PHYSICS 801 Lecture 2 Addendum

Two-component spinors

(well-defined Lorentz transf properties)

$$\{ Q_a, Q_b^* \} = 2\, \sigma^{\mu}_{ab} P_\mu$$

$$\{ Q_a, Q_b \} = 0$$

$$\{ Q_a^*, Q_b^* \} = 0$$

Or more generally algebra for <u>extended SUSY</u>

$$\{ Q_{ar}, Q_{bs}^* \} = 2\, \delta_{rs} \sigma^{\mu}_{ab} P_\mu$$

$$\{ Q_{ar}, Q_{bs} \} = \varepsilon_{ab} Z_{rs} \qquad \text{central charge}$$

$$\{ Q_{ar}^*, Q_{bs}^* \} = \varepsilon_{ab} Z^*_{rs}$$

(See Weinberg)

We can argue the form of SUSY algebra by staring at rep. of $SU(2)_L \times SU(2)_R$ Lorentz group

$SO(4) \simeq SU(2)_L \times SU(2)_R$

$J_i \pm i K_i$

rotation boost

$Q_a \qquad (0, \frac{1}{2})$

$Q_a^* \qquad (\frac{1}{2}, 0)$

$(0, \frac{1}{2}) \otimes (\frac{1}{2}, 0) = (\frac{1}{2}, \frac{1}{2})$ vector

$(0, \frac{1}{2}) \otimes (0, \frac{1}{2}) = (0, 0) + (0, 1)$

scalar tensor

Coleman-Mandula thm $\Rightarrow$ only $(0,1)$ generators are linear combination of proper Lorentz generators

$$[Q_a, P^\mu] = 0 \implies [\{Q_a, Q_b\}, P^\mu] = 0$$

$$\implies \text{no } (0,1) \text{ tensor on RHS}$$

$$\therefore \quad \{Q_{ar}, Q_{bs}\} = \varepsilon_{ab} Z_{rs}$$

(0, 0) representation

Since LHS is symmetric wrt $\{a, r\} \Leftrightarrow \{b, s\}$

and ε_{ab} is antisymmetric

$$Z_{rs} = -Z_{sr}$$

only appears in extended SUSY

We will be using 4-component Dirac notation.

Weyl representation

$$\psi = \begin{pmatrix} \psi_s^* \\ \psi_r \end{pmatrix}$$

transform as $(\frac{1}{2}, 0)$ & $(0, \frac{1}{2})$ rep. respectively

Q_a is a Majorana spinor

$$Q = \begin{pmatrix} \varepsilon Q_r^* \\ Q_r \end{pmatrix}$$

In 4-component notation

$$\{Q_{ar}, \bar{Q}_{bs}\} = 2\gamma^\mu_{ab} P_\mu \delta_{rs} + \left(\frac{1+\gamma_5}{2}\right) Z_{sr}^\dagger + \left(\frac{1-\gamma_5}{2}\right) Z_{rs}$$

Will focus on N=1 SUSY $\quad \{Q_a, \bar{Q}_b\} = 2\gamma^\mu_{ab} P^\mu$

Note that $\{Q_a, Q_b\} \neq 0$
$\{\bar{Q}_a, \bar{Q}_b\} \neq 0$ in 4 component notation

In the above, $\bar{Q}_a = Q_a^+ \gamma^0$

However there exists a different basis called Majorana basis

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$$

$\boxed{\gamma \text{ matrices purely imaginary}}$

These γ matrices satisfy Clifford algebra, just like the Weyl basis.

In this basis, Majorana spinor

$$\psi = \psi^*$$

Hence, in this basis $\bar{Q}_a = Q^T \gamma^0$

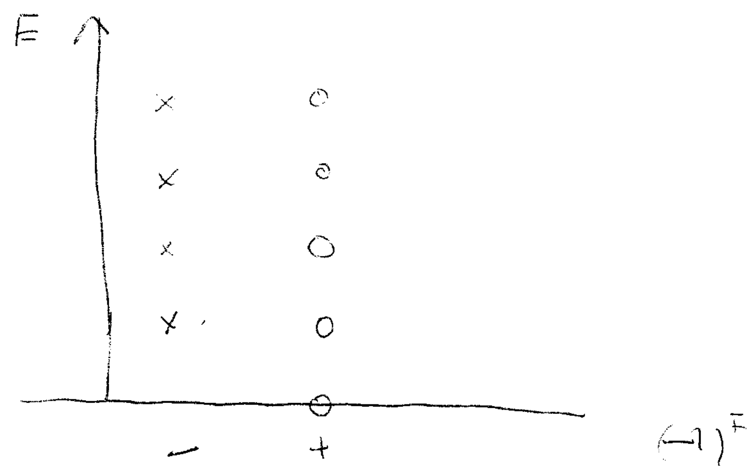
To avoid confusion, let's not change basis will write $\bar{Q} = Q^+ \gamma^0$, Discussions in last lecture won't change.

Witten Index

$$I = n_B - n_F = \mathrm{Tr}\left((-1)^F\right)$$

$(-1)^F = +1$ bosons
$\quad\quad\; -1$ fermions

Start from the vacuum $|\Omega\rangle$, one can construct excited states by bosonic & fermionic creation operators



$Q_\alpha |b\rangle = |f\rangle$ & vice versa

except for ground state $Q_\alpha |\Omega\rangle = 0$ — a bosonic SUSY vacuum

If $I \neq 0 \Rightarrow$ SUSY unbroken — unpaired bosonic state

$I = 0 \Rightarrow$ SUSY broken

Topological quantity : can compute in convenient limits to deduce behavior at strong coupling.