Review of the MSSM

Field content: SM + superpartners + extra Higgs & Higgsino

Non-gauge interactions from superpotential

\[ W = \mu \hat{H}_d^a \hat{H}_u^a + \frac{2}{3} \sum_{i,j} (f_{ij})_{ij} e_{ab} Q_i^a H_u^b L_j^c + (\bar{f}_{ij})_{ij} e_{ab} Q_i^a H_d^b D_j^c \]

so far only

Also soft-breaking terms.

\[ \mathcal{L}_{\text{soft}} = - \frac{1}{2} \left( M_1 \tilde{g} \tilde{g} + M_2 \tilde{w} \tilde{w} + M_3 \tilde{b} \tilde{b} + \text{c.c.} \right) \]

\[- \left( \tilde{u} a_d \tilde{Q} H_u - \tilde{d} a_u \tilde{Q} H_d - \tilde{e} a_e \tilde{L} H_u + \text{c.c.} \right) \]

\[ -m_\tilde{Q} \tilde{Q} \tilde{Q} + m_\tilde{u} \tilde{u} \tilde{u} + m_\tilde{d} \tilde{d} \tilde{d} + m_\tilde{e} \tilde{e} \tilde{e} + \left( \text{Higgs terms} \right) \]

\[ \text{real} \Rightarrow \text{Hermitean.} \]

\[ b = B_f \]

characteristic scale \( \mu_{\text{soft}} \approx 1 \text{ TeV} \)

105 free parameters.
105 parameters of soft MSSM. Is it hopeless?

Not right away. Many constraints from Higgs, severely restrict low-energy physics.
One hopes that eqpt constraints + high scale theoretical input is sufficient
to say interesting things (too optimistic?)

Constraint Class #1: FCNC — weak in SM, but generic MSSM soft terms
give large FCNC.

Ex. \( \mu \rightarrow e\gamma \) as SM 1-loop diagram.

But if \( (M_e^2)_{ij} \) is not diagonal in the same basis as for
the ordinary leptons, get nontrivial slepton mixing.

\[
\begin{align*}
\mu \rightarrow e\gamma & \quad \text{off-diagonal } (M_e^2) \\
\text{off-diagonal } (M_e^2) & \quad \text{off-diagonal } (M_e^2)
\end{align*}
\]

\[\text{Exp.: } Br(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}\]

MSSM: \[Br(\mu \rightarrow e\gamma) \propto \frac{(m_{\mu}^2 p_{\mu}^2 - m_e^2 p_e^2)}{m_{\mu}^2} \left( \frac{100 \text{ GeV}}{m_{\mu}} \right)^2 \times 10^{-5} \times \theta (1-10) \]

\( \tilde{\nu} \sim 1 \text{ TeV} \) gives \( Br \) too large!
Ex. Neutral Kaons.

\[ s \rightarrow u_{c,t} u_{c,t}^{*} \]

SM \( E^{+} \rightarrow E^{0} \)

Suppressed by GIM (cancellation due to opposite CKM phases)

Nonzero because \( c \) quark is massive \((m_{c} \sim 0)\)

\[
\Delta m_{c} \propto \frac{G_{F}}{\sqrt{2}} \frac{f_{c}^{2} m_{c}}{\sin \theta_{W} \cos \theta_{W} \sin \theta_{c}} \frac{m_{c}}{M_{W}}
\]

\((\Delta m_{c})_{\text{exp}} = 3.5 \times 10^{-12} \text{ MeV}\)

MSSM generates new diagrams.

\[
\begin{align*}
\text{Limit:} & \quad \left| \frac{\text{Re}(m_{\tilde{g}}^{2})}{(m_{\tilde{g}})^{2}} \right| \times 10^{-12} < 0.05 \\
\text{Even stronger limit with left + right handed squarks:} & \quad \frac{\text{Re}(m_{\tilde{q}}^{2} \tilde{g} \tilde{d})}{(m_{\tilde{q}})^{2}} \left( \frac{1}{500 \text{ GeV}} \right) < 0.01
\end{align*}
\]
Suppress FCNC

1. Defy tour make all the squark, slepton matrices & identity, This eliminates FCNC by hand.

2. Aperiodic $x_i$ not == 0, but are acting (the quark mass matrices)
diagonalized by the same unitary transf. as the quark mass matrices.

3. Heavy squarkes – "decupl" – unless the squarkes heavy enough
   to suppress the loop diagrams, A K sector gives tight constraint,
   then $M \geq 40$ TeV — leads to a hierarchy problem.

Constraints on A term

Higgs ren induces quadratic coupling $\bar{\ell}_i (h) \bar{Q}_j (H_u)$, etc.

so $a_{ij}$ should be cleanly disposed to avoid FCNC.
CP Violating

So far our analysis has shown that off-diagonal elements must be small. Even if this is satisfied, recall that \( \tilde{m}^2 \) is in general complex which can introduce CP-violating phases.

(Note - an even simpler example occurs for the gauginos!)

\[
M_{LL} + M_{LL} \bar{X}_{\gamma} \quad \text{Normally just set } M' = 0
\]

These phases bounded by low-energy experiments like electron dipole moment.

\[
\tilde{e} \times \tilde{\nu}_L
\]

\[
\text{Ex. } |\text{Im}(a_e)|v_e < 6 \times 10^{-4} \text{ m/e}
\]

Similar for neutron dipole with \( |\text{Im}(a_n)|v_e < 0.002 \text{ m/e} \)

Solution? No good one, really. Just make the assumption that sfermion matrices are real and diagonal.

\[
\left( \tilde{m}_a \right)_{ij} = m_{ij} \quad \text{Diag}\left( \tilde{m}_a \right)
\]

\& a-terms aligned w/ Yukawa: \( a_u = A_{uu} \), ...
Renormalization Group

General formula

\[ \frac{1}{\Delta y} \frac{dg}{dy} = \beta(g) = -\frac{g^3}{\alpha_s} \left( \frac{11}{3} C_G - \frac{2}{3} n_f S(R_F) - \frac{1}{3} n_f S(R_H) \right) \]

# of fermion species (right/left count separated)

\[ \alpha_s = \frac{\pi}{\text{physical \ mass \ of \ quark}} \]

# of complex scalars

\[ C(G) = N \text{ for } SU(N) \]

Special case: \( u(1)_Y \), \( s(Y) = 1 \)

\[ SU(3): \beta(g_3) = -\frac{3g_3^2}{16\pi^2} \left[ \frac{11}{3} \times 3 - \frac{2}{3} (4 \times 3) + \frac{1}{3} (N_H = 1) \times (0) \right] \]

\[ = -\frac{(g_3)^2}{16\pi^2} \left[ 11 - 4 \right] = \frac{3g_3^2}{16\pi^2} (-7) \]

\[ SU(2): \beta(g_2) = -\frac{3g_2^2}{16\pi^2} \left[ \frac{11}{3} \times 2 - \frac{2}{3} (4 \times 3) + \frac{1}{3} (1) \times (0) \right] \]

\[ = -\frac{3g_2^2}{16\pi^2} \left[ \frac{22}{3} - 4 - \frac{1}{16} \right] = -\frac{3g_2^2}{16\pi^2} \left( \frac{19}{16} \right) \]

\[ U(1)_Y: \beta(g_Y) = -\frac{3g_Y^2}{16\pi^2} \left[ 0 - \frac{2}{5}(n_f) \left( \frac{2}{3} \right)^2 - \frac{1}{5} (2) \left( \frac{1}{3} \right)^2 \right] \]

\[ = -\frac{3g_Y^2}{16\pi^2} \left[ -2 \times 3 - \frac{1}{16} \right] = -\frac{3g_Y^2}{16\pi^2} \left[ -\frac{40}{16} - \frac{1}{16} \right] \]

\[ g_Y = \frac{3}{5} g_1, \quad \beta(g_Y) = -\frac{3g_Y^2}{16\pi^2} \left( \frac{41}{16} \right) \]
**Summary**

\[ \beta_i = \frac{3 g_i^2}{8 \pi^2} \quad (b_1, b_2, b_3) = \left( \frac{4}{10}, -\frac{11}{6}, -7 \right) \]

**Example for MSSM**

\[ (b_1, b_2, b_3) = \frac{33}{5}, 1, 3 \]

With these RGEs,

**SM**

\[ \alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1} \]

**Next MSSM**

A stringy unification at 
\[ \sim 10^{16} \text{ GeV}, \text{ the traditional GUT scale} \]

MSSM - 2 Higgses.
Yukawa RGE:

Powerful NR theorem - superpotential not renormalized, only

\[ \Rightarrow \beta_{ij} = \frac{d}{d\log \mu} y_{ij} = \chi_i y_{nj} + \chi_j y_{nik} + \chi_k y_{ijn} \]

\[ \delta_i j = \frac{1}{16\pi^2} \left[ \frac{1}{2} y_{imn} y_{jmn} - 2 g^2 C_i (i) \delta_i j \right] \]

Yukawa RGE

\[ \frac{d y_{ij}}{d \log \mu} = \frac{f^4}{16\pi^2} \left( -\frac{5}{2} c_i \tilde{g}^2 + 6 \tilde{f}^2 + \tilde{f}^2 \right) \]

etc.

Typically for Yukawa ~ 1, others small, even \( Y_u, Y_d \).

usually ignore 1st & 2nd generation

\[ m_{t \bar{t}} \sim 125 \text{ GeV} \]

\[ m^v \sim 5 \text{ GeV} \ldots \text{ others even smaller} \]