Nongeometric Flux Compactifications

hep-th/0508133:

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+ work in progress

The Basic Setup

- Type II on $T^6/\mathbb{Z}_2$ with fluxes

- Basic ingredients:

$$T^6 + \left\{ \begin{matrix} \text{NS-NS} \\ \text{fluxes} \end{matrix} \right\} + \left\{ \begin{matrix} \text{R-R} \\ \text{fluxes} \end{matrix} \right\} + \text{O-plane}$$

- Generalise NS-NS fluxes:

$$\bar{H}_{abc} \xleftrightarrow{T_a} f^a{}_{bc} \xleftrightarrow{T_b} Q^{ab}{}_c \xleftrightarrow{T_c} R^{abc}$$

$$\left(\text{compare } \bar{F}_{xa_1\cdots a_p} \xleftrightarrow{T_x} \bar{F}_{a_1\cdots a_p} \right)$$

systematically incorporate into flux compactifications

- Understand generic properties of resulting vacua

for simplicity, SUSY sols: $D_\alpha W = 0$

Type II Flux Compactifications on $T^6/\mathbb{Z}_2 \ldots$

IIB: (O3) $W_{GVW} = \int (F - \tau H) \wedge \Omega$

Kachru, Schulz, Trived
DGKT (1)
⋮

- generates potential for axiodilaton, c.s. moduli
- "no-scale structure" $\Rightarrow$ SUSY vacua Minkowski
- tadpole cancellation $\Rightarrow$ finite # of vacua

IIA: (O6) $W = \int e^{J_c} \wedge F + \int (H + dJ_c) \wedge \tilde{\Omega}$

Villadoro + Zwirner
Derendinger et al
DGKT (2)
⋮

- dJ_c: "geometric flux"
- depends on all moduli at tree level
- SUSY vacua generically AdS
- tadpole cancellation does not constrain all fluxes
  $\Rightarrow$ infinite # of vacua

— Physically inequivalent classes of solutions

"Geometric Flux"

$$\bar{H}_{xyz} \xleftrightarrow{T_x} f^x{}_{yz}$$

Explicitly, consider T^3 with $\bar{H}_{xyz} = N$:

$$ds^2 = dx^2 + dy^2 + dz^2$$

$$B_{xy} = Nz$$

T-dualise on x: "twisted torus"

Scherk-Schwarz
Kachru Schulz Tripathy + Trivedi

$$ds^2 = (dx - Nz\,dy)^2 + dy^2 + dz^2$$

$$B_{xy} = 0$$

Useful description in terms of vielbeins η^a:

$$\{\eta^x = dx - Nz\,dy \;;\; \eta^y = dy \;;\; \eta^z = dz\}$$

$$d\eta^x = N\,dy\,dz = f^x{}_{yz}\,dy\,dz$$

Can systematically incorporate into flux compactifications:

- constraints
- (super)potential

Villadoro + Zwirner
DKPZ
⋮

Another T-duality

$$H_{xyz} \xleftrightarrow{T_x} f^x{}_{yz} \xleftrightarrow{T_y} Q^{xy}{}_z$$

Performing a T-duality on y in toy example gives KST[2]

$$ds^2 = \frac{1}{1+N^2z^2}\left(dx^2+dy^2\right) + dz^2$$

$$B_{xy} = \frac{Nz}{1+N^2z^2}$$

In terms of $\rho = b_{xy} + i\,\mathrm{vol}_{xy}$, boundary conditions around S^1_z are simple:

$$\frac{1}{\rho} \to \frac{1}{\rho} + N$$

transition functions mix $g + B$: <u>nongeometric</u>

"T-fold" Hull

Characterise this structure by

$$Q^{xy}{}_z = N$$

mixing of T^2_{xy} dof's around z-cycle

A final, formal T-duality

$$\bar{H}_{xyz} \xleftrightarrow{T_x} f^{x}{}_{yz} \xleftrightarrow{T_y} Q^{xy}{}_{z} \xleftrightarrow{T_z} R^{xyz}$$

No remaining isometries: cannot use Buscher rules

can nevertheless define formally

$$Q^{xy}{}_{z} \xleftrightarrow{T_z} R^{xyz}$$

analogous to $F^{(0)} \xleftrightarrow{T_x} F_x$

R-flux: no locally geometric description

Next: $\Rightarrow$ what combinations of fluxes yield a consistent theory?

$\Rightarrow$ how do these fluxes couple to moduli + what do the resulting solutions look like?

Constraints

- $Q, R \rightarrow$ new terms in RR tadpoles
  RR, NS Bianchi identities

- derive using T-duality

NS-NS

$$0 = \bar{H}_{x[ab} f^{x}{}_{cd]}$$

$$0 = f^{a}{}_{x[b} f^{x}{}_{cd]} + \bar{H}_{x[bc} Q^{ax}{}_{d]}$$

$$0 = Q^{[ab]}{}_{x} f^{x}{}_{[cd]} - 4 f^{[a}{}_{x[c} Q^{b]x}{}_{d]} + H_{x[cd]} R^{[ab]x}$$

$$0 = Q^{[ab}{}_{x} Q^{c]x}{}_{d} + f^{[a}{}_{xd} R^{bc]x}$$

$$0 = Q^{[ab}{}_{x} R^{cd]x}$$

(also, $f^{x}{}_{xa} = 0 = Q^{ax}{}_{x}$.)

R-R Constraints

$$* \quad 0 = \bar{F}_{[abc}\, \bar{H}_{def]}$$

$$0 = \bar{F}_{x[abc}\, f^{x}{}_{de]} - \bar{F}_{[ab}\, \bar{H}_{cde]}$$

$$0 = \bar{F}_{xy[abc}\, Q^{xy}{}_{d]} - 3\bar{F}_{x[ab}\, f^{x}{}_{cd]} - 2\bar{F}_{[a}\, \bar{H}_{bcd]}$$

$$* \quad 0 = \bar{F}_{xyz[abc]}\, R^{xyz} - 9\bar{F}_{xy[ab}\, Q^{xy}{}_{c]} - 18\bar{F}_{x[a}\, f^{x}{}_{bc]} + 6F^{(0)} H_{[abc]}$$

$$0 = \bar{F}_{xyz[ab]}\, R^{xyz} + 6\bar{F}_{xy[a}\, Q^{xy}{}_{b]} - 6\bar{F}_{x}\, f^{x}{}_{[ab]}$$

$$0 = \bar{F}_{xyza}\, R^{xyz} - 3\bar{F}_{xy}\, Q^{xy}{}_{a}$$

$$0 = \bar{F}_{xyz}\, R^{xyz}$$

* Particular model we consider:
orientifold planes set LHS to 16

Constraints, Compactly.

Define a twisted differential operator

$$\tilde{d} \equiv d + H\cdot + f\cdot + Q\cdot + R\llcorner$$

where

$$f\cdot\omega^{(p)} \equiv \frac{(p+2)!}{p!\,2!}\, f^{a_1}{}_{[a_2 a_3}\,\omega_{a_1| i_2 \ldots i_p]}, \text{ etc.}$$

Then ALL NS-NS constraints come from

$$(\tilde{d})^2 = 0,$$

and all NS-RR constraints come from

$$\tilde{d}\mathcal{F} = 0.$$

This naturally suggests an avenue for defining these NS-NS fluxes on more general manifolds.

A Simple Model: $(T^2)^3$

- work on a symmetric T^6

$$\tau^{ij} = \tau\,\delta^{ij}$$

IIB : O3 IIA : O6

- three complex moduli:

$$\tau = \begin{cases} \tau_1 + i\,\tau_2 & \text{complex (IIB)} \\ b + i\,\mathrm{vol} & \text{Kähler (IIA)} \end{cases}$$

$$S = \begin{cases} C_0 + i\,e^{-\phi} \\ \xi_0 + i\,e^{-\phi} \end{cases} \quad \text{axiodilaton}$$

$$\mathcal{U} = \begin{cases} \xi + i e^{-\phi}(\mathrm{vol})^2 & \text{Kähler} \\ \xi + i e^{-\phi}\,\tau_2^{\,2} & \text{complex} \end{cases}$$

- Kähler potential

$$K = -3\ln\left(-i(\tau-\bar{\tau})\right) - \ln\left(-i(S-\bar{S})\right) - 3\ln\left(-i(\mathcal{U}-\bar{\mathcal{U}})\right)$$

- flux-induced superpotential

Superpotential

T-dualise on α, β, γ to relate IIA, IIB

$$\begin{array}{lccccl} & \bar F_{ijk} & \bar F_{ij\gamma} & \bar F_{i\beta\gamma} & \bar F_{\alpha\beta\gamma} & \leftarrow \mathrm{IIB} \\ W = & (a_0 & - 3a_1\tau & + 3a_2\tau^2 & - a_3\tau^3) & \\ & \bar F^{(6)} & \bar F_{\alpha i\beta j} & \bar F_{\alpha i} & F^{(0)} & \leftarrow \mathrm{IIA} \end{array}$$

$$\begin{array}{lcccc} & \bar H_{ijk} & \bar H_{\alpha jk} & \bar H_{\alpha\beta k} & \bar H_{\alpha\beta\gamma} \\ + S\,(& -b_0 & + 3b_1\tau & - 3b_2\tau^2 & + b_3\tau^3) \\ & \bar H_{ijk} & f^{\alpha}{}_{jk} & Q^{\alpha\beta}{}_{k} & R^{\alpha\beta\gamma} \end{array}$$

$$\begin{array}{lcccccccc} & Q^{\alpha\beta}{}_k & & Q^{\alpha j}{}_k & Q^{\alpha\beta}{}_\gamma & & Q^{\alpha j}{}_\gamma & Q^{ij}{}_k & Q^{ij}{}_\gamma \\ + 3\mathcal{U}\,(& c_0 & + (& 2c_1 & - \tilde c_1)\tau & - (& 2c_2 & - \tilde c_2)\tau^2 & - c_3\tau^3) \\ & \bar H_{\alpha\beta k} & & f^{i}{}_{\beta k} & f^{\alpha}{}_{\beta\gamma} & & Q^{i\beta}{}_\gamma & Q^{ij}{}_k & R^{ij\gamma} \end{array}$$

$$\equiv P_1(\tau) + S P_2(\tau) + \mathcal{U} P_3(\tau)$$

- 14 fluxes − 6 constraints = 8 independent fluxes
- study solutions of $D_\alpha W = 0$

Two Technical Points

- take flux integers all even:
  avoid complications of exotic O-planes

- mod out by action of modular group Γ

  $(T^2)^3$: Γ generated by

  $S \to S + n$ $\qquad$ $w_i \to -w_i$

  $\mathcal{U} \to \mathcal{U} + n$

  $SL(2, \mathbb{Z})_\tau$ $\qquad$ $w_i \to -\bar{w}_i$

  with corresponding action on fluxes:

  $\cancel{\text{eg}}\ S \to S + n, \quad P_1 \to P_1 - P_2 \cdot n$

- gauge-fix:

  τ: require root of P_2 in $\mathcal{F}_0$

  $S, \mathcal{U}$: put P_1 in canonical form

  (signs)

An infinite set of vacua

- There are an infinite number of flux configurations (tentative: $\sim n^4$)

- A fraction of $\mathcal{O}(.1)$ of flux configurations admit physical SUSY solutions

$$\mathrm{Im}\,\tau,\ \mathrm{Im}\,\mathcal{U},\ \mathrm{Im}\,S > 0$$

$\Rightarrow$ infinite number of physically distinct vacua

A generic solution:

→ has all moduli stabilised

→ is nongeometric in any duality frame

Properties of Solutions

Moduli generically stabilised

- at string scale
  not surprising: essentially treating KK, winding on equal footing
- (in fraction examined) at $g > 1$, $|\lambda| > 1$

$\Rightarrow$ EFT analysis is unreliable (usual caveats)

However: enough to indicate qualitatively new behaviour:

IIB: finite number of solutions

IIA: infinite # of solutions, but accumulate at small g, large vol.

Here, infinite number of solutions in any region of parameter space (g, λ)

– Have reached limit of SUGRA

Open Questions

* stringy description of backgrounds with Q-, R-flux

* what additional kinds of structures are needed to make contact with heterotic compactifications?

* how do these nongeometric structures generalise beyond the torus?