PHY 801: SUSY - Spring 2008

Homework 1

Due Feb 15, 2008

1. Verify that

$$\left(1+\frac{\vec{\eta}\cdot\vec{\sigma}}{2}\right)(E-\vec{\sigma}\cdot\vec{p})\left(1+\frac{\vec{\eta}\cdot\vec{\sigma}}{2}\right)=(E-\vec{\eta}\cdot\vec{p})-\vec{\sigma}\cdot(\vec{p}-E\vec{\eta})$$

   where $\vec{\eta}=(\eta_1,\eta_2,\eta_3)$ is an infinitesimal velocity vector, $\vec{\sigma}=(\sigma_1,\sigma_2,\sigma_3)$ are Pauli matrices.

2. Show that $\psi^\dagger\sigma^\mu\psi$ and $\chi^\dagger\overline{\sigma}^\mu\chi$ transform separately as a 4-vector. Here $\sigma^\mu\equiv(1,\vec{\sigma})$, $\overline{\sigma}^\mu\equiv(1,-\vec{\sigma})$.

3. Consider two-component spinors with undotted indices, ζ and χ.
   (a) What is $\zeta\cdot\chi$ in terms of $\chi\cdot\zeta$?
   (b) What is $\chi_a\zeta^a$ in terms of $\chi^a\zeta_a$?

4. If χ is a two-component spinor with undotted indices,
   (a) Show that $i\sigma_2\chi$ transform as V^*.
   (b) Use (a) to show that $(i\sigma_2\chi)^T\zeta$ is Lorentz invariant.