSV; Dijkgraaf,Gukov,Neitzke,Vafa

- $H[\rho]$ might serve as a space-time action for topological string theory on $\mathcal{M}_6$ Dijkgraaf,Gukov,Neitzke,Vafa; Pestun,Witten

Mirror symmetry / T-duality with fluxes: a conjecture

➪ The Question:

What is the mirror dual of type IIB Calabi-Yau $O3/O7$ orientifolds with background fluxes?

- Mirror symmetry/T-duality is believed to map H-flux to a non-trivial mirror geometry.

- In type IIB Calabi-Yau orientifolds fluxes induce the Gukov-Vafa-Witten superpotential

$$W_{GVW} = \int_Y (F_3 - \tau H_3) \wedge \Omega(z)$$

- Let us restrict to a simple case: one complex structure modulus z
  $F_3 = 0$ and $H_3 = m\alpha_1 + e\beta^1$
  In large complex structure limit:

$$W_{GVW} = -\tau(e\, z + m\, z^2)$$

Has the H-flux e and m an $SU(3) \times SU(3)$ mirror geometry?

➪ Perform mirror symmetry (T-dulality in three directions SYZ):
H_3 has maximally two legs into the T-dualized directions (the 'Q-space' Shelton,Taylor,Wecht)

- Mirror deformation due to electric flux e: The complex three-form Ω_3 is not anymore closed. $SU(3)$ structure mirror ('half-flat')

  Kachru,Schulz,Tripathy,Trivedi; Gurrieri,Louis,Micu,Waldram; Fidanza,Minasian,Tomasiello

$$\Phi^{\rm odd} = \Omega_3 \qquad\qquad d\mathrm{Re}(\Omega_3) \ \propto \ e$$

- Mirror deformation due to magnetic flux m: A non-trivial one-form Ω_1 on the mirror space is needed:

$$\Phi^{\rm odd} = \Omega_1 + \Omega_3 + \Omega_5 \qquad\qquad d\mathrm{Re}(\Omega_1) \ \propto \ m$$

  Such a one-form is present on an appropriate $SU(3) \times SU(3)$ manifold!
  Can use the superpotential calculated above

$$W_{SU(3)\times SU(3)} = \int \langle d\varphi_c^{\rm odd}, \varphi^{\rm ev}(t)\rangle = -N^0(e\,t + m\,t^2)$$

- Origin of Ω_1? Recall $so(6,6) \cong \Lambda^2 T^* \oplus \Lambda^2 T \oplus \mathrm{End}T \quad \Rightarrow \quad \beta \in \Lambda^2 T$ two-vector

  $\Omega_1 + \Omega_3 = e^{-\beta}\Omega_3$: β correspond to non-commutativity of $\mathcal{M}_6$ for open strings

  Kapustin,Mathai,Rosenberg

Conclusions

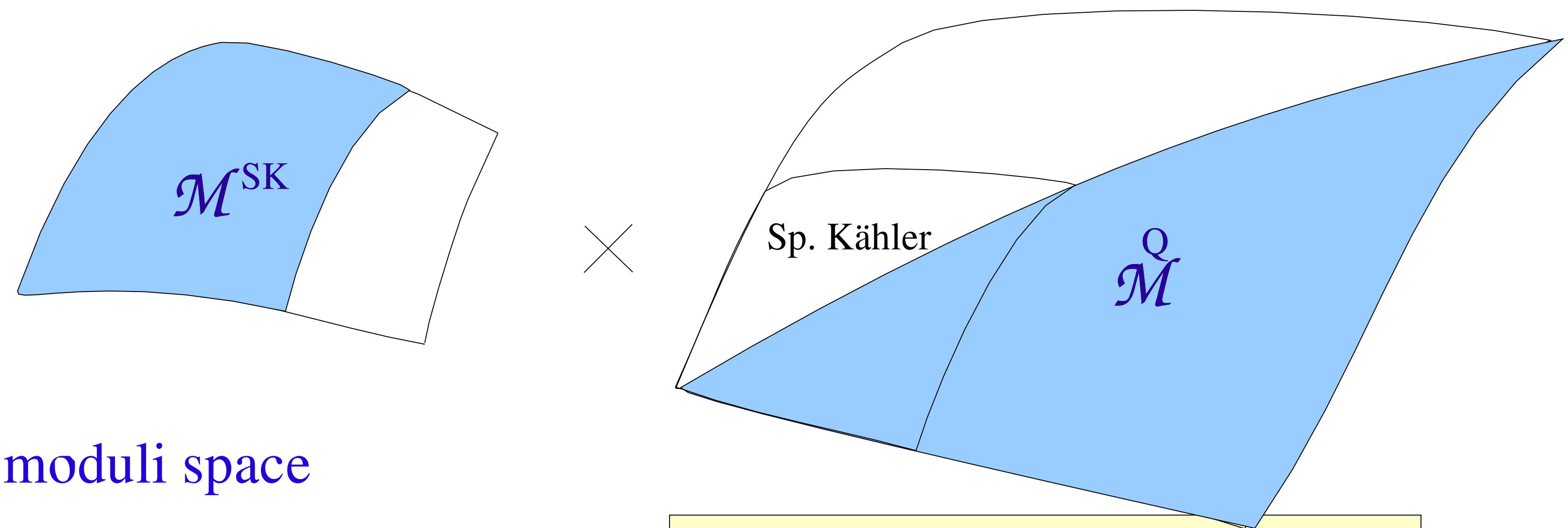
- discussed compactification of type II supergravity on orientifolds of $SU(3) \times SU(3)$ manifolds
  - determined four-dimensional spectrum without finite truncation, NS-NS and R-R sector is encoded by specific odd and even forms on $\mathcal{M}_6$
  - Kähler potential consists of the two Hitchin functionals on $\mathcal{M}_6$
  - holomorphic superpotentials for the geometry due to fluxes and non-Calabi-Yau geometry

- commented on mirror symmetry/T-duality of flux compactifications
  - mirror spaces of Calabi-Yau compactifications with H-flux are generically generalized $SU(3) \times SU(3)$ manifolds

- open questions:
  - determination of light modes for non-Calabi-Yau compactifications
  - effective four-dimensional theories for warped compactifications
  - interesting compact examples with space-time filling D-branes

N=2 moduli space

Special Kähler (vector multiplets) Quaternionic (hyper multiplets)



N=1 moduli space

$\mathcal{M}^{SK}$ remains special Kähler (same complex structure)

$\mathcal{M}^{Q}$ Kähler – half of Quaternionic