

# PHY 801: SUSY - Spring 2008

## Homework 3

Due March 28, 2008

1. Show that the Wess Zumino Model corresponds to the superpotential:

$$f(\hat{\mathcal{S}}_L) = \frac{1}{2}m\hat{\mathcal{S}}_L^2 + \frac{1}{3}g\hat{\mathcal{S}}_L^3$$

- a) Derive the Lagrangian in terms of the component fields  $\mathcal{S}, \psi_L, \mathcal{F}$ .  
b) Next rewrite the Lagrangian in terms of the fields  $A, B, \psi, F, G$ , where

$$\begin{aligned}\mathcal{S} &= \frac{1}{\sqrt{2}}(A + iB) \\ \psi_L &= \frac{1 - \gamma_5}{2}\psi \\ \mathcal{F} &= \frac{1}{\sqrt{2}}(F + iG)\end{aligned}$$

where  $\psi$  is Majorana. Show that you reproduce Eqs. (3.1c) and (3.43) of Baer and Tata.

- c) Going back to part (a), find the equations of motion for  $\mathcal{F}, \mathcal{F}^\dagger$ , and eliminate them from the Lagrangian. This yields the analogue of Baer and Tata (3.45).
2. Starting from the curl superfield  $\hat{W}_A$  in the Wess Zumino gauge, Eq. (6.31) in Baer and Tata, show that  $\widehat{W}_A^c$  is given by their Eq. (6.32).
3. Show that the term that we drop in the derivation of the gauge kinetic term, i.e.,

$$\frac{1}{4}\epsilon^{\nu\mu\mu'\nu'} F_{A\mu\nu} F_{A,\mu'\nu'}$$

is a total derivative.

4. Consider a simple model with a single left-chiral superfield  $\hat{\mathcal{S}}_L$  with a superpotential

$$f(\hat{\mathcal{S}}_L) = \alpha + \beta\hat{\mathcal{S}}_L + \gamma\hat{\mathcal{S}}_L^2 + \delta\hat{\mathcal{S}}_L^3$$

where  $\alpha, \beta, \gamma, \delta$  are real constants.

- a) Evaluate the scalar potential  $V(\mathcal{S})$ . Find the value of  $\langle \mathcal{S} \rangle$  that minimizes the potential. What is  $V(\langle \mathcal{S} \rangle)$ ? Is SUSY broken or unbroken?  
b) Expand around the minimum of the potential  $s \equiv \mathcal{S} - \langle \mathcal{S} \rangle$ . What is the mass of the  $s$  particle?  
c) Evaluate the mass of  $\psi_L$ . Is it the same as the mass of  $s$ ? Why does this make sense?