Microscopic Formulation of Puff Field Theory

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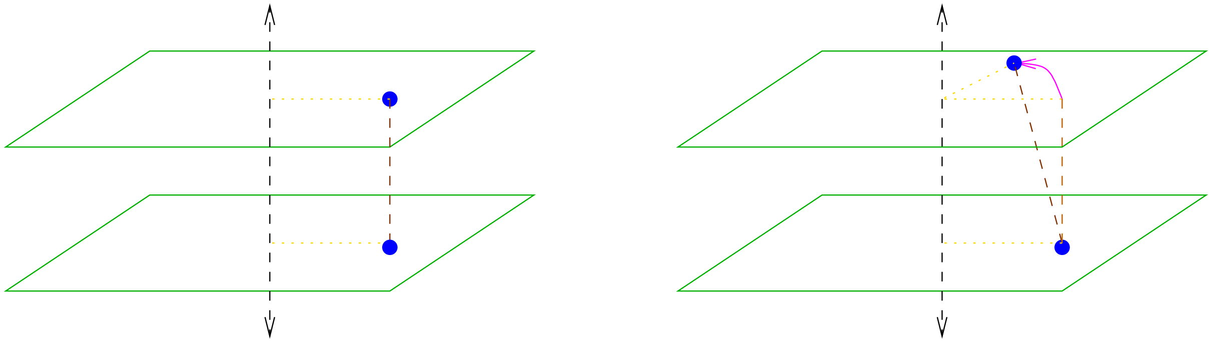
Based on work with Ganor, Jue, Kim and Ndirango
hep-th/0702030, and some on going work.

See also Ganor, hep-th/0609107

Melvin Universe: Axially symmetric solution of a system with gravity, gauge field, and possibly some scalars, with some magnetic flux along the axially symetric plane.

Melvin Universe in String Theory

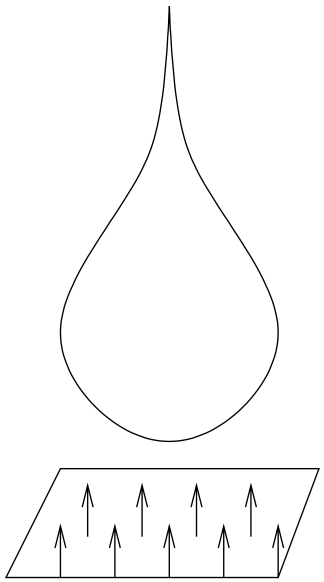
Compactification with a twist



KK-reduction/T-duality and its U-dual gives rise to Melvin solution of SUGRA

- Flat space: $ds^2 = -dt^2 + dr^2 + r^2 d\phi^2 + dz^2$
- Twist: $ds^2 = -dt^2 + dr^2 + r^2(d\phi + \eta dz)^2 + dz^2$
- T-dualize

$$
\begin{aligned}
ds^2 &= -dt^2 + dr^2 + \frac{1}{1+\eta^2 r^2}(r^2 d\phi^2 + dz^2) \\
B &= \frac{\eta r^2}{1+\eta^2 r^2} d\phi \wedge dz \\
e^{2\phi} &= \frac{g^2}{1+\eta^2 r^2}
\end{aligned}
$$



Topology: $R^{1,3}$

- Melvin Universes are secretly “flat.”
- It is just an orbifold.
- The world sheet theory is therefore exactly solvable.
- Lots of possible connections to recent work on integrable structures
- One can add D-branes and consider open strings

Lots of interesting things happen if one adds a D3-brane and take the decoupling limit.[1]

Type of Twist	Model
Melvin Twist: (z, ϕ)	Hashimoto-Thomas model • •
Melvin Shift Twist	Seiberg-Witten Model
Null Melvin Shift Twist	Aharony-Gomis-Mehen model
Null Melvin Twist	Dolan-Nappi model
Melvin Null Twist	Hashimoto-Sethi model
Melvin R Twist: (z)	Bergman-Ganor model •
Null Melvin R Twist	Ganor-Varadarajan model
R Melvin R Twist: $(\cdot)$	Lunin-Maldacena model •

[1] AH and Thomas, hep-th/0410123

Mostly non-local field theories: non-commutative gauge theories, dipole theories

Puff **F**ield **T**heory: a novel extension to this list

- $SO(d) \in SO(d,1)$
- $d = 3$ theory invariant under strong/weak coupling duality
- Simple SUGRA dual

- Consider a D0 in type IIA
- Lift to M-theory
- Twist
- Reduce back to IIA

$$ds^2 = -dt^2 + dr^2 + r^2(d\phi + \eta dz)^2 + d\vec{y}^2 + dz^2$$

$$z \sim z + g_s l_s = g_{YM0}^2 \alpha'^2, \qquad \eta = \frac{\Delta^3}{\alpha'^2}$$

$$\nu = g_{YM0}^2 \Delta^3 = \text{dimensionless, finite}$$

$$ds^2 = (1 + \eta^2 r^2)^{1/2} \left(-dt^2 + dr^2 + \frac{r^2}{1 + \eta^2 r^2} d\phi^2 + d\vec{y}^2 \right)$$

Easy to derive the SUGRA dual

Let's do the SUSY case of Melvin twists in two planes

Start with M-theory lift of D0-brane, twist:

$$ds_{11}^2 = -h^{-1}dt^2 + h(d\tilde{z} - v\,dt)^2 + d\rho^2 + \rho^2\left(ds_{B(2)}^2 + (d\phi + \eta\, d\tilde{z} + \mathcal{A})^2\right) + \sum_{i=1}^{5} dy_i^2$$

$$h(\rho, y) = 1 + \frac{gN\alpha'^{7/2}}{(\rho^2 + \vec{y}^2)^{7/2}}, \qquad v = h^{-1}\ .$$

Reduce, Decouple: $U = r/\alpha' =$ fixed

In terms of scaled variables:

$$\frac{ds^2}{\alpha'} = \sqrt{H+\Delta^6 U^2}\Bigg(-H^{-1}dt^2 + dU^2 + U^2 ds^2_{B(2)} + U^2\left(d\phi + \mathcal{A} + \frac{\Delta^3}{H}dt\right)^2 + d\vec{Y}^2\Bigg)$$

$$\frac{A}{\alpha'^2} = \frac{1}{H+\Delta^6 U^2}\left(-dt + U^2\Delta^3 d\phi\right)$$

$$e^{\phi} = g^2_{YM}(H+\Delta^6 U^2)^{3/4}$$

$$U = \frac{\rho}{\alpha'}, \quad \vec{Y} = \frac{y}{\alpha'}, \quad H(U,\vec{Y}) = \alpha'^2 h(\rho,\vec{y}) = \frac{g^2_{YM0}N}{(U^2+Y^2)^{7/2}}$$

Puff **Q**uanutm **M**echanics

Straight forward to generalize to (3+1)-dimensions

$$
\begin{aligned}
\frac{ds^2}{\alpha'} &= \sqrt{H+\Delta^6 U^2}\left(-H^{-1}dt^2 + \frac{1}{H+\Delta^6 U^2}d\vec{x}^2\right. \\
&\quad \left. + dU^2 + U^2 ds^2_{B(2)} + U^2\left(d\phi + \frac{\Delta^3}{H}dt\right)^2 + d\vec{Y}^2\right) \\
H &= \frac{g^2_{YM3} N}{U^4}
\end{aligned}
$$

Puff **F**ield **T**heory

- Unbroken $SO(3) \in SO(3,1)$
- Constant dilaton and RR 5-form flux: S-dual

These **P**uff **F**ield **T**heories can be defined as a decoupled field theory on Dp-branes for any $p \leq 5$ by T-dualizing different number of times.

Thermodynamics of Puff Field Theory

- Easy: repeat the construction starting with non-extremal D0
- $S = \left(\frac{T^{3-p}}{g^2_{YMp} N}\right)^{\frac{3-p}{5-p}} N^2 V T^p$
- Area of the horizon in Einstein frame is invariant under twists and dualities
- Same number of degrees of freedom as ordinary SYM

Microscopic formulation of Puff Field Theory

- What is the action?

Go back to Puff Quantum Mechanics

$$ds_{11}^2 = -h^{-1}dt^2 + h(d\tilde{z} - vdt)^2 + d\rho^2$$
$$+\rho^2(ds_{B(2)}^2 + (d\phi + \eta d\tilde{z} + \mathcal{A})^2) + \sum_{i=1}^{5} dy_i^2$$

If $g_{YM0}^2 \Delta^3 = \tilde{R}\eta = -b/d =$ rational, there is an $SL(2,Z)$

$$\begin{pmatrix} d\phi \\ \frac{d\tilde{z}}{\tilde{R}} \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d\phi \\ \frac{d\tilde{z}}{\tilde{R}} \end{pmatrix}$$

Then, the new IIA description is

$$
\begin{aligned}
ds^2 &= -h^{-1/2}dt^2 + h^{1/2}\left(d\rho^2 + \rho^2 ds^2_{B(2)} + \rho^2\left(\frac{d\phi}{d} + \mathcal{A}\right)^2 + d\vec{y}^2\right) \\
A &= -c\tilde{R}d\phi - vdt \\
e^{\phi} &= h^{3/4}
\end{aligned}
$$

Other than the seemingly innocent 1-form $c\tilde{R}d\phi$, this is just a Z_d orbifold of decoupled D0 $\longrightarrow$ local theory

$SL(2, Z)$ also acts on the rank of the gauge group $N \longrightarrow d^2N$, as well as coupling, etc.

Twisted Quiver Quantum Mechanics

• Such duality between local and non-local field is familiar from non-commutative gauge theories: Morita Equivalence

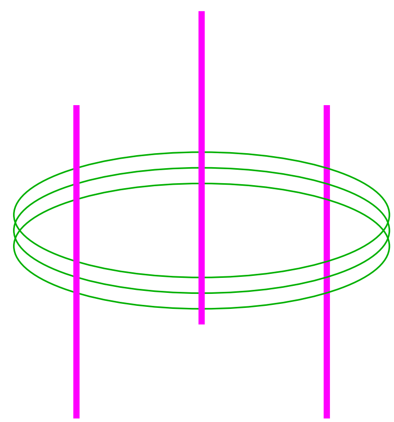
• Not to be confused with Seiberg-Witten correspondence

• What does $\frac{c}{d}(d\tilde{R})d\phi$ do to the quiver quantum mechanics? (In the case of non-commutative geometry, the analogue was 't Hooft non-Abelian flux AH and Ithzaki)

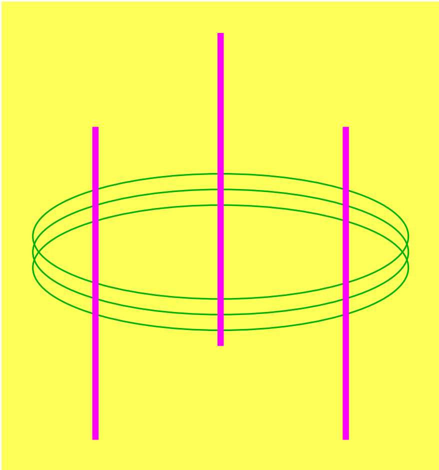
Strategy:

$$ds^2 = -h^{-1/2}dt^2 + h^{1/2}\left(d\rho^2 + \rho^2 ds^2_{B(2)} + \rho^2\left(\frac{d\phi}{d} + \mathcal{A}\right)^2 + d\vec{y}^2\right)$$

- Embed $\rho, B(2), \phi$ in Taub-NUT (can be removed later)
- Then, it is easier to visualize T-dualizing on ϕ
- Ignoring $\frac{c}{d}(d\tilde{R})d\phi$, the T-dual is decoupled dN D1 with d NS5 impurities



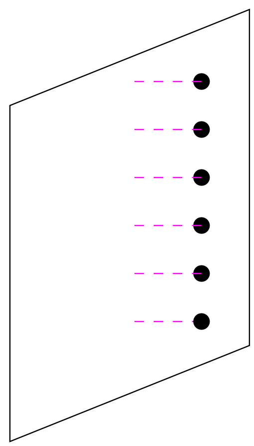
• In this picture $\frac{c}{d}(d\tilde{R})d\phi$ becomes the RR axion $\chi = \frac{c}{d}$
• 2 T-dualities ($\parallel$ NS5, $\perp$ D1) followed by S-duality maps this to dN D3-brane on T^2 with d D5 impurities with constant NSNS B-field $B = \frac{c}{d}$



• Simple NSNS background with D-branes: Seems definable microscopically (c.f. De Wolfe, Freedman, Ooguri)

Further T-dualize along the D3-brane (direction parallel to the original D1)

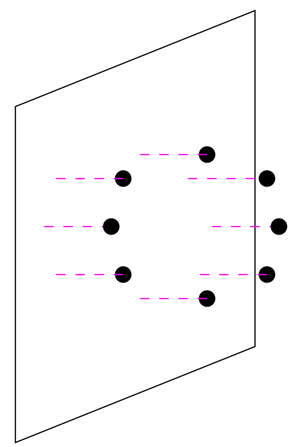
∞ array of D2-branes and D6-branes[2], in NSNS 2-form background



Usual $U(\infty)/Z_\infty$ gauge theory from compactifications of Matrix theory with flavor (2-6 strings)

[2] AH and Cherkis, hep-th/0210105

Useful to think as the $N \to \infty$ limit of $U(N)/Z_N$ quiver theory



This is essentially deconstruction

There is still the NSNS B-field: $B = \frac{c}{d}$ along the world volume of the D2

This is not Seiberg-Witten scaled: not non-commutativity

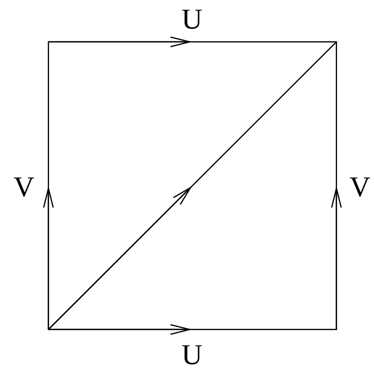
Instead, it corresponds to 't Hooft non-abelian flux

On S^1, gauge fields are identified up to gauge transformation

$$A \to A_U$$

On T^2,

$$A \to A_{UV} = A_{VU}$$



or $U^{-1}V^{-1}UV$ must act trivially on A. For adjoints, center $Z_N \in SU(N)$ acts trivially.

This gives rise to 't Hooft's non-abelian flux.

The flux $\frac{c}{d}$ is permissible since the rank of the gauge group is divisible by d.

The presence of D6 gives rise to matter in fundamental representation which is *not* invariant under center group Z_N.

If one also twists the flavor index, however, the (bi)fundamental does become invariant under the center group Z_N.

"Quark Fields In Twisted Reduced Large N QCD," Sumit R. Das *Phys. Lett.* **B132** (1983) 155

Conclusion:

PQM is a

- scaling limit
- of a strong coupling limit
- of a large N deconstruction limit
- of 2+1 dimensional SYM
- with 't Hooft flux
- and matter in fundamental representation
- with twisted flavor.

The statement is at the same level as that of Little String Theory in terms of deconstruction.[3]

[3] Arkani-Hamed, Cohen, Kaplan, Karch, Motl hep-th/0110146