PHY 801 HW # 2 Solutions

Note Title 3/4/2008

1 a.) $\bar{\psi}\gamma_5\chi = (\bar{\psi}\gamma_5\chi)^T$

$= -\chi^T\gamma_5\bar{\psi}^T$ $\quad$ ($\because \chi, \psi$ Grassman)

$= -\chi^T\gamma_5 C^{-1}\psi$ $\quad$ ψ = Majorana

$= \chi^T\gamma_5 C\psi$ $\quad$ $C^{-1} = -C$

$= \chi^T C\gamma_5\psi$ $\quad$ $[\gamma_5, C] = 0$

$= \bar{\chi}\gamma_5\psi$

b) $\bar{\psi}\gamma_\mu\chi = (\bar{\psi}\gamma_\mu\chi)^T = -\chi^T\gamma_\mu\bar{\psi}^T$

$= -\chi^T\gamma_\mu^T C^{-1}\psi$

$= -\chi^T C\gamma_\mu\psi$ $\quad$ $\left(\because C^{-1}\gamma_\mu^T C = -\gamma_\mu,\ C^{-1} = -C\right)$

$= -\bar{\chi}\gamma_\mu\psi$

c) $\bar{\psi}\gamma_\mu\gamma_5\chi = (\bar{\psi}\gamma_\mu\gamma_5\chi)^T = -\chi^T\gamma_5^T\gamma_\mu^T\bar{\psi}^T$

$= -\chi^T\gamma_5\gamma_\mu^T C^{-1}\psi$

$= -\chi^T\gamma_5 C\gamma_\mu\psi$

$= -\chi^T C\gamma_5\gamma_\mu\psi$

$= \chi^T C\gamma_\mu\gamma_5\psi$ $\quad$ $\{\gamma_\mu, \gamma_5\} = 0$

$= \bar{\chi}\gamma_\mu\gamma_5\psi$

d) $\bar{\psi}\sigma_{\mu\nu}\chi = \left[\bar{\psi}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\chi\right]^T$

$$= -\chi^T(\gamma_\nu^T\gamma_\mu^T - \gamma_\mu^T\gamma_\nu^T)\bar{\psi}^T$$

$$= \chi^T C(C^{-1}\gamma_\nu^T C C^{-1}\gamma_\mu^T - C^{-1}\gamma_\mu^T C C^{-1}\gamma_\nu^T) C\psi$$

$$= \bar{\chi}(\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu)\psi$$

$$= -\bar{\chi}\sigma_{\mu\nu}\psi$$

If $\psi = \chi$, the anticommutator is not zero

$$\{\psi_a^\dagger(x), \psi_c(x)\} = (2\pi)^3\delta^3(0)\delta_{ac}$$

The bilinears (where $\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$)

$$\bar{\psi}\Gamma\psi = \psi_a^\dagger\gamma^0_{ab}\Gamma_{bc}\psi_c$$

$$= \{\psi_a^\dagger, \psi_c\}\gamma^0_{ab}\Gamma_{bc} - \psi_c\psi_a^\dagger\gamma^0_{ab}\Gamma_{bc}$$

$$= (2\pi)^3\delta^3(0)\underbrace{\gamma^0_{ab}\Gamma_{ba}}_{\mathrm{Tr}(\gamma^0\Gamma)} + \text{stuff}$$

$\mathrm{Tr}(\gamma^0\Gamma) = 0$ except $\Gamma = \gamma^0$, so the only subtlety is the second one with $\mu = 0$.

The identity $:\bar{\psi}\gamma^0\psi: = -:\bar{\psi}\gamma^0\psi:$ remains valid since normal ordering drops $\delta^3(0)$ term.

2) $\mathcal{L}_{mass} = -m\left[\frac{1}{2}\bar{\Psi}\Psi - GA - FB\right]$

$$\delta\mathcal{L}_{mass} = -m\left[\frac{1}{2}\delta\bar{\Psi}\Psi + \frac{1}{2}\bar{\Psi}\delta\Psi - \delta G A - G\delta A - \delta F B - F\delta B\right]$$

$$= -m\left[\frac{1}{2}\left(-F\bar{\alpha} + iG\bar{\alpha}\gamma_5 - \bar{\alpha}\gamma_5\not{\partial}A - i\bar{\alpha}\not{\partial}B\right)\Psi\right.$$

$$+ \frac{1}{2}\bar{\Psi}\left(-F\alpha + iG\gamma_5\alpha + \not{\partial}\gamma_5 A\alpha + i\not{\partial}B\alpha\right)$$

$$- \bar{\alpha}\gamma_5\not{\partial}\Psi A - iG\bar{\alpha}\gamma_5\Psi$$

$$\left. - i\bar{\alpha}\not{\partial}\Psi B + F\bar{\alpha}\Psi\right]$$

Consider second term

$$\bar{\Psi}\left(-F\alpha + iG\gamma_5\alpha + \not{\partial}\gamma_5 A\alpha + i\not{\partial}B\alpha\right) = \left[\bar{\Psi}\left(-F\alpha + \cdots\right)\right]^T$$

$$= -\alpha^T\left(-F + i\gamma_5^T G\right)\gamma^0\Psi^* - \partial_\mu\left(\alpha^T\left(iB\gamma^{\mu T} + A\gamma_5^T\gamma^{\mu T}\right)\right)\gamma^0\Psi^*$$

Grassman # anticommute

$$= -\bar{\alpha}C^T\left(-F + i\gamma_5 G\right)C^T\Psi - \partial_\mu\left(\bar{\alpha}C^T\left(iB\gamma^{\mu T} + A\gamma_5\gamma^{\mu T}\right)\right)C^T\Psi$$

$$= \bar{\alpha}C\left(-F + i\gamma_5 G\right)C^{-1}\Psi + \partial_\mu\left(\bar{\alpha}C\left(iB\gamma^{\mu T} + A\gamma_5\gamma^{\mu T}\right)\right)C^{-1}\Psi$$

$$= \bar{\alpha}\left(-F + i\gamma_5 G\right)\Psi - \partial_\mu\left(\bar{\alpha}\left(iB\gamma^\mu + A\gamma_5\gamma^\mu\right)\right)\Psi$$

since $[F, C] = [G, C] = [\gamma_5, C] = 0$, $C\gamma^\mu C^{-1} = -\gamma^{\mu T}$

$\therefore$ We brought it to the form of the 1st term.

$$\delta \mathcal{L}_{mass} = -m \Big[F\bar{\alpha}\psi\left(-\tfrac{1}{2}-\tfrac{1}{2}+1\right) + iG\,\bar{\alpha}\gamma_5\psi\left(\tfrac{1}{2}+\tfrac{1}{2}-1\right)$$

$$-\tfrac{1}{2}\bar{\alpha}\gamma_5 \partial\!\!\!/ A\psi - \tfrac{1}{2}\partial_\mu(\bar{\alpha}A\gamma_5\gamma^\mu)\psi - \bar{\alpha}\gamma_5\partial\!\!\!/(\psi)A$$

$$-\frac{i}{2}\bar{\alpha}\partial\!\!\!/(B)\psi - \frac{i}{2}\partial_\mu(\bar{\alpha}B\gamma^\mu)\psi - i\bar{\alpha}\partial\!\!\!/\psi B \Big]$$

$$= -m\Big[-\partial_\mu(\bar{\alpha}\gamma_5\gamma^\mu A)\psi - \bar{\alpha}\gamma_5\gamma^\mu A\partial_\mu\psi$$

$$-i\partial_\mu(\bar{\alpha}B\gamma^\mu)\psi - i\bar{\alpha}B\gamma^\mu\partial_\mu\psi\Big]$$

$$= +m\,\partial_\mu\left(\bar{\alpha}\gamma_5\gamma^\mu A\psi + i\bar{\alpha}B\gamma^\mu\psi\right)$$

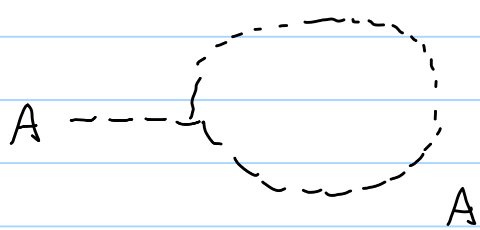
3) a) Under a SUSY Transformation

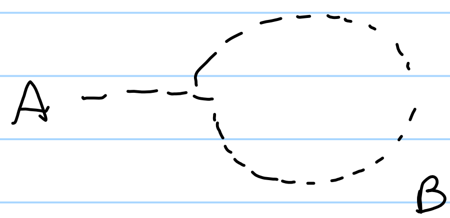
$$\delta \mathcal{L} = k\left(3A^2 \delta A - 3\delta A B^2 - 6AB\delta B\right)$$

$$= 3k\left(\bar{i} A^2 \bar{\alpha} \gamma_5 \psi - \bar{i}\, \bar{\alpha} \gamma_5 \psi B^2 + 2AB \bar{\alpha} \psi\right)$$

not a total derivative, hence SUSY non-inv.

b) Contribution to one-point function of A:





$$\sim \underset{\uparrow}{3k} \int \frac{d^4p}{p^2 - m_A^2} \qquad \sim \underset{\uparrow}{-3k} \int \frac{d^4p}{p^2 - m_B^2}$$

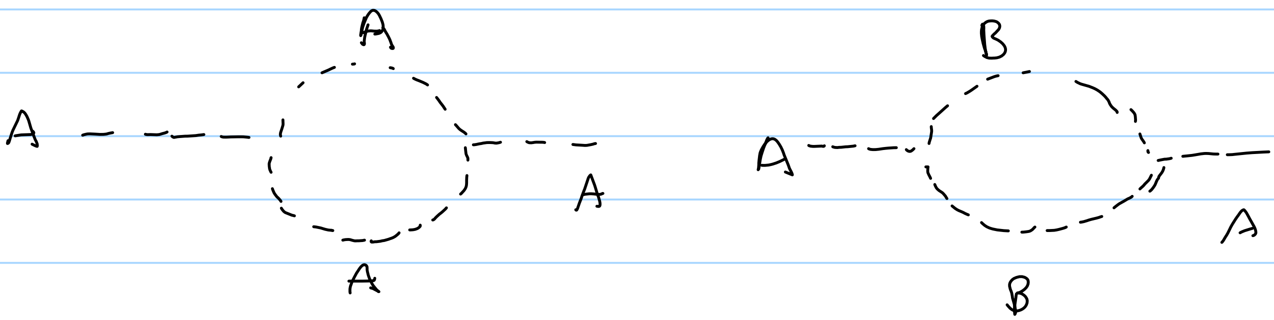
Symmetry factor $\qquad$ Coupling $3AB^2$

$$3k \int d^4p \left[\frac{1}{p^2 - m_A^2} - \frac{1}{p^2 - m_B^2} \right]$$

$\Rightarrow$ No new quadratic divergence even if $m_A \neq m_B$

c) Two point function of A recieves corrections from two sets of diagrams

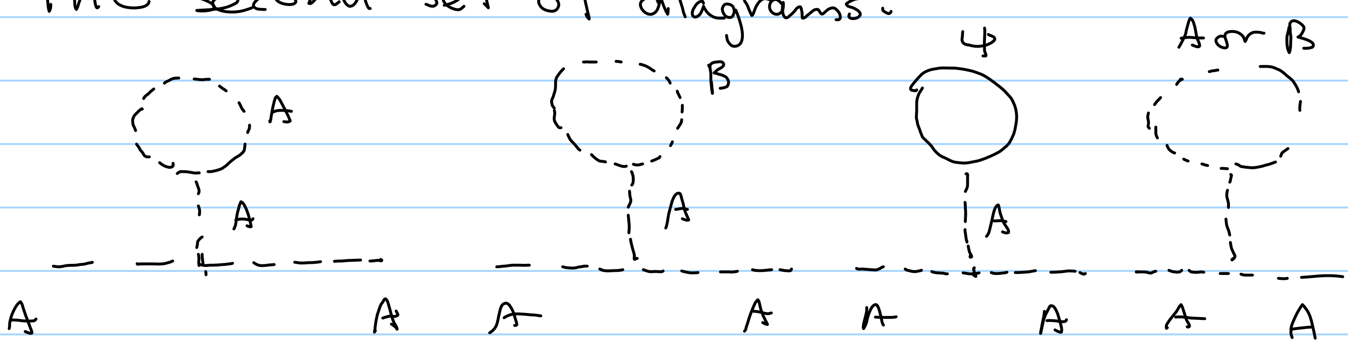
The First Set of diagrams



$$\sim \int d^4p \, \frac{1}{p^2-m_A^2} \, \frac{1}{(p-q)^2-m_A^2} \qquad \sim \int d^4q \, \frac{1}{p^2-m_B^2} \, \frac{1}{(p-q)^2-m_B^2}$$

These diagrams contribute to logarithmically divergence

The second set of diagrams:



These diagrams do not add to new quadratic divergence because of the vanishing one-pt quadratic divergence of

$$\underset{\uparrow \text{ Baer \& Tata}}{\mathcal{L}_{WZ}} + \underbrace{k\,(A^3 - 3AB^2)}_{\text{part (a)}}$$