Nongeometric Flux Compactifications

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+ work in progress
The Basic Setup

- Type II on $T^6/Z_2$ with fluxes

- Basic ingredients:

$$ T^6 + \{ \text{NS-NS fluxes} \} + \{ \text{R-R fluxes} \} + \text{O-plane} $$

- Generalise NS-NS fluxes:

$$ H_{abc} \leftrightarrow T_a \rightarrow f^a_{\ bc} \leftrightarrow T_b \rightarrow Q^{ab\ c} \leftrightarrow T_c \rightarrow R^{abc} $$

(compare $F_{axa_1\ldots a_p} \leftrightarrow T_x \rightarrow \tilde{F}_{a_1\ldots a_p} $)

systematically incorporate into flux compactifications

- Understand generic properties of resulting vacua

For simplicity, SUSY sols: $D\chi \omega = 0$
Type II Flux Compactifications on $\mathbb{C}^3/\mathbb{Z}_n$

**II B**: (03) $W_{\text{II} B} = \int (F - \Phi)^2$

- generates potential for axion dilaton, c.s. moduli
- "no-scale structure" $\Rightarrow$ SUSY vacua Minkowski
- tadpole cancellation $\Rightarrow$ finite # of vacua

**II A**: (06) $W = \int e^{-\Phi} \sqrt{F} + \int (H + d\Omega)_c \wedge \tilde{\Omega}$

- $d\Omega_c$: "geometric flux"
- depends on all moduli at tree level
- SUSY vacua generically AdS
- tadpole cancellation does not constrain all fluxes
  $\Rightarrow$ infinite # of vacua
- Physically inequivalent classes of solutions
"Geometric Flux"

\[ H_{xyz} \xleftarrow{T_x} f^x_{yz} \]

Explicitly, consider \( T^3 \) with \( H_{xyz} = N \):

\[ ds^2 = dx^2 + dy^2 + dz^2 \]

\[ B_{xy} = N \]

\( T \)-dualise on \( x \): "twisted torus"

\[ ds^2 = (dx - Nz dy)^2 + dy^2 + dz^2 \]

\[ B_{xy} = 0 \]

Useful description in terms of vielbeins \( \eta^a \):

\[ \{ \eta^x = dx - Nz dy ; \eta^y = dy ; \eta^z = dz \} \]

\[ d\eta^x = Nd_y dz = f^x_{yz} dydz \]

Can systematically incorporate into flux compactifications:

- constraints
- (super)potential
Another T-duality

\[ H_{xy} \leftrightarrow T_x \xrightarrow{f_y} T_y \xrightarrow{Q^{xy}} \]

Performing a T-duality on \( y \) in toy example gives \( KST^2 \)

\[ ds^2 = \frac{1}{1 + N^2 z^2} (dx^2 + dy^2) + dz^2 \]

\[ B_{xy} = \frac{N^2}{1 + N^2 z^2} \]

In terms of \( \rho = b_{xy} + i \text{vol}_{xy} \), boundary conditions around \( S^1 \) are simple:

\[ \frac{1}{\rho} \to \frac{1}{\rho} + N \]

transition functions mix \( g + B \): non-geometric

"T-Fold"

Characterise this structure by

\[ Q^{xy}z = N \]

mixing of \( T^2_{x,y} \) dof's around \( z \)-cycle

Hull
A Final, Formal T-duality:

\[
\begin{align*}
\mathcal{H}_{xy} & \xleftarrow{\mathcal{T}_x} \mathcal{E}^x_{y^*} & \xrightarrow{\mathcal{T}_x} \mathcal{Q}_{xy} & \xleftarrow{\mathcal{T}_p} \mathcal{R}^{xy*}
\end{align*}
\]

No remaining isometries: cannot use Buscher rules
can nevertheless define formally

\[
\begin{align*}
\mathcal{Q}_{xy} & \xleftarrow{\mathcal{T}_2} \mathcal{R}^{xy*}
\end{align*}
\]

analogous to \( F^{(o)} \xrightarrow{\mathcal{T}_x} F_x \)

R-Flux: no locally geometric description

Next: what combinations of fluxes yield a consistent theory?

\( \Rightarrow \) how do these fluxes couple to moduli + what do the resulting solutions look like?
Constraints

- $Q_R \to$ new terms in RR tadpoles
  
  \[ \text{RR, NS Bianchi identities} \]

- derive using T-duality

\[
\begin{align*}
0 &= \bar{H}_{x[ab} \, f^x \, cd]} \\
0 &= f^a_x \{b \, f^x \, cd]} + \bar{H}_{x[bc} \, Q^{ax} \, d]} \\
0 &= Q^{[ab} \, f^x \, cd]} - 4 f^a_{x[bc} \, Q^{bdx} \, d]} + \bar{H}_{x[cd]} \, R^{ab]x} \\
0 &= Q^{abx} \, Q^{cdx} \, d] + f^a_{xd} \, R^{bc]x} \\
0 &= Q^{ab} \, R^{cd]} \, x}
\end{align*}
\]

(also, $f^x_x = 0 = Q^{ax}_x$.)
R-R Constraints

\[ 0 = \bar{F}^{abc} \bar{F}_{dcf} \]

\[ 0 = \bar{F}^{x}_{abc} F^{x}_{dcf} - \bar{F}^{x}_{abc} H_{dcf} \]

\[ 0 = \bar{F}^{x}_{abc} \left( Q^{x}_{d} - 3 F^{x}_{abc} f^{x}_{cd} - 2 \bar{F}^{x}_{abc} H_{cde} \right) \]

\[ 0 = \bar{F}^{x}_{abc} R^{xyz} - 9 \bar{F}^{x}_{abc} Q^{xy}_{dz} - 18 \bar{F}^{x}_{abc} F^{x}_{bc} + 6 F^{(a)} H_{[abc]} \]

\[ 0 = \bar{F}^{x}_{abc} R^{xyz} + 6 \bar{F}^{x}_{abc} Q^{x}_{z} - 6 \bar{F}^{x}_{abc} f^{x}_{z} \]

\[ 0 = \bar{F}^{x}_{abc} R^{xyz} - 3 \bar{F}^{x}_{abc} Q^{xy}_{z} \]

\[ 0 = \bar{F}^{x}_{abc} R^{xyz} \]

* Particular model we consider:
orientifold planes set LHS to 16
Constraints, Compactly.

Define a twisted differential operator

\[ d = d + H \cdot + \bar{f} \cdot + G \cdot + \Omega \]

where

\[ f \cdot \omega^{(p)} \equiv \frac{(p+2)!}{p! \cdot 2!} f^a \left[ a_1 a_2 a_3 \omega_{a_1 i_2 \ldots i_p} \right], \text{ etc.} \]

Then all NS-NS constraints come from

\[ (\tilde{d})^\dagger = 0, \]

and all NS-RR constraints come from

\[ \tilde{d} F = 0. \]

This naturally suggests an avenue for defining these NS-NS fluxes on more general manifolds.
A Simple Model: \((T^2)^3\)

- Work on a symmetric \(T^4\)

\[ \tau^{ij} = \tau \delta^{ij} \]

\[ \text{II}B : 03 \quad \text{II}A : 06 \]

- Three complex moduli:

\[ \tau = \begin{cases} \tau_1 + i \tau_2 & \text{complex (II}B) \\ b + i \text{vol} & \text{Kähler (II}A) \end{cases} \]

\[ S = \begin{cases} \zeta + i e^{-\eta} & \text{axiodilaton} \\ \bar{\zeta}_0 + i e^{-\eta} \end{cases} \]

\[ U = \begin{cases} \bar{\zeta} + i e^{-\eta} (\text{vol})^2 & \text{Kähler} \\ \zeta + i e^{-\eta} \tau_1^2 & \text{complex} \end{cases} \]

- Kähler potential

\[ K = -3 \ln \left( -i (\zeta - \bar{\zeta}) \right) - \ln \left( -i (\bar{\zeta} - \zeta) \right) - 3 \ln \left( -i (\tau_1 - \bar{\tau}_1) \right) \]

- Flux-induced superpotential
Superpotential

T-dualise on $\alpha, \beta, \gamma$ to relate IIA, IIB

$$W = \frac{F_{ijk}^* F_{i\gamma}^* F_{i\beta \gamma}^* F_{i\alpha \beta \gamma}^*}{F_{i\alpha \beta \gamma}^* F_{i\alpha \beta \gamma}^*} \quad \leftarrow \text{IIA}$$

$$= \frac{H_{ijk}^* H_{i\gamma}^* H_{i\beta \gamma}^* H_{i\alpha \beta \gamma}^*}{H_{i\alpha \beta \gamma}^* H_{i\alpha \beta \gamma}^*} \quad \leftarrow \text{IIB}$$

$$+ S \left( -b_0 + 3b_1 \tau - 3b_2 \tau^2 + b_3 \tau^3 \right)$$

$$\quad \frac{Q_{\alpha k}^* Q_{\beta k}^* Q_{i \gamma}^* Q_{i \gamma}^*}{Q_{i \gamma}^* Q_{i \gamma}^*} \quad R_{i \gamma}^{* \beta \gamma}$$

$$+ 3U \left( c_0 + (2c_1 - c_2) \tau - (2c_2 - c_3) \tau^2 - c_3 \tau^3 \right)$$

$$\quad \frac{Q_{i \gamma}^{* \alpha} Q_{i \gamma}^{* \beta} Q_{i \gamma}^{* \gamma} Q_{i \gamma}^{* \gamma}}{Q_{i \gamma}^{* \gamma} Q_{i \gamma}^{* \gamma}} \quad R_{i \gamma}^{* \gamma}$$

$$\equiv P_i(\tau) + SP_2(\tau) + UP_3(\tau)$$

- 14 fluxes - 6 constraints = 8 independent fluxes
- study solutions of $D_\alpha \omega = 0$
Two Technical Points

- take flux integers all even:
  avoid complications of exotic O-planes

- mod out by action of modular group $\Gamma$

  $(T^2)^3: \Gamma$ generated by

  $S \rightarrow S + n, \quad \omega_i \rightarrow -1, i$

  $U \rightarrow U + n, \quad \omega_i \rightarrow -\omega_i$

  $SL(2, \mathbb{Z})_c$

  with corresponding action on fluxes:

  $S \rightarrow S + n, \quad P_i \rightarrow P_i - P_2 \cdot n$

- gauge-fix:

  2: require root of $P_2$ in $F_0$

  $S, U$: put $P_i$ in canonical form (signs)
An infinite set of vacua

- There are an infinite number of flux configurations (tentative: \( \sim n^4 \))

- A fraction of \( \mathcal{O}(1) \) of flux configurations admit physical SUSY solutions
  
  \[ \text{Im } \tau, \text{Im } \kappa, \text{Im } S > 0 \]

  \[ \Rightarrow \text{ infinite number of physically distinct vacua} \]

A generic solution:

- has all moduli stabilised

- is nongeometric in any duality frame
Properties of Solutions

Moduli generically stabilised

- at string scale
  not surprising: essentially treating KK, winding on equal footing

  * (in fraction examined) at \( g > 1 \)
  \[ |\lambda| > 1 \]

\( \Rightarrow \) EFT analysis is unreliable (usual caveats)

However: enough to indicate qualitatively new behaviour:

IIIB: finite number of solutions

IIA: infinite # of solutions, but accumulate at small \( g \), large vol.

Here, infinite number of solutions in any region of parameter space \((g, \lambda)\)

- Have reached limit of SUGRA
Open Questions

* stringy description of backgrounds with $Q$, $R$-flux

* what additional kinds of structures are needed to make contact with heterotic compactifications?

* how do these nongeometric structures generalise beyond the torus?