String theory effects on 5D black strings

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hep-th/0702072, hep-th/0703087, 0705.1847[hep-th], 0801.1863 [hep-th]

Great Lakes String Conference 2008
University of Wisconsin, Madison

Motivation

Black hole entropy

General relativity → Thermodynamics

Bekenstein-Hawking area law for entropy

Corrections from
higher dimension operators

String theory → Statistical mechanics

Degeneracy of D-branes bound states

Weakly coupled regime
(large charge limit)

Maldacena, Strominger & Witten

Small black holes

- **Classical description:** Singular solutions in SUGRA with vanishing area at horizon.
- **Quantum description:** Two-charge system (momentum + winding) with non-zero entropy

Higher order corrections should provide a string/Planck scale horizon and reproduce entropy according to Wald's formula

Outline

- Motivation
- Challenges and progress of higher derivative corrections
- Theoretical framework: 5D conformal supergravity
- Black strings: detailed analysis
- Small black holes
- Conclusion

Why five dimensions?

HD corrections: Large versus small black holes

First determine all corrections up to a given order (very hard task, but doable!)

For **large black hole** at leading order the solution is regular.
Expansion is systematic and higher orders are small

For **small black holes** the leading order solution is singular. Size of the horizon is of the same order as corrections. All terms in expansion contribute. (very hard task!)

Anomaly inflow: Take advantage of AdS_3/CFT_2

Gauge and Gravitational anomalies:
Entropy formula controlled by left & right moving **central charges** of 1+1 CFT.

$c_{IJK} A^I F^J F^K$

$c_{2I}\, A^I \, Tr(R^2)$

KEY: Central charge governed by the coefficients of **Chern-Simons** terms in supergravity.

Kraus & Larsen

The theory and the victim…

Theoretical framework: 5D Supergravity

- N=2 supergravity coupled to vector multiplets
- Can be obtained as arising from a reduction of **M-theory on CY_3.** Four-form flux describing M-branes wrap cycles in CY_3 leading to sources in 5D.

M2-branes source of electric charge -- q_I.

M5-branes source of magnetic charge -- p^I.

- New feature: **off-shell formalism** -- Auxiliary fields are introduced to make susy transformations independent of action.

Laboratory: Five dimensional black strings

- Extended one dimensional objects carrying magnetic charge.
- Near horizon geometry: **AdS_3 x S^2.**
- M5-branes wrapping four-cycles. **Scales** at the horizon are set by $\mathbf{c_{IJK}p^Ip^Jp^K}$.
- **Small black hole:** $c_{IJK}p^Ip^Jp^K=0$.

Precision Measurements

Off-shell conformal supergravity

Bergshoeff et al
Fujita et al
Hanaki et al

- Supersymmetry transformations close off-shell

$$\delta\psi_\mu = \left(\mathcal{D}_\mu + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}\gamma_\mu\gamma\cdot v\right)\epsilon = 0$$

$$\delta\Omega^I = \left(-\frac{1}{4}\gamma\cdot F^I - \frac{1}{2}\gamma^a\partial_a M^I - \frac{1}{3}M^I\gamma\cdot v\right)\epsilon = 0$$

$$\delta\chi = \left(D - 2\gamma^c\gamma^{ab}\mathcal{D}_a v_{bc} - 2\gamma^a\epsilon_{abcde}v^{bc}v^{de} + \frac{4}{3}(\gamma\cdot v)^2\right)\epsilon = 0$$

- Inclusion of a scalar (**D**) and two-form ($\mathbf{v}_{ab}$) auxiliary field
- This formalisms allows **unambiguously** to find supersymmetric completion of four derivative mixed gravitational Chern-Simons term

$$\mathcal{L}_0 = -\frac{1}{2}D - \frac{3}{4}R + v^2 + \mathcal{N}\left(\frac{1}{2}D - \frac{1}{4}R + 3v^2\right) + 2\mathcal{N}_I v^{ab}F^I_{ab}$$
$$+\mathcal{N}_{IJ}\left(\frac{1}{4}F^I_{ab}F^{Jab} + \frac{1}{2}\partial_a M^I\partial^a M^J\right) + \frac{1}{24e}c_{IJK}A^I_a F^J_{bc}F^K_{de}\epsilon^{abcde}$$

$$e\mathcal{L}_1 = \frac{c_{2I}}{24\cdot 16}\epsilon_{abcde}A^{Ia}R^{bcfg}R^{de}{}_{fg} + \ldots$$

Attractors & maximal SUSY

Strategy

• Near-horizon **enhancement** of **supersymmetry: AdS$_3$ x S^2**

•**Entropy** depends only on **near-horizon** data, given uniquely by the **charges** carried by the BH

Ferrara, Kallosh & Strominger

•. **Extremization** principles: Minimizing central charge and/or entropy function.

AdS$_3$ (central charge) AdS$_2$ (entropy function)

Kraus & Larsen
A. Sen

c-extremization

Starting from 2d Conformal Anomaly: $T_i^i = -\frac{c}{12} R^{(2)}$

$$I = \frac{1}{4\pi^2} \int d^5x \sqrt{g} \mathcal{L} + S_{CS} + S_{\text{bndy}} \longrightarrow \boxed{c = -6\ell_A^3 \ell_S^2 \mathcal{L}}$$

Extremize: $\boxed{\frac{\partial c}{\partial \phi^i} = 0}$ with $\phi^i = \{\ell_A, \ell_S, M^I, \ldots\}$

Example: Black string

SUSY: $AdS_3 \times S^2$

$$ds^2 = \ell_A^2 ds^2_{AdS} + \ell_S^2 d\Omega_2^2$$

$$\ell_S = \frac{1}{2}\ell_A \ , \quad M^I = \frac{p^I}{\ell_A}$$

c-extremization

$$\ell_A^3 = p^3 + \frac{1}{12}c_2 \cdot p$$

Central charge

$$c = 6p^3 + \frac{3}{4}c_2 \cdot p$$

- **Thermodynamics** of a black string excited to level $\mathbf{q_0}$

Cardy's Formula

$$S = 2\pi\sqrt{\frac{1}{6}c_L L_0}$$

+

5D Attractor

$$c = \frac{1}{2}(c_L + c_R)$$

$$c_R = 6p^3 + \frac{1}{2}c_2 \cdot p$$

$$c_L = 6p^3 + c_2 \cdot p$$

$$S = 2\pi\sqrt{\frac{1}{6}|q_0|(c_{IJK}p^Ip^Jp^K + c_2 \cdot p)}$$

Agreement with microscopic analysis and results obtained by looking at a subsector of possible 4D higher derivative corrections.

We can do better…

Full black string geometry

- Magnetic charges are topological and determined by the Bianchi Identity

$$F^I = -\frac{p^I}{2}\epsilon_2$$

- Most of the geometry is captured by solving the gravitino variation

$$ds^2 = e^{2U(r)}\left(dt^2 - dx_5^2\right) - e^{-4U(r)}\left(dr^2 + r^2 d\Omega_2^2\right)$$

- Solving gaugino variation relates F^I with the scalars M^I. Imposing the BI for this solutions, determines the harmonic functions

$$M^I e^{-2U} = H^I = M_\infty^I + \frac{p^I}{2r}$$

- Special geometry constraint (eom for **D**), governs the profile U(r)

$$e^{-6U} = \frac{1}{6}c_{IJK}H^I H^J H^K + \frac{c_{2I}}{24}\left(\nabla H^I \nabla U + 2H^I \nabla^2 U\right)$$

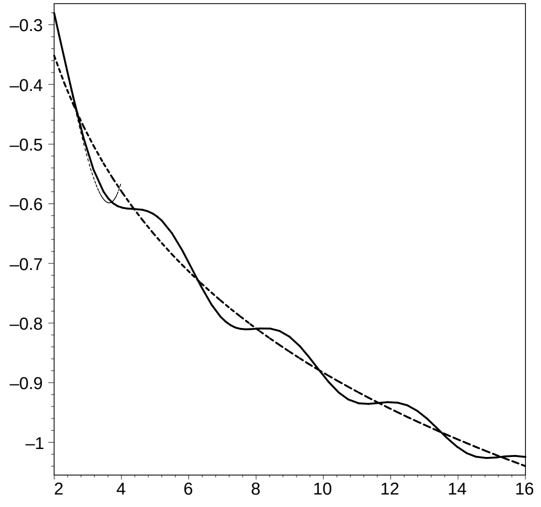
Small but significant

- **Small black string:** $CY_3=T^2$ x K3 where $c_2(K3)=24$, **p** M5-branes wraps cycle of **K3**.

Dual to **p** heterotic string with **8 bosons + 8 fermions** in right movers and **24 bosons** in left movers

$$c_R = 6p^3 + \frac{1}{2}c_2 \cdot p \qquad c_L = 6p^3 + c_2 \cdot p$$

$c_R = 12p$ $\qquad$ $c_L = 24p$



- Plot displays numerical solution and asymptotic solutions of the metric function for a small black string.
- Asymptotic flat boundary conditions match the near horizon geometry.

Conclusions

• Exploiting **symmetries** and **anomalies** of 5D black strings, we can obtain and explain agreement between macroscopic and microscopic entropy.

Off-shell supergravity	→	Geometry + Fields
Chern-Simons terms	→	Anomalies
Attractor mechanism	→	Near horizon

All the ingredients combined allow us to study in detail the full geometry and compute the relevant thermodynamic quantities (central charge in 5D, entropy in 4D).

• **Other scenarios:** Static and rotating **black holes** on asymptotically flat space and Taub-NUT spaces. Comparison with claims from **OSV** conjecture and **4D-5D lift.**

↑ Degeneracy formula for dyons