Lecture 1

1/29/08

Motivation: Why SUSY?

① May be discovered in near future (important for theory + expts)

② Useful for QFT exact results (duality, holomorphy vs lattice)

③ Good for the soul (revisit issues in QFT, SM physics; also important tools in particle physics, cosmology, string theory)

BSM and SUSY

One of the motivations for BSM is the hierarchy problem. Historically, recognizing UV divergences & understanding how the divergences are resolved has shown to pay off (SM and the weak bosons)

Electroweak sector

$$v \approx 246 \text{ GeV}$$

sets the scale & masses of theory

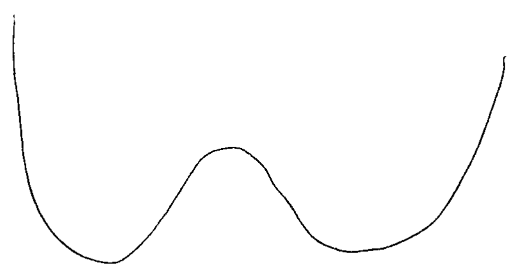
e.g. $M_W = gv/2 \sim 80$ GeV $g =$ SU(2) gauge coupling

$$M_H = v\sqrt{\frac{\lambda}{2}}$$ where $\lambda =$ Higgs self coupling

$$V = -\mu^2 \phi^\dagger \phi + \frac{\lambda}{4}(\phi^\dagger \phi)^2 \qquad \lambda > 0, \quad \mu^2 > 0$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad SU(2) \text{ doublet}$$

Negative sign of mass term essential for



Wine-bottle potential
↓
Spontaneous Symmetry breaking

Classical minimum $\Rightarrow$ $|\phi| = \sqrt{2}\,\mu / \sqrt{\lambda} \equiv v/\sqrt{2}$

$\Rightarrow$ important to preserve both the sign & magnitude of the $-\mu^2 \phi^\dagger \phi$ term.

Loop level: will encounter integrals of the form

$$\int_0^\Lambda d^4k \; f(k, \text{external momenta})$$

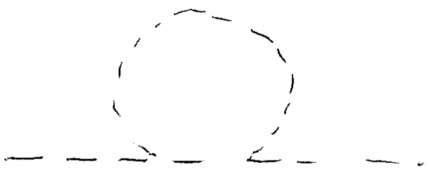
SM is renormalizable $\rightarrow$ can take $\Lambda \to \infty$

but more reasonably, regard SM as an effective theory. At the very least

$$M_P = (G_N)^{-1/2} \simeq 1.2 \times 10^{19} \text{ GeV}$$

If there is <u>any</u> scale of new physics $\Lambda \Rightarrow$ a problem

Consider 4-boson self-interaction



A contribution to $\phi^\dagger \phi$

$$\sim \lambda \int_0^\Lambda d^4k \, \frac{1}{k^2 - M_H^2}$$

divergent as $\Lambda \to \infty$

$$\Rightarrow \lambda \Lambda^2 \phi^\dagger \phi$$

Now μ^2 is fixed by v

$$\frac{\mu^2}{\lambda} = \frac{V}{4}$$

perturbativity $\Rightarrow \lambda < 1$
$\Rightarrow \mu$ at most a few hundred GeV

But if $\Lambda \sim 10^{19}$ GeV

$\Rightarrow$ severe fine-tuning to get the resulting $-\mu^2 \phi^\dagger \phi$ term to be negative and $\sim (100\,\text{GeV})^2$

<u>Note</u>: Not a problem for SM in isolation (no second scale, no Λ, no v)

can choose $-\mu^2_{ren} = -$ (a few hundred GeV)^2

as often do for mass terms in renormalizable theories.

"Fine-tuning" problem affects not only mass of Higgs

$$M_H = \sqrt{2}\,\mu$$

but also mass of W-boson

$$M_W = g\mu/\sqrt{\lambda}$$

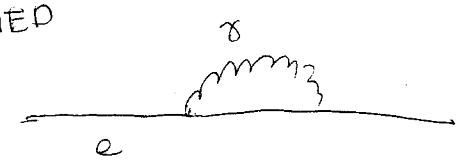
and ultimately all masses of SM derived from μ

Is this a generic problem of mass terms in any renormalizable field theories, why a big fuss about it now when discussing SUSY?

Specific to fundamental scalars

Consider instead a fermion, e.g. an electron

QED



naively

$$\delta m \sim \alpha \int^{\Lambda} \frac{d^4k}{\not{k}\, k^2}$$

$$\sim \alpha \Lambda$$

However, when calculation is done properly, one finds

$$\delta m \sim \alpha\, m \ln \Lambda$$

even if $\Lambda \sim 10^{19}$ GeV correction is not enormous

due to chiral symmetry

$$\psi \to e^{i\alpha\gamma_5}\psi \quad \text{for } U(1)$$

$$\psi \to e^{i\vec{\alpha}\cdot\vec{\tau}/2\,\gamma_5}\psi \quad \text{for } SU(2)$$

Can show this from the Dirac Action

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \qquad \bar{\psi} \equiv \psi^\dagger \gamma^0$$

and $\{\gamma^\mu, \gamma^5\} = 0$ that the chiral rotation is a symmetry of the derivative terms but not the mass term

$\Rightarrow \quad \delta m \propto m \qquad$ (vanishes as $m \to 0$)

Dimension analysis $\Rightarrow \delta m \sim m \ln \Lambda$

<u>What about gauge bosons?</u>

Gauge Symmetry $\Rightarrow m_\gamma^2 A^\mu A_\mu \equiv 0$

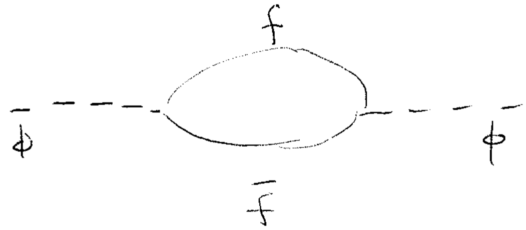
For W, Z where gauge symmetry is broken

$M_W \sim \mu$ just like M_H $\rightarrow$ same Λ dependence as M_H

Fermions & Gauge bosons are what we need to worry about for the SM

Can we find a symmetry (analogous to chiral symmetry & gauge symmetry) that protects δm^2 for M_H ?

Remember there are also fermion loops



$$\sim \quad -g_f^2 \int_0^{\Lambda} \frac{d^4k}{\not k \not k} \phi^\dagger \phi$$

$$\sim \quad -g_f^2 \phi^\dagger \phi \Lambda^2$$

↑ sign is crucial

Combining with self-interaction contribution

$$(\lambda - g_f^2)\, \phi^\dagger \phi$$

If $\quad \lambda = g_f^2 \quad \Rightarrow$ quadratic sensitivity to Λ cancels

Such a relation between boson self-interaction and boson-fermion coupling is characteristic of susy.

The leading contribution is then logarithmic

$$\sim \lambda \, (M_H^2 - M_f^2) \ln \Lambda$$

provided that all bosons and fermions have masses not much heavier than M_H, contribution $\sim M_H^2$ (otherwise some fine-tuning)

Bose - Fermi symmetry protects M_H^2 from Λ^2 corrections!

SUSY is not the <u>only</u> solution to the hierachy problem
Other solutions:

(1) Technicolor — strong dynamics ⇒ composite Higgs

(2) Extra Dimensions — weakness of gravity a result of large / warped dimensions

However, SUSY is the most well-developed framework for BSM

Also, a framework that can be studied <u>perturbatively</u>

Additional Positive Indications of SUSY

(1) Precision EW ⇒ $M_H \lesssim 200$ GeV at 99% CL

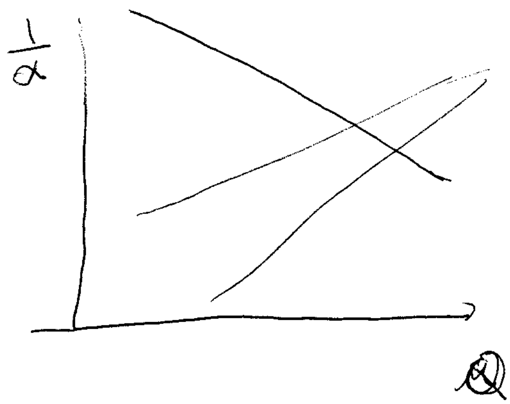
MSSM has 2 Higgs doublets and <u>predicts</u> lightest Higgs $M_H \lesssim 140$ GeV

SM has no (to be precise, very weak) constraint on M_H.

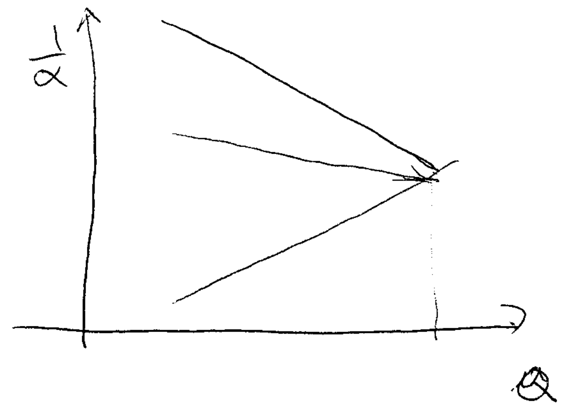
$$M_H = v\sqrt{\frac{\lambda}{2}}$$

perturbativity ⇒ upper bound on M_H $\lesssim$ a few-hundred GeVs
but bound is weaker than MSSM

② Gauge Coupling Unification



SM



MSSM

also unification scale is higher – helps proton stability

③ Electroweak Symmetry breaking

Mass parameter $m^2 \phi^\dagger \phi$ also "run"

Starting from positive m^2, RGE takes m^2 to a negative value at EW scale.

Top quark has large Yukawa coupling

Why not SM itself?

Initial conditions more natural within SUSY

④ Cold dark Matter – LSP

⑤ . . . (See Baer – Tata for other additional good indications of SUSY)