A1

Lecture 5 Addendum

① $$W_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} P^\nu M^{\rho\sigma}$$

$$\downarrow$$

$$M^{\rho\sigma} = \underset{\text{spin}}{S^{\rho\sigma}} + \underset{\text{orbital}}{i(x^\rho\partial^\sigma - x^\sigma\partial^\rho)}$$

Orbital part drops out

$$W_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} P^\nu S^{\rho\sigma}$$

For massive states $W^2 = -m^2 s(s+1)$ intrinsic spin

② For massless states $W^2 = P^2 = 0$ & $W_\mu P^\mu = 0$

$$\Rightarrow \quad W_\mu = \lambda P_\mu$$

Consider W_0 component:

$$\lambda = \frac{\vec{P}\cdot\vec{S}}{P_0} = \hat{P}\cdot\vec{S}$$ def of helicity

λ has magnitude $= |\vec{S}|$ and is real

$$\therefore \quad \lambda = \pm S$$

(3) Recall in two-component form

$$\{ Q_\alpha, \bar{Q}_{\dot\beta} \} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$\{ Q_\alpha, Q_\beta \} = \{ \bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta} \} = 0$$

Up to normalization, Q_α and $\bar{Q}_{\dot\beta}$ are the annihilation & creation operators of Clifford algebra

Last time, we constructed a state

$$|p, \lambda'\rangle = Q_1 Q_2 |p, \lambda\rangle$$

This is the Clifford vacuum.

For the same reason that $\bar{Q}_{\dot i}$ changes the helicity by $1/2$, Q_1 & Q_2 also changes the helicity by $1/2$ unit.

This follows from

two component form
$$\begin{cases} [M_{\mu\nu}, Q_\alpha] = -\frac{1}{2} (\sigma_{\mu\nu})_{\alpha\beta} Q_\beta \\ [M_{\mu\nu}, \bar{Q}_{\dot\alpha}] = -\frac{1}{2} (\bar\sigma_{\mu\nu})_{\dot\alpha\dot\beta} Q_{\dot\beta} \end{cases}$$

massless state
$$[W_0, \bar{Q}^{\dot\alpha}] |p\lambda\rangle = -\frac{1}{2} P_0 (\sigma^3 Q)^{\dot\alpha} |p, \lambda\rangle$$

etc

A3

(4) Particles with spin $> 1/2$ related to gauge sym to decouple ghosts in S-matrix

Spin 1 : Yang-Mills

Spin 2 : general coordinate inv.

Spin 3/2 : local SUSY

Given a spin 3/2 field (Rarita-Schwinger field) construct a current

$$J_\mu = \varepsilon_{\mu\nu\rho\sigma}\gamma^\nu\partial^\rho\psi^\sigma$$

suppress spinorial index

Spin 3/2 field

$$\partial^\mu J_\mu = 0$$

Introduce a term $\mathcal{L}' = -i\bar{\psi}_\mu J^\mu$ to cancel extra terms since we promote SUSY to local $\alpha \to \alpha(x)$

But $\delta_{\alpha(x)}\mathcal{L}' = ik\bar{\psi}_\mu\gamma_\nu T^{\mu\nu}\alpha + \ldots$

stress tensor

Need $\mathcal{L}_{Grav} = -g_{\mu\nu}T^{\mu\nu}$ to cancel these terms

$\Rightarrow$ SUGRA. We need to include spin 2 field in multiplet.