Towards a Holographic Model of Color Flavor Locking Phase

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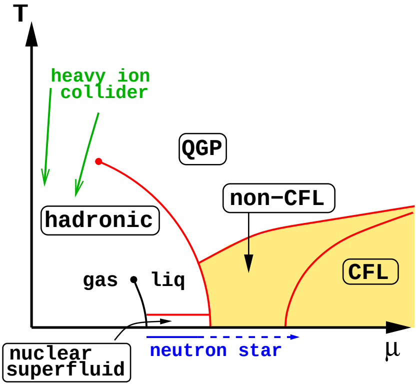
- Based on 0909.1296 [hep-th] + work in progress, with Koji Hashimoto (RIKEN) and Shunji Matsuura (KITP).

1. Motivation/Introduction
2. D3-D7 system in $AdS_5 \times S^5$.
3. Color-Flavor Locking phase (CFL) from baryon density backreaction.
4. Finite temperature generalization.
5. Future directions.



- At asymptotically large chemical potential μ, perturbative QCD calculation helps.
- At very small chemical potential μ, lattice QCD simulation is useful.
- However at intermediate $\mu \sim 10^2$ Mev, QCD is strongly coupled, perhaps gauge/string duality may give us a new handle.

- Due to asymptotic freedom, when $E_F \gg 1$ and $g_{QCD}(E_F) \ll 1$, there is a Fermi surface of weakly interacting quarks.
- Near Fermi surface $E_F \sim \mu$, it costs no free energy $F = E - \mu N$ to add/subtract a particle/hole.
- Including the weakly attractive gluon exchange between a pair of quarks with antisymmetric color wave functions, the pair formation is inevitable!
- Contrasting with conventional BCS theory of superconductivity, Coulombic interaction between a pair of electrons is repulsive, need phonon exchange for Cooper pair. We need to understand complicated band structures.

- Specifically, the di-quark state can be decomposed into:

$$< \psi_i^\alpha C\gamma_5 \psi_j^\beta > \propto \Delta_{CFL} \epsilon^{\alpha\beta A} \epsilon_{ijA} + \kappa \Delta_{CFL} (\delta_i^\alpha \delta_j^\beta + \delta_j^\alpha \delta_i^\beta), \quad \kappa \ll 1, \quad (1)$$

  where color indices α, β and flavor indices i, j run from 1 to 3. The gap parameter Δ_{CFL} can be computed perturbatively.

- The $\bar{3}_c$ state ($\epsilon^{\alpha\beta A}\epsilon_{ijA}$) is energetically favored, through single gluon exchange and instanton induced interaction.

- The $\bar{3}_c$ state is only invariant under combined/locked color + flavor rotation, the symmetry breaking pattern is:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \longrightarrow SU(3)_{c+L+R} \times \mathbb{Z}_2 . \quad (2)$$

The $SU(3)_c$ symmetry is spontaneously broken, CFL phase appears as the "Higgs phase" of QCD. The $SU(3)_L \times SU(3)_R$ chiral symmetry is also broken by $< \psi_L \psi_L >$ or $< \psi_R \psi_R >$, differs from $< \overline{\psi}_R \psi_L >$.

- If we lower μ, strange quark mass M_s cannot be ignored, such effect can split the Fermi momentum p_f, and pairing of strange quarks with lighter up/down quarks can be disfavored if $M_s^2/\mu \gtrsim \Delta_{CFL}$.
- More exotic phases such as 2 SC phase, crystalline color superconductivity or even nuclear superfluid can appear.
- These regimes are generally strongly coupled $g(\mu) \gg 1$, perhaps holography would offer new understanding here, such as computing the Δ_{CFL}.

- Following Weinberg (1986), we can associate superconductivity with spontaneous breaking of local symmetries.
- Holographic superconductors/superfluidity:
  Spontaneous breaking of certain local gauge symmetries in the bulk $AdS_{d+1} \times X \longrightarrow$ spontaneous breaking of global symmetries in boundary d-dimensional theory.
- For example, "p-wave" superconductor, involving a $SU(2)$ gauge field $A^a_\mu \tau^a$ and identifying τ^3 as EM $U(1)$:

$$A = \phi(r)\tau^3 dt + w(r)(\tau^- dz + \tau^+ d\bar{z}). \tag{3}$$

- If the components $A^\pm_\mu$ condense through thermal effects, this spontaneously breaks the EM $U(1)$, and gives rise to Meissner effect.

Let us begin by considering the case with zero temperature.

- Consider usual $AdS_5 \times S^5$ background generated by N_c D3 branes:

$$ds^2 = (r_6^2/R^2)\eta_{\mu\nu}dx^\mu dx^\nu + (R^2/r_6^2)\left(dr_6^2 + r_6^2 ds_5^2\right),$$
$$g_s C_4 = r_6^4/R^4 d^4x, \quad R^4/\alpha'^2 = 4\pi g_s N_c = \lambda.$$

- Rewriting $dr_6^2 + r_6^2 ds_5^2 = dr^2 + r^2 ds_3^2 + dy^2 + dz^2$, we also introduce $N_f \ll N_c$ flavor D7 branes wrapping a 4 cycle $dr^2 + r^2 ds_3^2$.
- In static gauge, the transverse directions $(y(r), z(r))$ are D7 fields, using $U(1)$ symmetry to set $z(r) = 0$ and $r_6^2(r) = r^2 + y(r)^2$, the D7 pullback metric is:

$$G_{ab}d\xi^a d\xi^b = \frac{r_6^2}{R^2}(\eta_{\mu\nu}dx^\mu dx^\nu) + \frac{R^2}{r_6^2}\left((1 + (y'(r))^2)dr^2 + r^2 ds_3^2\right). \tag{4}$$

- We also introduce the "Baryon Chemical Potential", which is identified with a $U(1)$ gauge field $A_t(r)$, the D7 DBI action is

$$S_{\mathrm{DBI}}^{\mathrm{D7}}/V_4 = -\mathcal{N}\int dr\, r^3\sqrt{(1+(y'(r)^2)-(2\pi\alpha' A_t'(r))^2}, \tag{5}$$

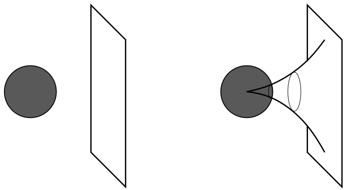
where $\mathcal{N} = N_f \mathcal{T}_{\mathrm{D7}}(2\pi^2)g_s^{-1}$. We can define constants of motion $\mathbf{c}$ and $\mathbf{d}$ for $y'(r)$ and $2\pi\alpha' A_t'(r)$, and solve them exactly:

$$y(r) = \frac{\mathbf{c}}{2\,3^{1/4}\mathcal{N} r_0^2}\mathbb{F}\left(\varphi(r), \frac{2+\sqrt{3}}{4}\right), \quad 2\pi\alpha' A_t(r) = \frac{\mathbf{d}}{\mathbf{c}}y(r),$$

$$\varphi(r) = \arccos\left(\frac{1-(\sqrt{3}-1)(r/r_0)^2}{1+(\sqrt{3}+1)(r/r_0^2)^2}\right), \; r_0^6 = \frac{\mathbf{d}^2-\mathbf{c}^2}{\mathcal{N}^2}. \tag{6}$$

- Asymptotically, we can also relate the quark mass m and μ with $\mathbf{c}$ and $\mathbf{d}$:

$$\mathbf{c} = \gamma\mathcal{N}(2\pi\alpha')^3(\mu^2-m^2)m, \quad \mathbf{d} = \gamma\mathcal{N}(2\pi\alpha')^3(\mu^2-m^2)\mu. \tag{7}$$



- The D7 brane is deformed into a “Spiky configuration”, known as the Black Hole Embedding. On the deformed 4-cycle with induced metric:

$$ds_4^2 = \left(1 + (y'^2 - (2\pi\alpha' A_t')^2)\right) dr^2 + r^2 ds_3^2 = \left(\frac{r^6}{r^6 + r_0^6}\right) dr^2 + r^2 ds_3^2 \,. \tag{8}$$

  now turn on $SU(N_f)$ field strength, and look for the instanton solution, which are n_c D3 branes.

- The n_c instantons arise from splitting n_c D3 branes from the boundary, with $SU(N_c) -> S(U(N_c - n_c) \times U(n_c))$.

- It turns out that the 4-cycle can be mapped to a comformally flat metric using the coordinate transformation $r = \xi(1 - r_0^6/4\xi^6)^{1/3}$, and one-instanton is:

$$A_i^{\text{inst.}}(\xi) = \frac{2\rho^2 \xi^j \tau_{ji}}{\xi^2(\xi^2+\rho^2)}, \quad F_{ij} = *_4 F_{ij}. \tag{9}$$

  where $\tau_{ij} = \frac{1}{4}(\overline{\tau}_i \tau_j - \overline{\tau}_j \tau_i)$, and ρ is the volume modulus.

- The instanton moduli space of n_c instantons $\mathscr{I}_{n_c}$ is isomorphic to the Higgs branch ($< q > \neq 0$) of n_c D3 branes $\mathscr{M}_{\text{Higgs}}$.

- There also exists Coulomb branch $\mathscr{M}_{\text{Coulomb}}$ on n_c D3 branes ($< q >= 0$), we are interested in obtaining moduli potential, and making Higgs branch energetically favorable.

- In $AdS_5 \times S^5$ background (1), we can evaluate the potential on the volume modulus ρ by considering both DBI and CS actions. For self-dual configuration $F_{ij} = *_4 F_{ij}$, D7-DBI action simplifies into:

$$S_{\rm DBI} = -\mathcal{T}_7 \int d^4x d^4\xi \left(1 + \frac{(2\pi\alpha')^2}{4} \frac{r_6^4}{R^4} (F_{ij}(\rho) *_4 F^{ij}(\rho)) \right). \qquad (10)$$

- While the CS action from the pull back of C_4 yields:

$$S_{\rm CS} = \mathcal{T}_7 \int d^4x d^4\xi \frac{(2\pi\alpha')^2}{4} \frac{r_6^4}{R^4} (F_{ij}(\rho) *_4 F^{ij}(\rho)). \qquad (11)$$

- In such case, the instanton parts from DBI and CS identically canceled, no potential is generated and no energetically favored Higgs branch/CFL phase.

- The baryon density $A_t(r)$: Identified with fundamental strings dissolved in D7s, which carries B_2 field.
- Taking into small backreaction due to F1s, which source bulk NS-NS H_3 and through SUGRA eoms, the RR F_3.
- Consider only linearized perturbation in B_2, the source can be derived from expanding D7 DBI action:

$$-\mathbf{d}\int d^4x dr \left(B_{0y}y' + B_{0r}\right). \tag{12}$$

- Adding this to the bulk IIB supergravity action, and restrict to the background C_4 of $AdS_5 \times S^5$:

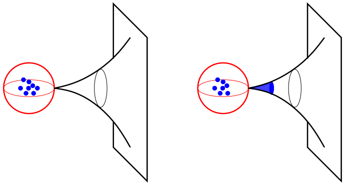
$$S_{\mathrm{B}} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x(-g_{10})^{1/2}(e^{-2\Phi}|H_3|^2 + \frac{1}{2}|G_5|^2) + \frac{1}{4\kappa_{10}^2}\int F_5 \wedge B_2 \wedge F_3,$$

where $G_5 = F_5 - 1/2C_2 \wedge H_3 + 1/2B_2 \wedge F_3$.

- Near $r \sim 0$, the linearized equation of motion for F_3 and H_3 then yield:

$$F^{(3)}_{123} = \frac{8\pi^3 \alpha'^2 \mathbf{d}}{N_c} \,. \tag{13}$$

- A complementary description: F_3 is sourced by D5 Baryon vertices at $r_6 = 0$, as required by charge conservation of D3-D7 strings ending at $r_6 = 0$.



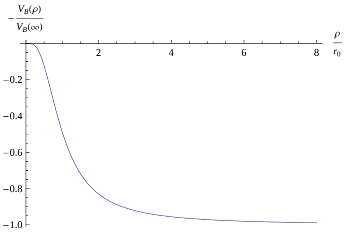
An analogous computation using D5 as localized source also yields (13).

- Given the F_3, we can compute the additional potential $V_B(\rho)$ for the volume modulus ρ through the CS term:

$$\mathcal{T}_7 \int (2\pi\alpha')^3 A\wedge F_3 \wedge F \wedge F = \frac{1}{8(2\pi)^4\alpha'} \int d^4x F^{(3)}_{123} \int d^4\xi \,\mathrm{tr}\left[A_0 \epsilon^{ijkl} F_{ij} F_{kl}\right]. \tag{14}$$

- Substituting the explicit expression for F_3 and instanton profile F_{ij}, the potential $V_B(\rho)$ is:

$$V_B(\rho) \quad = \quad -\frac{2\pi\alpha' \mathbf{d}}{\mathcal{N} N_c} \int_0^\infty dr \, \frac{\mathbf{d}}{\sqrt{r^6 + r_0^6}} \frac{\rho^4(3\xi^2(r)+\rho^2)}{(\xi^2(r)+\rho^2)^3}. \tag{15}$$



- The potential $V_B(\rho)$ drives ρ to non-zero value and makes Higgs branch energetically favorable.
- The three form flux F_3 here is only valid for $r \to 0$. However at small ρ, $V_B(\rho)$ is cut-off independent, and should be considered as the beginning of the condensation.
- At large r, we expect F_3 to decay to zero; for $\rho \to \infty$, $F_{ij}(\rho) *_4 F^{ij}(\rho) \to 0$, we expect $V_B(\rho \to \infty) \to 0$. We speculate that finite minimum for ρ, and can be verifed by complete F_3.

To understand the exact symmetry breaking/locking pattern, focus on $U(n_c) \times U(N_f)$ color-flavor groups, and the ADHM data. Equivalent to the Higgs branch of $U(n_c)$ theory.

- The ADHM data is encoded in two $n_c \times n_c$ complex matrices $\mathbf{B}_{1,2}$, and two $n_c \times N_f$ complex matrices $\mathbf{I}$ and $\mathbf{J}$. The $U(n_c) \times U(N_f)$ symmetry acts as:

$$\mathbf{I}^\dagger \longrightarrow U\mathbf{I}^\dagger V^{-1}, \quad \mathbf{J} \longrightarrow U\mathbf{J}V^{-1} \tag{16}$$

  where $U \in U(N_f)$ and $V \in U(n_c)$.

- For simplicity, consider $N_f = 2$ and the 't Hooft ansatz with all n_c instantons at the origin, $\mathbf{B}_{1,2}$ are diagonal and ADHM constraints reduce to

$$\mathbf{I}\mathbf{I}^\dagger = \mathbf{J}^\dagger\mathbf{J}, \quad \mathbf{I}\mathbf{J} = 0, \tag{17}$$

  they are equivalent to (part of) D-term and F-term conditions on n_c D3s.

- The constraints can be solved by the following matrices:

$$I^\dagger = \begin{pmatrix} \rho_1 & \rho_2 & \cdots & \rho_{n_c} \\ 0 & 0 & \cdots & 0 \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \rho_1 & \rho_2 & \cdots & \rho_{n_c} \end{pmatrix}, \qquad (18)$$

  where $\{\rho_1, \ldots, \rho_{n_c}\}$ are the volume moduli of n_c instantons.

- To identify the unbroken symmetry, consider $n_c = 1$ so that $\mathbf{I}^\dagger = (\rho, 0)^T$ and $\mathbf{J} = (0, \rho)^T$ and the transformations leaving $\mathbf{I}^\dagger$ and $\mathbf{J}$ invariant are:

$$U = \begin{pmatrix} e^{i\alpha_1} & 0 \\ 0 & e^{i\alpha_2} \end{pmatrix}, \quad \mathbf{I}^\dagger : V = e^{i\alpha_1}, \; \mathbf{J} : V = e^{i\alpha_2}, \qquad (19)$$

  where $\alpha_{1,2} \in \mathbb{R}$, therefore we need $\alpha_1 = \alpha_2$.

- The global part of $U(1)_{\text{color}}$ is locked with the diagonal $U(2)_{\text{flavor}}$:

$$U(1)_{\text{color}} \times U(2)_{\text{flavor}} \longrightarrow U(1)_{\text{CFL}} \qquad (20)$$

- The local part of $U(1)_{\text{color}}$ and $SU(2)_{\text{flavor}} \subset U(2)_{\text{flavor}}$ are spontaneously broken, we expect boundary theory to exhibit both superconductivity and superfluidity.
- We can extend the analysis to n_c instantons and N_f D7 branes, the symmetry breaking pattern in our simplified set up is:

$$U(n_c)_{\text{color}} \times U(N_f)_{\text{flavor}} \to U(1)_{\text{CFL}} \times U(N_f - 2)_{\text{flavor}} . \qquad (21)$$

- Notice that we have restricted our analysis to 't Hooft instantons, which does not cover the entire moduli space, there maybe enhanced residual symmetries.

Contrasting with earlier papers on Holographic superconductors/superfluidity, we have achieved:

- An explicit mechanism/potential for spontaneously breaking of both color and flavor symmetries through the backreaction of baryon density.
- The field theory dual is explicitly known, such that condensed scalar and its canonical normalization can be identified.
- We established the onset of scalar condensation, given complete solution for F_3, we would be able to answer about the stability of the vacuum.

We can extend our analysis for color-flavor locking phase to $T \neq 0$ case, by considering D7 branes in AdS-Black Hole geometry.

- In the Poincare coordinates, the metric and C_4 are given by:

$$
\begin{aligned}
ds^2 &= (u^2/R^2)\left(-(f_-^2/f_+)dt^2 + f_+ dx_3^2\right) + (R^2/u^2)\left(du^2 + u^2 ds_5^2\right), \\
C_4 &= \frac{1}{4R^4}\left(u^2 + u_0^4/u^2\right)^2 d^4x. \qquad (22)
\end{aligned}
$$

  where $f_\pm = 1 \pm u_0^4/u^4$ with u_0 the location of the horizon.

- Turning on the baryon density $A_t(u)$, and rewriting $ds_5^2 = d\theta^2 + \sin^2\theta ds_3^2 + \cos^2\theta d\phi^2$ and $\chi = \cos\theta$, the D7 action $S_{DBI}/V4$ is:

$$
-\mathcal{N}\int du \frac{u^3 f_- f_+ (1-\chi^2)}{4}\sqrt{1-\chi^2+u^2\chi'^2-(2\pi\alpha' A_t')^2\frac{2f_+(1-\chi^2)}{f_-^2}}\,. \qquad (23)
$$

- Near $u = u_0$, the D7 embedding can be approximated by:

$$\chi = \chi_0 \,, \quad \chi' = 0 \,. \tag{24}$$

We can similarly show that the resultant 4-cycle is conformally flat, and write down the instanton solution.

- At $T \neq 0$, the instanton volume modulus ρ receives two different potentials: 1.Thermal effects. 2.Backreaction due to baryon density.

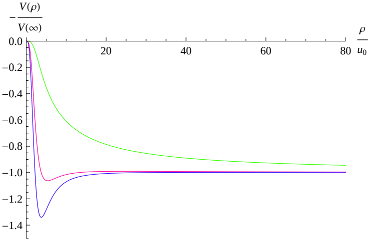
- The thermal effect $V_T(\rho)$ can be computed from combining $S_{\rm DBI}$ and CS term $\int C_4 \wedge F \wedge F$, they no longer cancel and give:

$$V_T(\rho) \quad = \quad -\frac{\mathcal{N}}{4R^4} \int d\zeta \, u^4 f_+ (f_+ - f_-) \frac{(2\pi\alpha')^2}{8} \frac{192\rho^4\zeta^3}{(\zeta^2+\rho^2)^4} \,. \tag{25}$$

- While for the backreaction, we can perform similar computation to obtain $F^{(3)}_{123} = \frac{8\pi^3\alpha'^2\mathbf{D}}{N_c}$, and the induced potential $V_B(\rho)$ is

$$V_B(\rho) = -\frac{\mathbf{D}^2}{N_c\mathcal{N}}\int du \frac{2f\rho^4}{\sqrt{f_+}\sqrt{u^6 f_+^3(1-\chi_0^2)^3 + 8(\mathbf{D}/\mathcal{N})^2}} \frac{(3\zeta^2+\rho^2)}{(\zeta^2+\rho^2)^3}.$$

- Here we plotted the combined potential $V_T(\rho) + V_B(\rho)$ on the instanton volume modulus ρ



- Given the CFL phase, it would be interesting to consider the holographic superfluid in this setup, by considering the fluctuations.
- Constructing the explicit vortex solution carrying quantized magnetic fields and study their dynamics.
- Generalize to other set up such as D4/D6 system, which flows to pure bosonic Yang-Mills at low energy, here we study the expansion of D4-monopole on D6 branes.
- Searching for the complete backreacted SUGRA solution involving D5 baryon vertices + F1 strings, this would help understanding the fate of CFL phase.