D-branes at singularities and SUSY breaking

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References

- Wijnholt, *"Large Volume Perspective on Branes at Singularities"*: quiver gauge theories and superpotentials for the D-branes at del Pezzo singularities.
  Verlinde, Wijnholt, *"Building the Standard Model on a D3-brane"*: an example of SM-like model on a D3-brane.
  Buican, Malyshev, Morrison, Wijnholt, Verlinde, *"D-branes at Singularities, Compactification, and Hypercharge"*: a review of the model building on D-branes, some compactification issues.
  Malyshev, *"Del Pezzo singularities and SUSY breaking"*: construction of compact CY manifolds with del Pezzo singularities and the ISS type of SUSY breaking.
- Intriligator, Seiberg, Shih, *"Dynamical SUSY Breaking in Meta-Stable Vacua"*.

Is String Theory right?

Is it possible to falsify String Theory?

Example: suppose someone proves that SUSY requires a particle below 1TeV but LHC finds no such particle.

Then the Superstring Theory is not a fundamental theory of the world.

We will assume that SUSY exists and is broken by some mechanism.

The question is whether this mechanism can be realized in String Theory.

A possible scenario is

$$\text{observed particles} \longrightarrow \text{field theory} \longrightarrow \text{string theory model}$$

This question is very hard to answer in general.

The strategy could be to study some examples to get intuition about the possibilities.

I will describe the realization of Intriligator, Seiberg, Shih (ISS) construction on D-branes at the tip of the cone over del Pezzo surfaces.

ISS – is a field theory that admits a (meta)stable SUSY breaking vacuum.

Outline

1. Motivation for D-branes at del Pezzo singularities
2. Review of ISS
3. The ISS at del Pezzo 6 singularity

D-branes at singularities (type IIB)

The "+" sides:

- Many possible gauge theories
- Control over moduli
- Some specific information is known (e.g., superpotential)

The "–" sides:

- Too many possible gauge theories
- Extra fields (e.g., Higgs multiplets)
- Some information is unknown (e.g., Kahler potential)

Del Pezzo singularities

There are infinitely many singularities of Calabi-Yau (CY) three-folds.

Demand that the singularity is

1. Gorenstein – the resolution preserves the CY condition;
2. Primitive – it can be resolved by a single blowup;
3. Isolated – point-like,

then there are only 11 possibilities: the conifold and the cones over del Pezzo surfaces (the $\mathbb{P}^2$, the $\mathbb{P}^1 \times \mathbb{P}^1$, and the $\mathbb{P}^2$ blown up at $k = 1, \ldots, 8$ points).

The corresponding gauge theories are reach enough and capture many essential features.

ISS

The field theory contains chiral fields Φ_{ij}, φ^i_c, $\tilde{\varphi}^{ic}$, where $i, j = 1 \ldots N_f$ are flavor indices and $c = 1 \ldots N$ is the color index under the $SU(N)$ gauge group. The superpotential is

$$W = h\mathrm{Tr}\varphi\Phi\tilde{\varphi} - h\mu^2\mathrm{Tr}\Phi \quad (1)$$

the F-term equation for the Φ field is

$$\sum_c \tilde{\varphi}^{ic}\varphi^j_c = \mu^2\delta^{ij} \quad (2)$$

If $N < N_f$, then this equation cannot be satisfied and the SUSY is broken by the rank condition, since

$$\mathrm{rank}\ (\tilde{\varphi} \cdot \varphi) \leq N < N_f$$

Stability of SUSY breaking vacuum

The fluctuations of the scalar fields around this vacuum split into

- Massive fluctuations
- Goldstone bosons for broken $SU(N_f)$
- Classical pseudomoduli (get positive mass squared at one loop)

We also take $N_f > 3N$ so that $SU(N)$ is IR free and has a UV Landau pole at some scale Λ.

UV limit of ISS

The Seiberg dual theory above Λ is $SU(N_c)$ SQCD, $N_c = N_f - N$, with N_f massive flavors

$$m = \mu^2/\Lambda \tag{3}$$

This theory has N_c SUSY vacua.

The SUSY breaking vacuum has positive vacuum energy, i.e. it is metastable and can tunnel to the SUSY vacuum.

It is long lived for

$$m << \Lambda$$

Thus the problem is to find an SQCD with massive quarks such that their mass is much smaller than Λ_{QCD}.

Some properties of del Pezzo surfaces

The del Pezzo surfaces are the complex projective plane $\mathbb{P}^2$, the $\mathbb{P}^1 \times \mathbb{P}^1$, and the $\mathbb{P}^2$ blown up at $k = 1, \ldots, 8$ points.

Denote by dP_k the $\mathbb{P}^2$ blown up at k points.

The complex projective plane has one four-cycle, $H_4(\mathbb{P}^2) = 1$, one two-cycle, $H_2(\mathbb{P}^2) = 1$, and one zero-cycle, $H_0(\mathbb{P}^2) = 1$.

Blowing up a point in $\mathbb{P}^2$ corresponds to inserting $\mathbb{P}^1$ instead of the point. This process increases the number of two-cycles by one. Thus

$$H_0(dP_k) = 1, \;\; H_2(dP_k) = k + 1, \;\; H_4(dP_k) = 1 \tag{4}$$

CY cone over del Pezzo

Consider a complex cone over del Pezzo surface such that the del Pezzo at the tip is slightly resolved.

There are two complex directions tangent to the del Pezzo and one normal complex direction.

The structure of the normal bundle is completely fixed by the condition of Ricci flatness. This line bundle is called the canonical line bundle.

The D-branes

I will talk about D-branes that span the 4-dimensional Minkowski space and wrap some cycles in the internal geometry.

Thus a D3-brane is a point in the internal space, a D5-brane wraps a two-cycle, and a D7-brane wraps a four-cycle.

The D-branes placed at the tip of the cone split into the so called fractional branes.

Fractional branes

A fractional brane is a bound state of branes. Typically it will be a D7-brane with some D5 and D3-brane charges that we write as a charge vector

$$\mathbf{Q} = (Q_7, \sum_i Q_5^i A_i, Q_3) \tag{5}$$

where A_i are the two-cycles on del Pezzo that the D5 component wraps.

The D-brane charges are measured by the interaction with the Ramond-Ramond fields C_n via the Chern-Simons action

$$S_{CS} = \int_{D7} \sum_n C_n e^F \tag{6}$$

where we put $B = 0$ and omit the curvature terms (which in fact present for the cones over del Pezzo).

Expanding the exponent we find that

$$Q_5^i = \int_{\tilde{A}_i} F$$

$$Q_3 = \frac{1}{2} \int_{dP_k} F \wedge F$$

This formula is a little naive because we omitted the curvature contributions. But it illustrates that a bound state of branes can be thought of as a D7-brane together with a nontrivial flux of the F-field in its world volume. The dimension of the linear space of charge vectors for the del Pezzo k surface is

$$H_0(dP_k) + H_2(dP_k) + H_4(dP_k) = k + 3 \tag{7}$$

For any configuration, this is the maximal number of fractional branes.

Quiver gauge theory

The D3-brane at the tip of the cone over dP_k is unstable and splits to a combination of stable fractional branes so that the charge vector conserves

$$(0,0,1) = \sum_{\alpha=1}^{k+3} N_\alpha \mathbf{Q}_\alpha \tag{8}$$

The corresponding quiver gauge theory has $k+3$ gauge groups $SU(N_\alpha)$.

The number of chiral matter fields in the bifundamental representation $(\bar{N}_\alpha, N_\beta)$ is given by the antisymmetric intersection between the fractional branes

$$N_{\alpha\beta} = (\mathbf{Q}_\alpha, \mathbf{Q}_\beta)_- \tag{9}$$

Gauge theory parameters

The parameters of the quiver gauge theory depend on the boundary value of the SUGRA fields (dilaton, metric and the Ramond-Ramond fields).

The gauge couplings and the FI parameters are given by the central charges of the fractional branes (the fractional branes preserve $\mathcal{N} = 1$ susy, hence they are BPS objects characterized by the central charge)

$$\begin{aligned} \frac{1}{g_\alpha^2} &\sim |Z_\alpha| \\ \xi_\alpha &\sim \arg(Z_\alpha) \end{aligned}$$

where Z_α depends on the dilaton and the periods of the Kahler form and the B-field. The theta angle depends on the periods of the B-field and RR-fields.

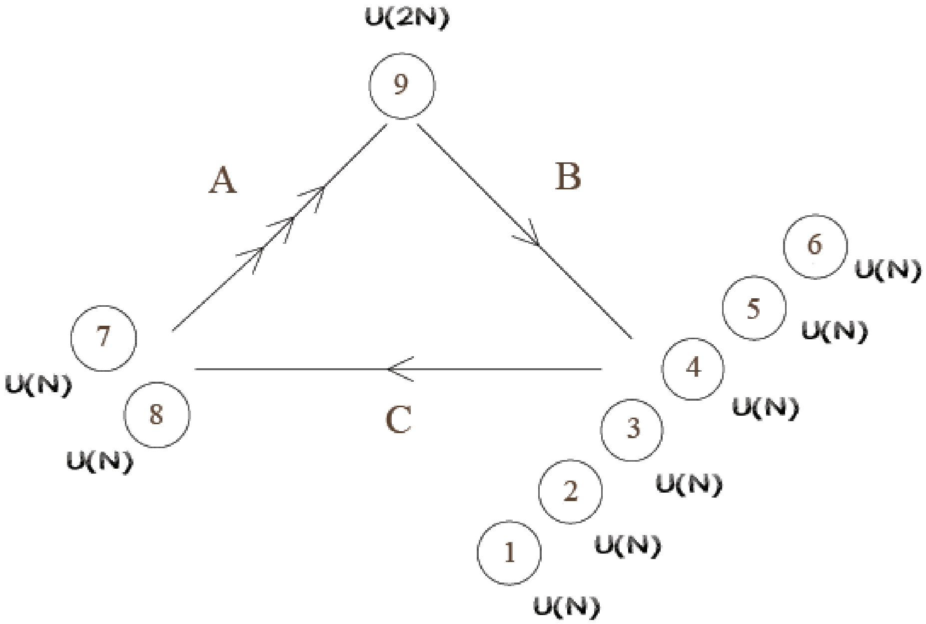
Matter lagrangian parameters

Marginal deformations: the superpotential (up to the Kahler deformations) depends on the complex structure deformations of the base of the cone, i.e. on the complex deformations of the del Pezzo itself. The dP_k surface has $2k-8$ complex structure deformations ($k>4$).

Relevant deformations and the vevs of the operators depend on the complex deformations of the cone that vanish at infinity, they deform the singularity, partially or completely.

The cone over dP_k surface has $c^\vee(E_k)-1$ complex deformations of the singularity, where $c^\vee(E_k)$ is the dual Coxeter number of E_k. For $k=3\ldots8$, it is $4,5,8,12,18,30$ respectively. The cone over $\mathbb{P}^1\times\mathbb{P}^1$ and the cone over dP_2 have 1 complex deformation.

Quiver gauge theory for the cone over dP_6



Quiver gauge theory for N D3-branes on the cone over dP_6.

dP_6 surface has one zero-cycle, one four-cycle and seven two-cycles, correspondingly there are 9 gauge groups in the theory. The matter fields are A_j^k, B_i, C_{ij}, where the indices $i = 1, \ldots, 6$ and $j = 7, 8$ label the gauge groups and $k = 1, 2, 3$ labels the three A fields.

Superpotential

The superpotential has the Yukawa couplings

$$W = \sum_{i,j,k} \lambda_{ij}^k A_j^k B_i C_{ij} \tag{10}$$

The couplings λ_{ij}^k are parameterized by the complex structure deformations of dP_6 which depend on the coordinates of the 6 blown up points $u_k^{(i)}$, $i = 1, \ldots, 6$ and $k = 1, 2, 3$.

We can choose the Yukawa couplings for $j = 7$ to be

$$\lambda_{i7}^k = u_k^{(i)} \tag{11}$$

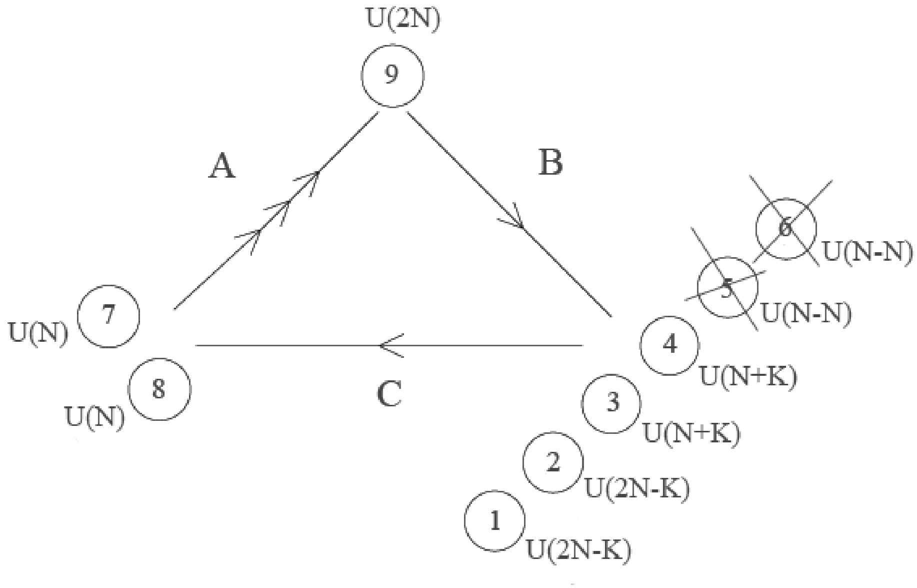
the Yukawas for $j = 8$ also depend on $u_k^{(i)}$ but in a more complicated way.

Four general points $u^{(i)}$ can be fixed by $SL(3)$ transformations of $\mathbb{P}^2$.

The remaining two points are parameterized by four complex numbers.

We will show that by adding some non-anomalous fractional branes with some choice of Yukawa couplings one can get an ISS vacuum in this quiver gauge theory.

Adding fractional branes



Restrict our attention to the fields going to the node 7 and do the Seiberg duality on the nodes $1, 2, 3$. The fields B and C going to the nodes $1, 2, 3$ form meson fields

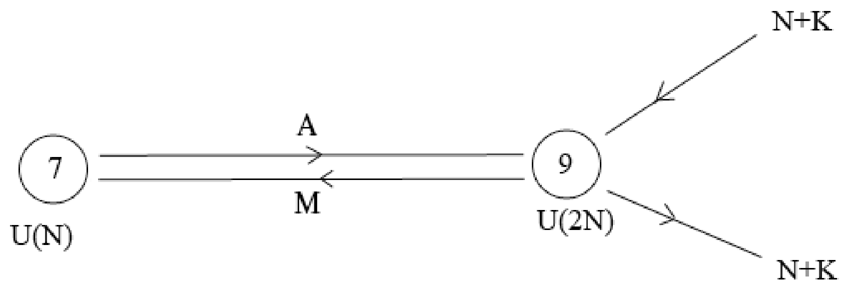
$$M_i = B_i C_{i7}/\Lambda_i, \quad i = 1, 2, 3 \tag{12}$$

The coupling of these meson fields to the A fields has the form of a mass term

$$W = \sum_{i,k} \lambda_i^k \Lambda_i A_i^k M_i \tag{13}$$

Tuning one of the complex parameters, we can make one of the masses small (this corresponds to putting the third blowup point close to the line through the first two points).

Integrating out the massive fields we get the following part in the quiver diagram



The superpotential is

$$W = m\mathrm{Tr}AM \tag{14}$$

where the mass parameter is proportional to one of the Yukawa couplings. If we tune this coupling to be small we get a small quark masses in a model similar to the UV limit of ISS.

The last step for the supersymmetry breaking is the Seiberg duality on the node 9. The new gauge group has $\widetilde{N}_c = N_c - N_f = 2N - (2N - K) = K$.

With appropriate redefinition of the fields and parameters, we get the superpotential

$$W = h\mathrm{Tr}\varphi\Phi\tilde{\varphi} - h\mu^2\mathrm{Tr}\Phi \quad (15)$$

where $N_f = N$ and $\widetilde{N}_c = K < N_f$.

Thus the SUSY is broken by the rank condition.

The differences with the original ISS construction are

- The ISS flavor symmetry is gauged.
- There are extra chiral fields going to the color node of ISS.

We can choose the gauged flavor symmetry to be weakly coupled, i.e. it will not affect the analysis.

The extra fields couple to the ISS fields through the higher dimensional operators that don't affect the SUSY breaking analysis either.

Future directions

- Find the spectrum of the masses after the SUSY breaking.
- Solve the R-symmetry problem (if the R-symmetry is unbroken, then the gaugino fields remain massless).

This is the topic of my current work with Ken Intriligator and Matt Sudano from UC San Diego.

The idea is that under the scale of SUSY breaking the gauged flavor symmetry may become strongly coupled. The condensation of the corresponding gaugino field breaks the R-symmetry.