Lecture 1

Motivation: Why SUSY?

1. May be discovered in near future (important for theory + expts)
2. Useful for QFT exact results (duality, holomorphy) vs lattice
3. Good for the soul (revisit issues in QFT, SM physics; also important tools in particle physics, cosmology, string theory)

BSM and SUSY

One of the motivations for BSM is the hierarchy problem. Historically, recognizing UV divergences & understanding how the divergences are resolved has shown to pay off (SM and the weak bosons)

Electroweak sector

\[ \nu \text{ hierarchy } \approx 246 \text{ GeV} \]

\[ \text{e.g. } M_W = g \sqrt{2} \approx 80 \text{ GeV} \]

\[ M_H = \nu \sqrt{2} \]

where \( \nu \) = Higgs self-coupling
\[ V = -\mu^2 \phi^+ \phi + \frac{\lambda}{4} (\phi^+ \phi)^2 \quad \lambda > 0 \quad \mu^2 > 0 \]

\[ \phi = \begin{pmatrix} \phi^+ \\ \phi^- \end{pmatrix} \quad \text{SU}(2) \text{ doublet} \]

Negative sign of mass term essential for

\[ \text{Wine-bottle potential} \]
\[ \downarrow \]
\[ \text{Spontaneous symmetry breaking} \]

Classical minimum \( \Rightarrow \) \[ |\phi| = \sqrt{2} \mu / \sqrt{\lambda} \equiv \sqrt[4]{2} \]

\( \Rightarrow \) important to preserve both the sign and magnitude of the \(-\mu^2 \phi^+ \phi\) term.

\underline{Loop level:} will encounter integrals of the form

\[ \int_0^\Lambda d^4 k \ f (k, \text{external momenta}) \]

SM is renormalizable \( \Rightarrow \) can take \( \Lambda \to \infty \)

but more reasonably, regard SM as an effective theory. At the very least

\[ M_p = (G_N)^{-1/2} \approx 1.2 \times 10^{19} \text{ GeV} \]
If there is any scale of new physics $\Lambda \Rightarrow$ a problem

Consider 4-boson self-interaction

A contribution to $\phi^4$

$\sim \lambda \int_0^\Lambda d^4k \frac{1}{k^2 - M_{12}^2}$

diverged as $\Lambda \to 0$

$\Rightarrow \lambda \Lambda^2 \phi^4$

Now $\mu^2\phi^4$ is fixed by $\nu$

$$\frac{\mu^2}{\nu} = \frac{\lambda}{4}$$

perturbativity $\Rightarrow \lambda < 1$

$\Rightarrow \mu$ at most a few hundred GeV

But if $\Lambda \sim 10^{19}$ GeV

$\Rightarrow$ severe fine-tuning to get the resulting $\mu^2\phi^4$

term to be negative and $\sim (100 \text{ GeV})^2$

Note: Not a problem for SM in isolation

(no second scale, no $\Lambda$, no $\phi^4$)

can choose $-\mu^2_{\text{ren}} = - (\text{a few hundred GeV})^2$

as often do for mass terms in renormalizable theories.
"Fine-tuning" problem affects not only mass of Higgs

\[ M_H = \sqrt{2} \mu \]

but also mass of W-boson

\[ M_W = 9 \mu / \sqrt{\lambda} \]

and ultimately all masses of SM derived from \( \mu \).

Is this a generic problem of mass terms in any renormalizable field theory, why a big fuss about it now when discussing SUSY?

Specific to fundamental scalars

Consider instead a fermion, e.g., an electron

\[ S_{\text{ED}} \]

\[ S_{\text{m}} \sim \alpha \int \frac{d^4k}{kk^2} \]

\[ \sim \alpha \Lambda \]

However, when calculation is done properly, one finds

\[ S_{\text{m}} \sim \alpha m \ln \Lambda \]

even if \( \Lambda \sim 10^{19} \text{ GeV} \)

correction is not enormous due to chiral symmetry

\[ \psi \rightarrow e^{i \alpha \cdot \gamma_5} \psi \quad \text{for } U(1) \]

\[ \psi \rightarrow e^{i \alpha \cdot \gamma_5 \cdot \frac{\gamma_5}{2}} \psi \quad \text{for } SU(2) \]
Can show this from the Dirac Action
\[ \mathcal{L}_{\text{Dirac}} = \bar{\Psi} (i \gamma^\mu \partial_\mu - m) \Psi \quad \bar{\Psi} = \Psi^\dagger \gamma^0 \]
and \( \{ \gamma^\mu, \gamma^5 \} = 0 \) that the chiral rotation is a symmetry of the derivative terms but not the mass term
\[ \implies \delta m \propto m \quad \text{(vanishes as } m \rightarrow 0) \]
Dimension analysis \( \implies \delta m \sim m \ln \Lambda \)

What about anomaly gauge bosons?

Gauge Symmetry \( \implies m^2 \gamma^\mu A_\mu A_\nu = 0 \)

For W, Z where gauge symmetry is broken
\[ M_W \sim \mu \quad \text{just like } M_H \implies \text{same } \Lambda \text{ dependence} \quad \text{as } M_H \]

Fermions & gauge bosons are what we need to worry about for the SM

Can we find a\* symmetry (analogous to chiral symmetry \& gauge symmetry) that produces \( \delta m^2 \) for \( M_H \)?
Remember there are also fermion loops

\[ \sim -g_f^2 \int_0^\Lambda \frac{d^4 k}{k} \phi^+ \phi \]

\[ \sim -g_f^2 \phi^+ \phi \Lambda \]

Sign is crucial

Combining with self-interaction contribution

\[ (\lambda - g_f^2) \phi^+ \phi \]

\[ \lambda = g_f^2 \Rightarrow \text{quadratic sensitivity to } \Lambda \text{ cancels} \]

Such a relation between boson self-interaction and boson-fermion coupling is characteristic of susy.

The leading contribution is then logarithmic

\[ \sim \Lambda (M_H^2 - M_f^2) \ln \Lambda \]

provided that all bosons and fermions have masses not much heavier than $M_H$, contribution $\sim M_H^2$ otherwise some fine-tuning.

Bose - Fermi symmetry protects $M_H^2$ from $\Lambda^2$ corrections!
SUSY is not the only solution to the hierarchy problem.

Other solutions:

1. Technicolor - strong dynamics \(\Rightarrow\) composite Higgs

2. Extra Dimensions - weakness of gravity a result of large (warped) dimensions

However, SUSY is the most well-developed framework for BSM.

Also, a framework that can be studied perturbatively.

Additional Positive Indications of SUSY

1. Precision EW \(\Rightarrow\) \(M_H < 200\text{ GeV}\) at 99\% CL

MSSM has 2 Higgs doublets and predicts lightest Higgs \(M_H < 140\text{ GeV}\).

SM has no (to be precise, very weak) constraint on \(M_H\).

\[ M_H = v \sqrt{2} \]

Perturbativity \(\Rightarrow\) upper bound on \(M_H\)

\(\approx\) a few hundred GeVs but bound is weaker than MSSM.
2. Gauge Coupling Unification

\[ \frac{1}{\alpha} \]

\[ Q \]

\[ \alpha \]

SM

3. Electroweak Symmetry breaking

Mass parameter \( M^2 \phi^\dagger \phi \) also "ran"

Starting from positive \( m^2 \), RGE takes \( m^2 \) to a negative value at EW scale.

Top quark has large Yukawa coupling

Why not SM itself?

Initial conditions more natural within SUSY

4. Cold dark Matter - LSP

5. ... (See Baer-Tata for other additional good indications of SUSY)