Nonlocal Operators in Gauge Theory and Holography

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Motivation

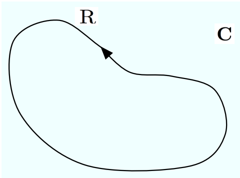
- The various phases of a gauge theory can be characterized by inserting an infinitely heavy probe and studying its response
- Operators supported on a curve like Wilson, Polyakov and 't Hooft operators insert a probe charged particle and are order parameters for the confined, deconfined and Higgs phase of gauge theory
- Goal is to construct operators that insert a probe string – a surface operator – and study whether they lead to new order parameters for phases of gauge theory. Surface operators are supported on a surface
- The physical content of holographic gauge theories is encoded in the correlation function of gauge invariant operators

$\Longrightarrow$ Need bulk description of *all* operators

- Gain intuition about these operators from bulk geometric description

Wilson Loops

- A Wilson loop inserts a charged probe particle



and measures the phase acquired by the particle

$$< W_R[C] >= \frac{1}{\text{Dim}R} \left\langle Tr_R\, P \exp\left(i \oint_C A \right) \right\rangle$$

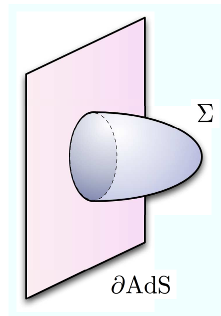
- A Wilson loop characterizes phases of gauge theories in terms of the potential between electric charges

  - Confinement
  - Higgs
  - Coulomb
  - Free magnetic

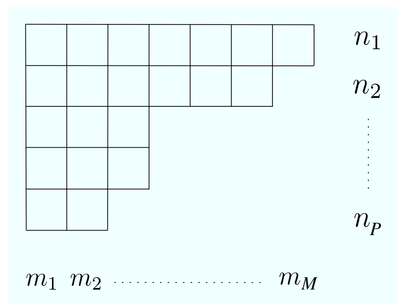
- Attempt to formulate gauge theories by using Wilson loops as the fundamental variables

Holographic Wilson Loops

- A Wilson loop corresponds to a string worldsheet that ends on the boundary of the bulk spacetime along the curve C Maldacena Rey & Yee



- A Wilson loop is labeled by a representation R of gauge group



- A Wilson loop corresponds to a configuration of M $D5$ branes *or* P $D3$ branes in AdS$_5\times$S^5 ending on C J.G & Passerini Drukker & Fiol Yamaguchi

- Circular Wilson loops in $\mathcal{N}=4$ SYM captured by a matrix model. Find exact all order agreement with the bulk D-brane computation!

- Gauge Theory excitations have a description in terms of bulk geometries which have the symmetries of the vacuum only asymptotically

- Can construct smooth asymptotically $AdS_5 \times S^5$ bubbling solutions for *all* half supersymmetric Wilson loops once a boundary condition is specified

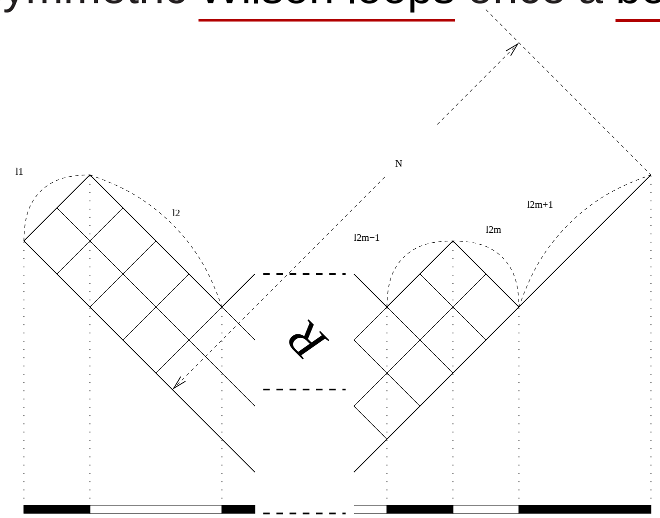
LLM

Yamaguchi

Lunin

J.G & Römelsberger

D'Hoker et al



- In the context of holography in topological string theory, one can construct bubbling Calabi-Yau manifolds whose closed string partition function yields precisely the expectation value of the Wilson loop

J.G & Okuda

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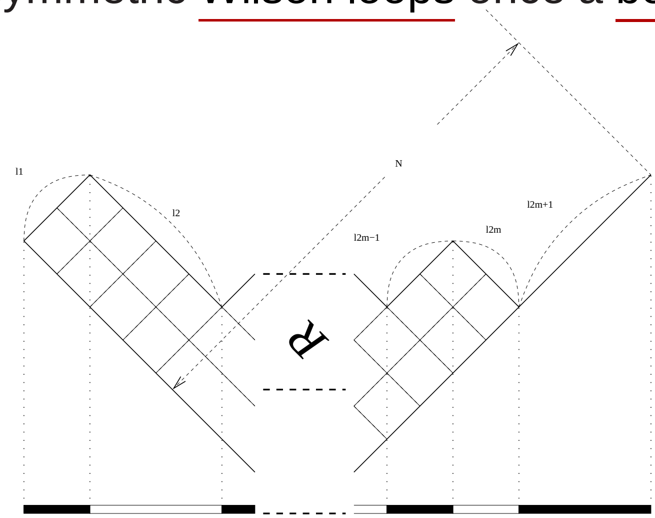

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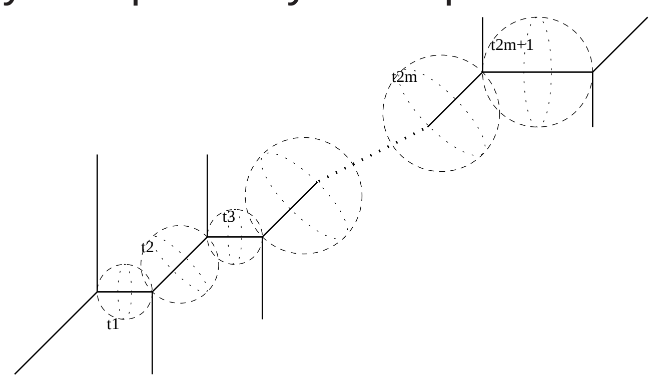


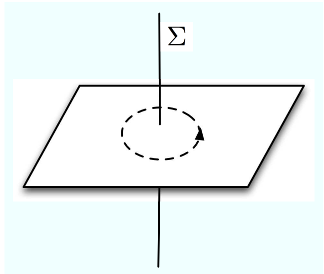
Surface Operators

- A Surface Operator inserts a probe string. Operator is supported on a surface Σ

- Operator characterized by Aharonov-Bohm phase:

Rohm

Alford et al



$$\Longrightarrow \qquad \Psi \rightarrow U\Psi \qquad U = P \exp i \oint A$$

Labeled by a conjugacy class $[U]$ of gauge group:

$$A = \begin{pmatrix} \alpha_1 \otimes 1_{N_1} & 0 & \dots & 0 \\ 0 & \alpha_2 \otimes 1_{N_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_M \otimes 1_{N_M} \end{pmatrix} d\theta$$

- Surface Operator breaks $U(N) \rightarrow L \equiv \prod_{l=1}^{M} U(N_l)$ along Σ.

  Can insert two-dimensional θ-angles:

Gukov & Witten

$$\exp\left(\sum_{l=1}^{M} \eta_l \int_\Sigma Tr\, \frac{F_l}{2\pi}\right)$$

- This operator *may* distinguish novel phases of gauge theories. Expect:

$$< \mathcal{O}_\Sigma > \simeq \exp(-TA) \qquad \text{or} \qquad < \mathcal{O}_\Sigma > \simeq \exp(-TV)$$

for a surface Σ of area A enclosing a volume V and $T = T(\alpha_l, \eta_l)$

- A surface operator in $\mathcal{N} = 4$ SYM can also acquire an L-invariant pole for Φ near Σ:

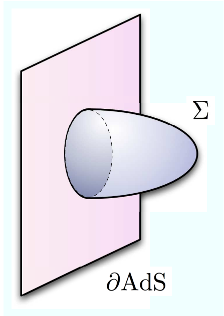
$$\Phi = \frac{1}{z} \begin{pmatrix} \beta_1 + i\gamma_1 \otimes 1_{N_1} & 0 & \dots & 0 \\ 0 & \beta_2 + i\gamma_2 \otimes 1_{N_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \beta_M + i\gamma_M \otimes 1_{N_M} \end{pmatrix}$$

- A supersymmetric surface operator in $\mathcal{N} = 4$ SYM depends on $4M$ parameters $(\alpha_l, \beta_l, \gamma_l, \eta_l)$. Operator becomes singular whenever parameters coincide and $\prod_{l=1}^{M} U(N_l)$ symmetry gets enhanced

- These operators can be studied in terms of a path integral with a codimension two singularity in R^4 *or* in terms non-trivial boundary conditions for $\mathcal{N} = 4$ SYM on $\text{AdS}_3 \times \text{S}^1$

Holographic Surface Operators

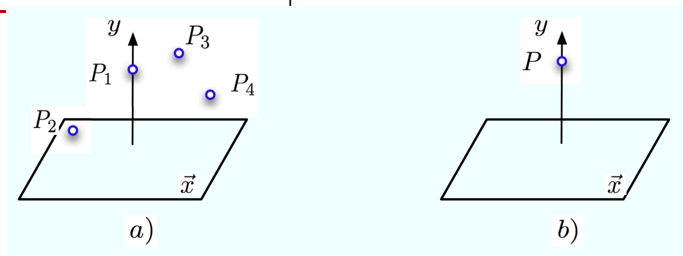
- A surface operator corresponds to a $D3$-brane that ends on the boundary of the bulk spacetime along a surface Σ



- Can construct the *exact* asymptotically $AdS_5 \times S^5$ bubbling solutions for all half supersymmetric surface operators

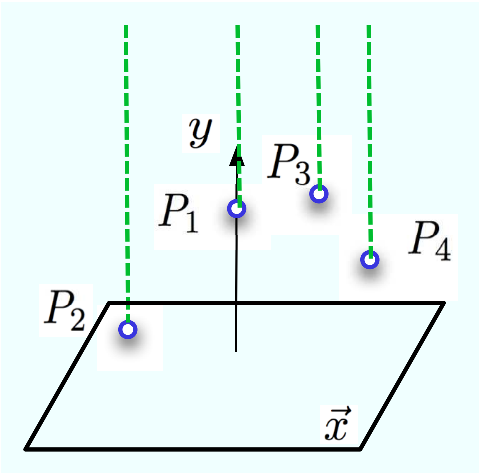
J.G & Matsuura
Lin & Maldacena

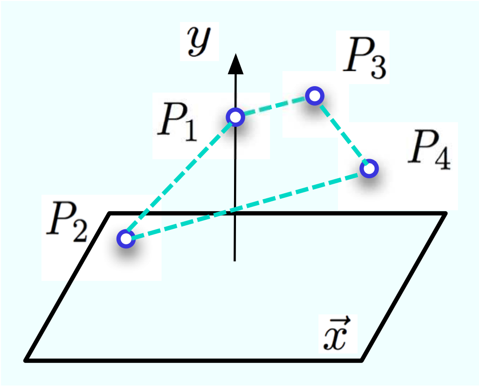
- The bulk gravitational description is given by a metric and five-form flux on $AdS_3 \times S^1 \times S^3 \times X$
- Metric and five-form flux is completely determined by the choice of a particle distribution in $X = R^3_+$



- Bulk data is the position of the charges $(\vec{x}_l, y_l)$ for $l = 1, \ldots, M$

- Bulk solutions exhibit rich topology. The geometry has surfaces D_l of disk topology as well as S^2's:

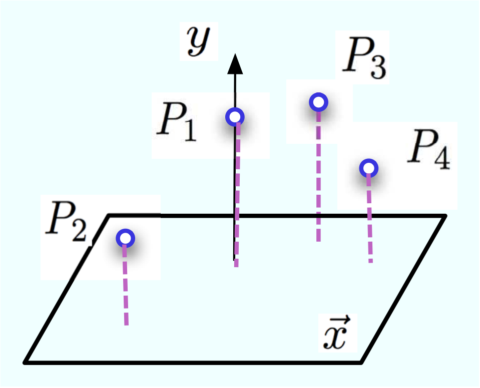




- Bulk solution is fully determined only once the periods of SUGRA two-forms are specified

$$\int_{D_l} \frac{B_{NS}}{2\pi} \qquad \int_{D_l} \frac{B_R}{2\pi} \qquad l = 1, \dots, M$$

- Bulk geometry also contains S^5's:



- Mapping of bulk solution to surface operator parameters:

$$
\begin{aligned}
(\beta_l, \gamma_l) &= \frac{\vec{x}_l}{l_s^2} \\
\alpha_l &= -\int_{D_l} \frac{B_{NS}}{2\pi} \\
\eta_l &= \int_{D_l} \frac{B_R}{2\pi} \\
N_l &= \frac{1}{4\pi^4 l_p^4} \int_{S_l^5} F_5
\end{aligned}
$$

- Bulk solution becomes singular whenever the corresponding surface operator also becomes singular

- SUGRA predicts the action of the S-duality group of $\mathcal{N} = 4$ SYM on surface operators. Agrees precisely with conjecture!

- Can compute the OPE of a surface operator with various local and nonlocal operators in various regimes in gauge theory, using probe branes and bubbling solutions

Drukker, J.G, Matsuura

- Compute the correlation function of local operators, Wilson loops, 't Hooft loops with a surface operator
- Get agreement between the different regimes:

  - Semiclassical gauge theory

  -Probe brane

  -Bubbling solutions
- The supergravity computation of the correlation functions requires understanding how to extract dynamics from the bubbling solutions. For example:

$$< \mathcal{O}_{\Delta=2} \cdot \mathcal{O}_{\Sigma} >= \frac{1}{\sqrt{\lambda}} \sum_{l=1}^{M} (\beta_l + i\gamma_l)^2$$

- Correlation functions of Wilson loops can also be computed using the bubbling geometries for Wilson loops
- Precise agreement is found between the matrix model and the supergravity computation

J.G, Matsuura, Okuda, Trancanelli

Surface Operators And Fermions

- Can construct a surface operator by introducing new degrees of freedom supported on a surface Σ and integrating them out

Buchbinder, J.G & Matsuura

- Consider the low energy description of the $D3/D7$ brane system:

	0	1	2	3	4	5	6	7	8	9
$N\ D3$	$\times$	$\times$	$\times$	$\times$						
$M\ D7$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$

- $3-7$ strings yield a chiral fermion χ localized on $\Sigma = R^{1,1}$

$$S_{defect} = \int dx^+ dx^- \; \bar{\chi}(\partial_+ + A_+)\chi$$

- Quantum mechanically the theory is anomalous but anomalies cancel by anomaly inflow

$$S = S_{\mathcal{N}=4} + S_{defect} + S_{CS}(A),$$

$$S_{CS}(A) = -\frac{(2\pi\alpha')^2 \tau_3}{2} \int G_1 \wedge Tr\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

- Take backreaction of $D7$'s to G_1 flux into account: $dG_1 = g_s M \delta^2(x)$

- The action:

$$S = S_{\mathcal{N}=4} + S_{defect} + S_{CS}(A),$$

does *not* have the symmetries of the $D3/D7$ intersection

- It is inconsistent to only replace the $D7$ brane by its RR flux. We must consider the field theory in the full $D7$ brane background!

Harvey & Royston

Buchbinder, J.G & Matsuura

- The $D7$ background is given by

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + dx^I dx^I + H_7 f\bar{f} dz d\bar{z} \quad (1)$$

$$H_7 = e^{-\Phi}, \qquad \tau = C + ie^{-\Phi}, \qquad \partial_{\bar{z}}\tau = 0 \quad (2)$$

where $H_7 f\bar{f} = e^{-\Phi}\eta^2\bar{\eta}^2|\prod_{i=1}^{M}(z - z_i)^{-1/12}|^2$

Greene et al

- We can construct the supersymmetric action of $\mathcal{N} = 4$ SYM on the $D7$ background

- Integrating out the localized fermions inserts surface operator into $\mathcal{N} = 4$ SYM path integral

$$\exp\left[iM\Gamma_{WZW}(A)\right]$$

Some Remarks

- In the regime $g^2 M << 1$ can describe a surface operator by AdS$_3 \times$S^5 probe $D7$ branes in AdS$_5 \times$S^5. In this regime the gauge anomaly is suppressed and can consider the gauge theory in flat space
- Can find the exact bulk gravitational solution describing the fully localized $D3/D7$ brane intersection, which is described by a metric on AdS$_3 \times$S$^5 \times X$
- $g^2 M$ corrections break the classical scaling symmetry of the gauge theory
- The bulk supergravity solution reveals that the holographic dual gauge theory does indeed live on the $D7$-brane background. The metric on the conformal boundary – where the gauge theory lives – is that of the $D7$-branes

Conclusions

- There is a very explicit description of interesting gauge theory operators in terms of the dual bulk description
- The bulk description provides us with a very geometrical way to gather intuition about novel nonlocal operators in gauge theory
- We have performed the first computations with these operators and found that there are different methods to perform the computations which are valid in different regimes
- Have used the bubbling solutions to extract dynamical information
- We have found that the gauge/gravity duality extends to situations where the holographic gauge theory lives in a non-trivial supergravity background