

String realization of 4-d $N=2$ S-duality

P.C. Argyres w/ N. Seiberg, J. Wittig

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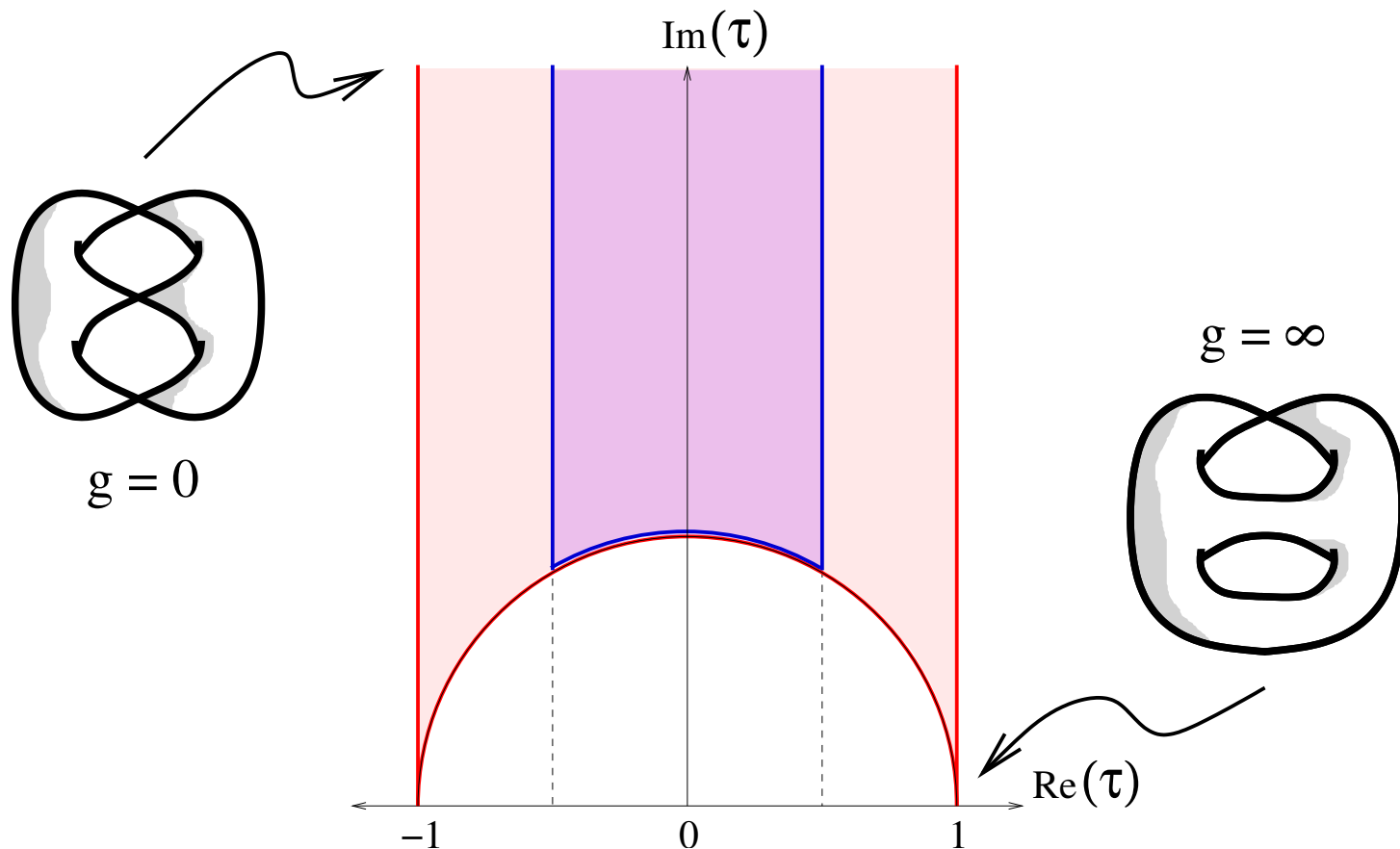
and to appear w/ J. Vázquez-Poritz

Outline

- I** Review of N=2 S-duality proposal
- II** Three examples
- III** IIA/M theory brane realizations
- IV** NS5-O6⁻ bound state and its M theory lift
- V** Connection of S-duality to $sl(2, \mathbb{Z})$ invariance of T^2/\mathbb{Z}_2
- VI** Summary

I Review of N=2 S-duality

- From PCA, N. Seiberg 0711.0054
- **What happens as $g \rightarrow \infty$ in N=2 Lagrangian theories?**
E.g., $su(3)$ w/ $6 \cdot 3$ has S-duality group $\Gamma^0(2) \subset sl(2, \mathbb{Z})$ w/ fundamental domain in τ -plane containing $\text{Im}(\tau) = 0$ ($g = \infty$):



Review of N=2 S-duality (cont)

- The physics at $g = \infty$ of an N=2 Lagrangian SCFT with gauge algebra \mathfrak{g} , with $\text{rank}(\mathfrak{g}) = r$, is a **weakly coupled** scale-invariant gauge theory with gauge algebra $\tilde{\mathfrak{g}}$ with smaller rank, $\text{rank}(\tilde{\mathfrak{g}}) = s < r$, which is coupled to an **isolated** rank- $(r-s)$ N=2 SCFT:

	\mathfrak{g}	w/ hyper-plets	\simeq	$\tilde{\mathfrak{g}}$	w/ (SCFT \oplus hyper-plets)
coupling:	g	—		$1/g$	fixed
rank:	r	0		s	$r-s$

- The **coupling** between $\tilde{\mathfrak{g}}$ and the SCFT is the standard gauge coupling: $\tilde{\mathfrak{g}}$ gauges a subalgebra of the **global symmetry algebra** of the SCFT \Rightarrow the SCFT provides “matter fields” charged under $\tilde{\mathfrak{g}}$.

II Three examples of $\mathfrak{g} \text{ w/ } \mathfrak{r} = \tilde{\mathfrak{g}} \text{ w/ } \tilde{\mathfrak{r}} \oplus \text{SCFT}[d : \mathfrak{h}]$

1. $\mathfrak{su}(3) \text{ w/ } 6 \cdot 3 = \mathfrak{su}(2) \text{ w/ } 1 \cdot 2 \oplus \text{SCFT}[3 : E_6]$

Here $\text{SCFT}[3 : E_6]$ is the isolated rank 1 SCFT with flavor symmetry E_6 and Coulomb vev of dimension 3 discovered by Minahan & Nemeschansky hep-th/9608047.

2. $\mathfrak{su}(4) \text{ w/ } 4 \cdot 4 \oplus 2 \cdot 6 = \mathfrak{sp}(2) \text{ w/ } 3 \cdot 4 \oplus \text{SCFT}[3 : E_6]$

3. $\mathfrak{su}(3) \text{ w/ } \text{SCFT}[3 : E_6] = ?$

Here $E_6 \supset \mathfrak{su}(3) \oplus G_2$, so must be a non-lagrangian theory in all limits since has an exceptional flavor symmetry.

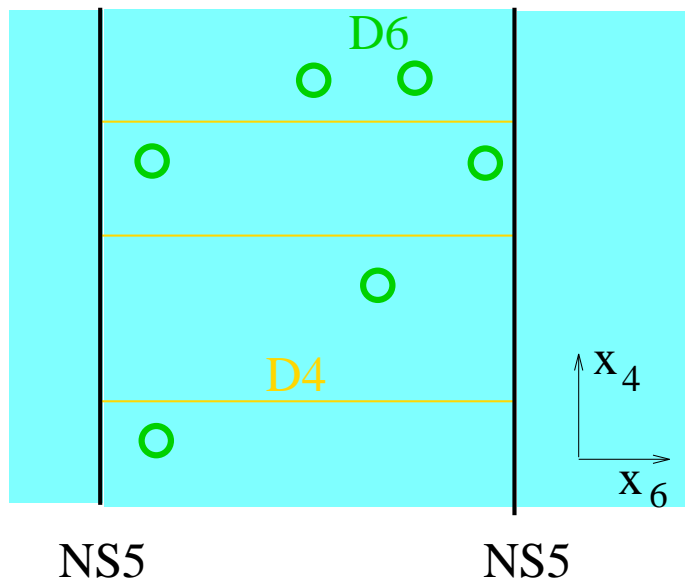
Many more examples ... (PCA, J. Wittig 0712.2028)

Some surprises about N=2 SCFTs (John's talk)

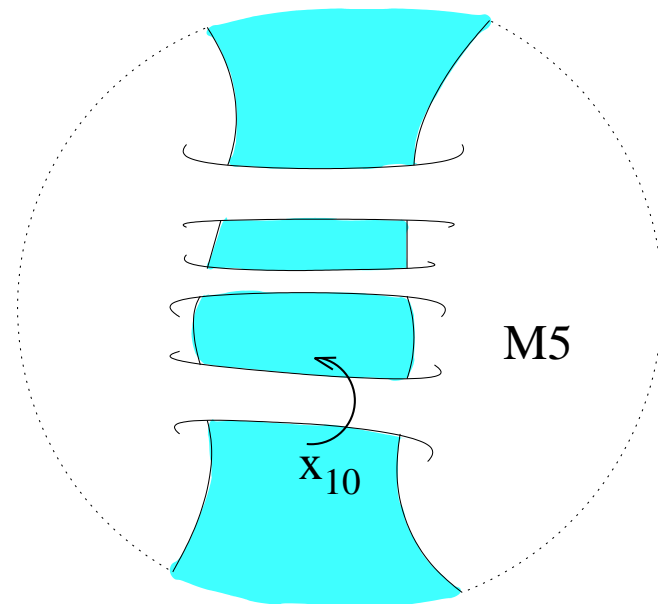
General, direct approach to N=2 SCFTs (AI's talk)

III IIA/M brane ladders

- We cannot prove this duality conjecture, since it relates strong coupling to “weak+strong” coupling in field theory. But many ft dualities are “lifted” to geometrical equivalences in string theory, at least giving more evidence.
- One construction of 4-d $N=2$ SCFTs is as a ladder of D4 branes suspended between NS5 branes (Witten, hep-th/9703166). *E.g.*, $su(3)$ w/ $6 \cdot 3$:



IIA string



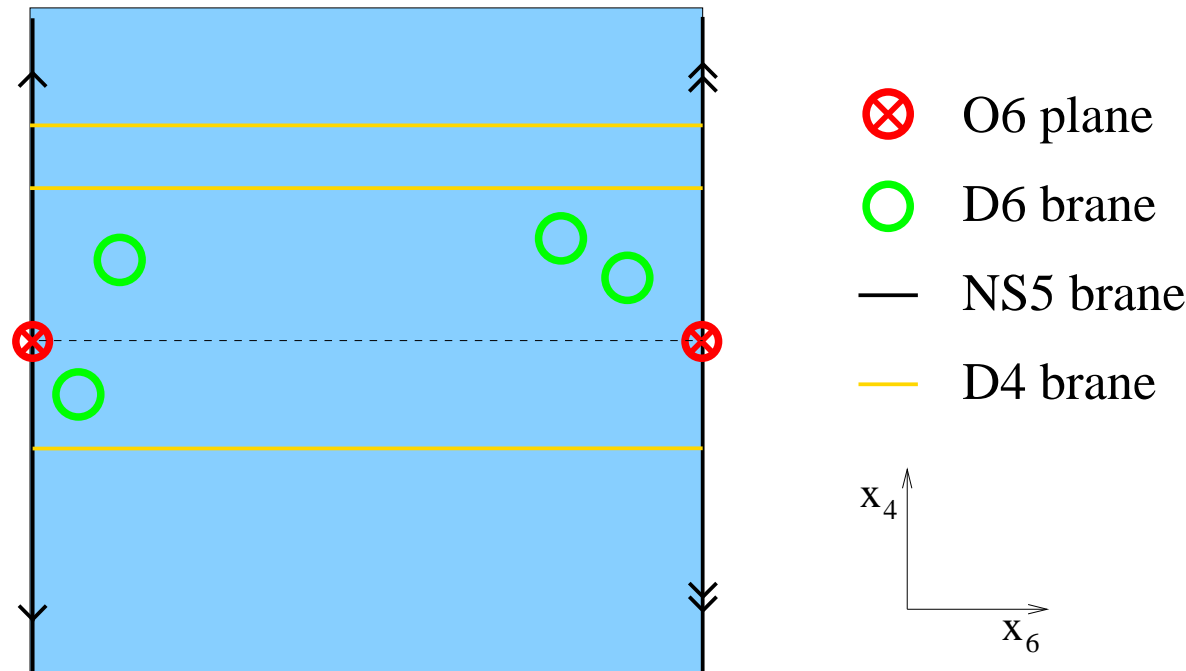
M theory

IIA/M brane ladders (cont.)

- In the M theory lift, the M5 brane describes the SW curve of the low energy effective action.
- The inverse gauge coupling, $\text{Im}(\tau)$, is the separation of the NS5 branes.
- So the infinite-coupling limit is given by coincident parallel 5 branes. The same is true in the M theory lift.
- Coincident 5 branes are strongly coupled configurations: the 4-d open string modes no longer decouple from the 6-d modes, and the low energy effective description breaks down.

IV Adding $O6^-$ planes

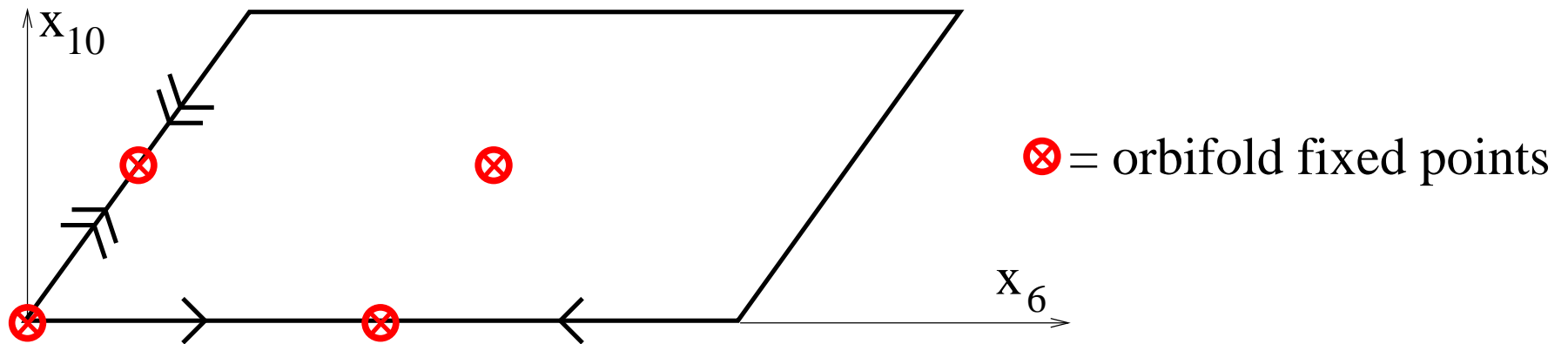
- The key to getting a string description which is well-behaved in the infinite-coupling limit is to add $O6^-$ orientifold planes to the mix.
- $su(n)$ w/ 4·fund \oplus 2·antisymm. (Landsteiner and Lopez hep-th/9708118):



- With zero or one NS5, it gives $sp(n)$ w/ 4·fund \oplus 2·traceless-antisymm.

Adding $O6^-$ planes (cont.)

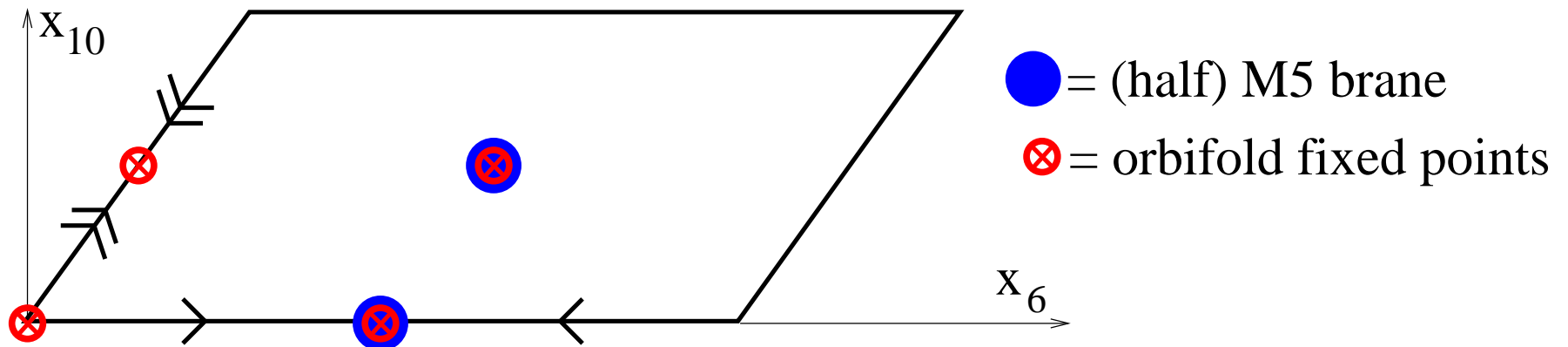
- In IIA, the 2 $O6^-$'s restrict to an interval along x_6 , equivalent to an S^1/\mathbb{Z}_2 orbifold.
- To lift the pair of $O6^-$'s to M theory, we add an x_{10} circle. The x_6-x_{10} directions becomes a T^2/\mathbb{Z}_2 orbifold. (PCA, Maimon, Pelland hep-th/0204127.)



- Now add in NS5 branes. A half-NS5 brane is stuck at an $O6^-$ orbifold point: it is its own orbifold image. Its M theory lift is a half-M5 brane stuck at one orbifold point.

Adding $O6^-$ planes (cont.)

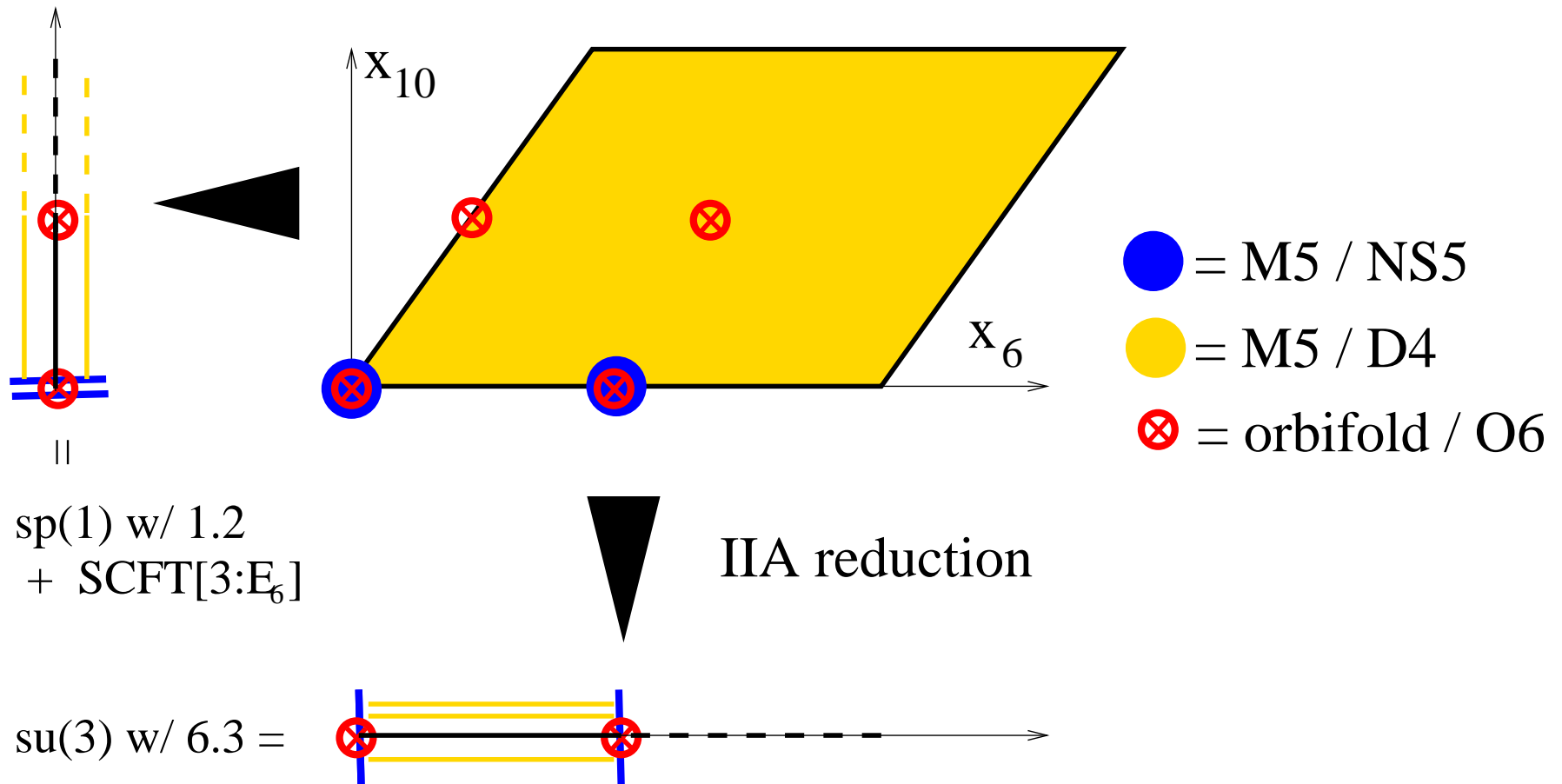
- Can now make a new configuration of 2 stuck half-M5 branes. In the IIA limit it looks like a whole NS5 bound to the $O6^-$: it has no moduli to move away.



- The curve holomorphically embedding an **M5 brane** with this new asymptotic behavior and wrapping the x_{10} circle n times **is the SW curve of an isolated rank- n SCFT $_n$** .
- SCFT $_n$ has $\{3, 5, \dots, 2n + 1\}$ as its spectrum of Coulomb branch dimensions. SCFT $_1 = \text{SCFT}[3 : E_6]$.

V Geometrical picture of S-duality

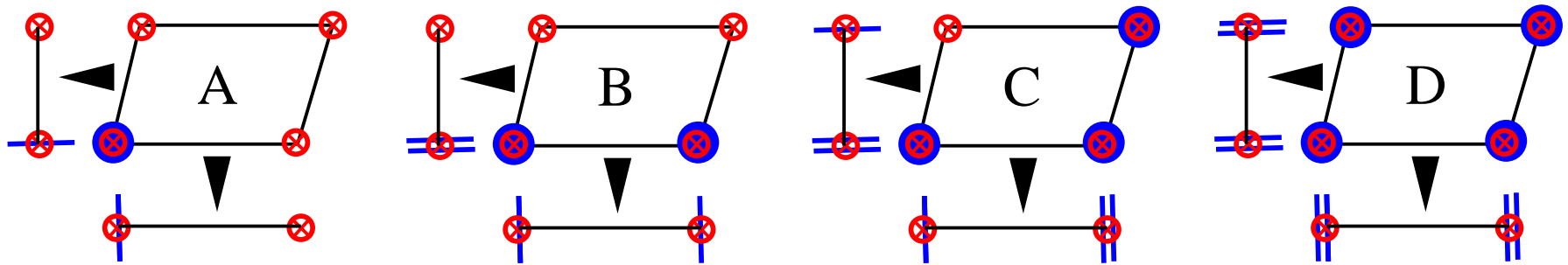
S-duality now realized as choice of circle on which to reduce M theory to IIA string theory:



With extra D4 brane, becomes the su(4) w/ $4 \cdot 4 \oplus 2 \cdot 6 = \text{sp}(2)$ w/ $3 \cdot 4 \oplus \text{SCFT}[3 : E_6]$ example mentioned earlier.

Geometrical picture of S-duality (cont)

This then gives 4 families of S-dualities:



A: Known self-dual $sp(n)$ w/ $4 \cdot \mathbf{f} + 1 \cdot \mathbf{a}$.

B: Gives $su(2n)$ w/ $4 \cdot \mathbf{f} + 2 \cdot \mathbf{a} = sp(n)$ w/ $SCFT_{n-1}$ equivalence.

Generalizes earlier examples.

C: Self-dual $su(n)$ w/ $SCFT_{n-1}$.

Includes $su(3)$ w/ $SCFT[E_6]$ found earlier?

D: New self-dual $su(n)$ w/ $(SCFT_n \oplus SCFT_n)$.

C and **D** have no purely weakly-coupled limits.

VI Summary

- Field theory S-duality of (a class of) $N=2$ SCFTs is realized geometrically in M theory.
- A class of isolated $N=2$ SCFTs is realized by open string modes on D4 branes in the presence of a novel NS5-O6⁻ bound state.
- New self-dual non-lagrangian $N=2$ SCFTs with exactly marginal operators and no lagrangian limits are constructed.