# Half-Twisted (0,2) Correlators from the Coulomb Branch

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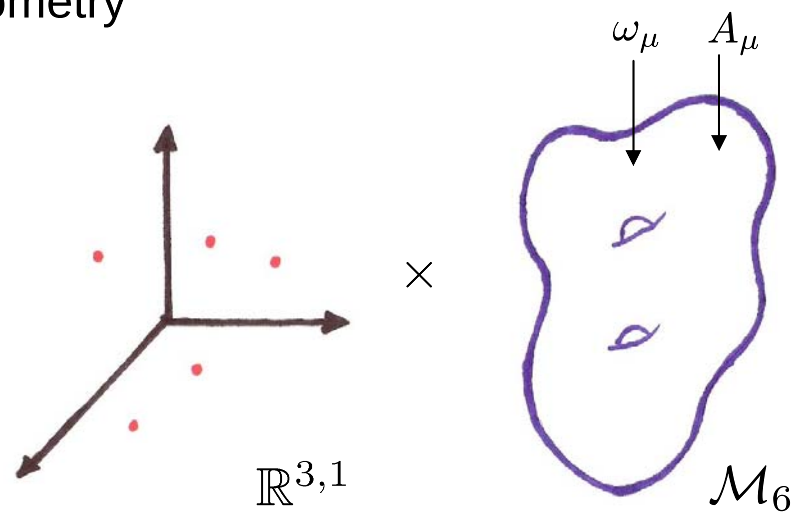
# Outline

1. Motivation: How much do we know about the Heterotic String?
2. (0,2) GLSMs with a (half-)twist
3. Computing Correlators
4. Examples
5. Conclusion

# Motivation: The Heterotic String

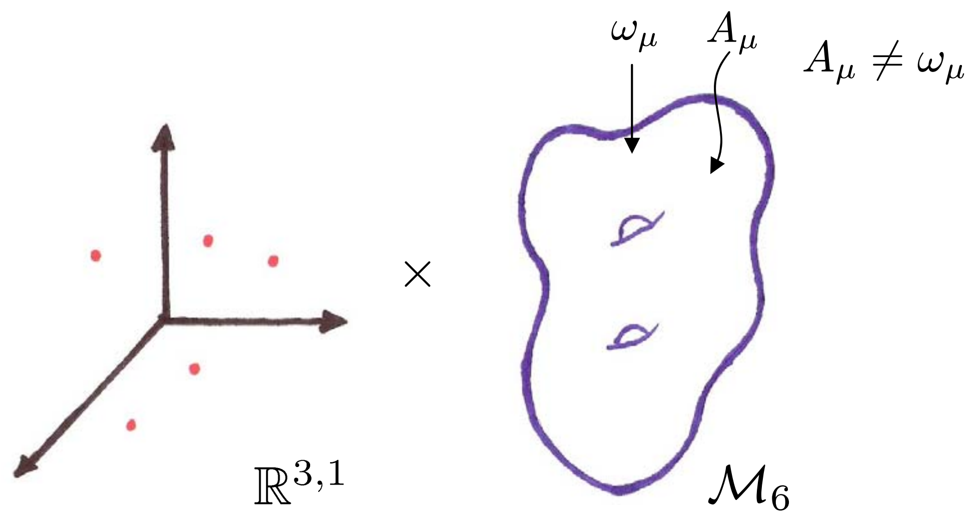
- $d=4,\ E_8\times E_8$ Heterotic compactification (without torsion) is specified by a gauge bundle on a geometry




- Typical example: $\mathcal{M}_6 = CY_3$ with standard embedding $A_\mu = \omega_\mu$
  - Worldsheet is a (2,2) SCFT
  - Spacetime low energy effective field theory:
    - Unbroken $E_6\times E_8$ gauge group, $27$ and $\overline{27}$ matter multiplets, moduli
    - Can compute $27^3$ and $\overline{27}^3$ Yukawa couplings, Mirror symmetry, Special Geometry

# Motivation: The Heterotic String

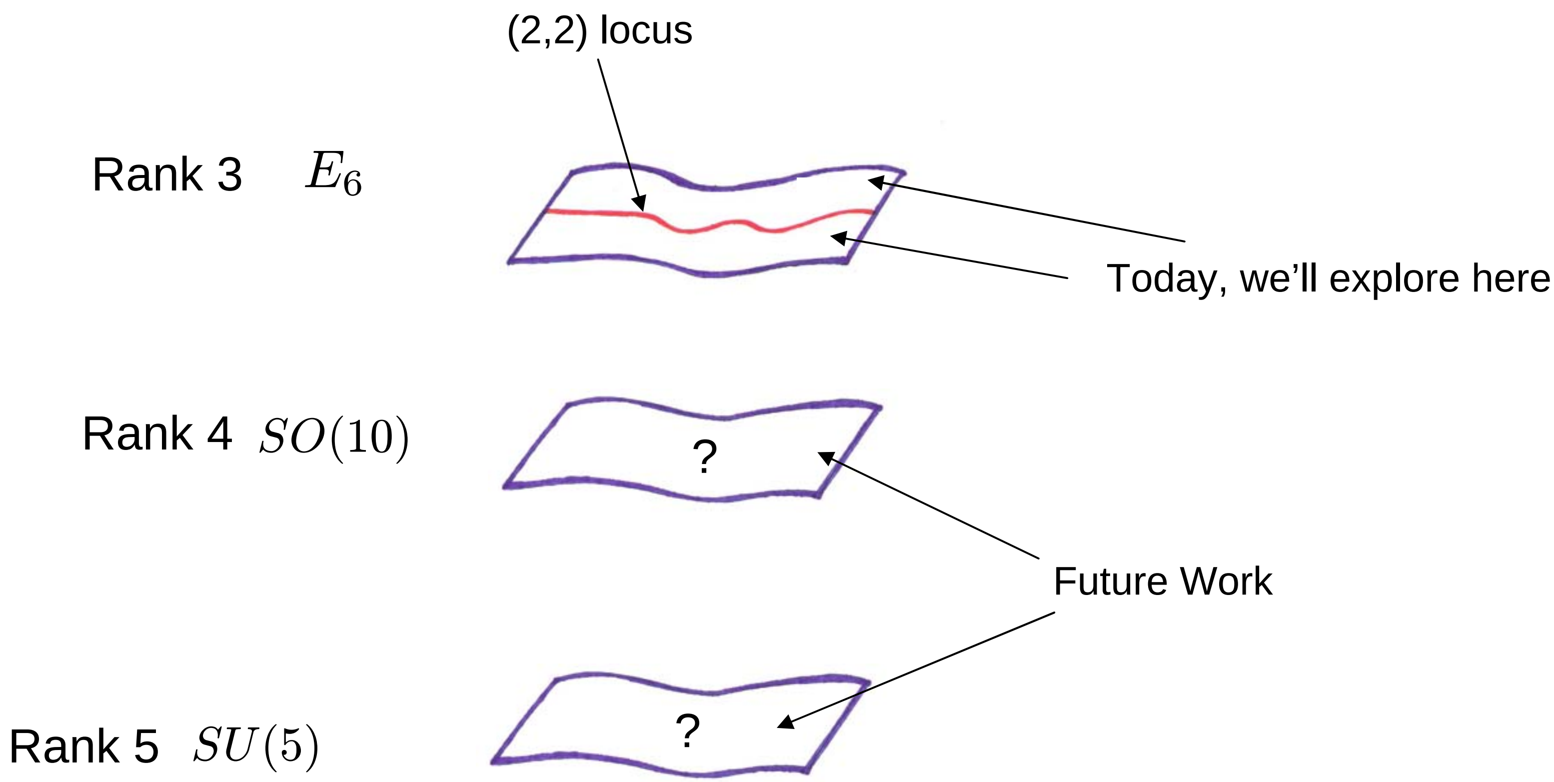
- How does this change with a more general choice of bundle?




  - Low energy phenomenology: more realistic gauge groups e.g. SO(10), SU(5)
- Want to understand the worldsheet description of such vacua:
  - (2,2) SUSY is reduced to (0,2)
  - Are Yukawa couplings computable?
  - Is there special geometry? Is there Mirror symmetry?

# Motivation: The Heterotic String

- Many unexplored moduli of the Heterotic String
- We really only understand a small slice of the Heterotic moduli space!



# Motivation: The Heterotic String

- Outline of our approach:
  - Study subclass (0,2) theories (ones with (2,2)-locus)
  - Use half-twisted GLSM (on Coulomb branch)
  - Compute *exact* (all-instanton) answers for correlators
- Our technique is easy to use

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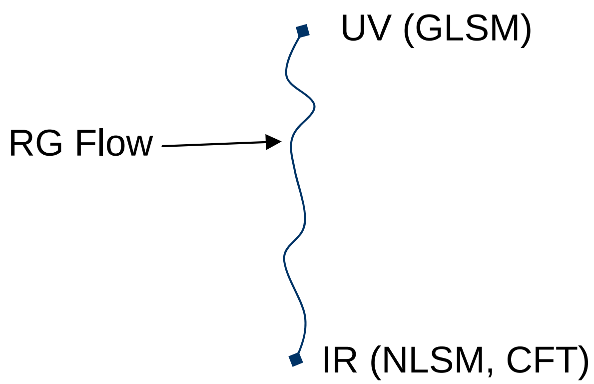
You shouldn’t be asleep yet……

# The (Twisted) GLSM is fun!

- Why is the GLSM useful?
  - Useful for generating NLSMs and CFTs




- Half twisted GLSM
  - (0,2) analogue of the A-model GLSM
  - Compute RG invariant properties of physical theories *exactly*

# Review of (0,2) GLSM

- Consider (0,2) theories with a (2,2) locus.
- Field content easily understood by decomposing (2,2) fields:

| (2,2) Field | Bosons | Fermions |
|---|---|---|
| Matter fields | $\Phi^i$ | $\Gamma^i$ |
| Vector multiplet | $V_{\pm,a}$ | |
| Field Strength | $\Sigma_a$ | $\Upsilon_a$ |

$i = 1, \ldots, n$

$a = 1, \ldots, n-d$

  - $\Gamma^i$ contain the left-moving Heterotic fermions (i.e. specify bundle)

- $\overline{\mathcal{D}}_+\Gamma^i = E^i(\Phi, \Sigma)$ with $E^i$ a holomorphic function of $(\Sigma_a, \Phi^i)$
- $E^i$ specifies bundle structure. We consider (0,2) bundle deformations

$$E^i = \sum_a \phi^i Q^i_a \sigma^a \rightarrow E^i = \sum_{a,j} M^{i\,a}_{\ j} \phi^j \sigma_a$$

(2,2) (0,2)

Matrix of complex parameters specifying bundle

# Review of (0,2) GLSM

- Action for (0,2) GLSM: $S = S_{\rm kin} + S_{\rm F-I} + S_{\rm J}$

$$\begin{aligned} S_{\rm kin} &= \int d^2y d^2\theta \left\{ -\frac{1}{8e_0^2}\overline{\Upsilon}_a \Upsilon_a - \frac{i}{2e_0^2}\overline{\Sigma}_a \partial_- \Sigma_a - \frac{i}{2}\overline{\Phi}^i(\partial_- + iQ_i^a V_{a,-})\Phi^i - \frac{1}{2}\overline{\Gamma}^i\Gamma^i \right\}, \\ S_{\rm F-I} &= \tfrac{1}{8\pi i}\int d^2y d\theta^+ \Upsilon_a \log(q_a)|_{\overline{\theta}^+=0} + \text{h.c.}, \\ S_J &= \int d^2y d\theta^+ \Gamma^i J_i(\Phi)|_{\overline{\theta}^+=0} + \text{h.c.}. \longleftarrow \text{ Matter superpotential} \end{aligned}$$

where $q^a = \exp(-2\pi r_a + i\theta_a)$ (Kähler moduli)

- We will mostly consider theories with $J_i = 0$.

- Bosonic Potential

$$U = 2\sum_\alpha \phi^\dagger_{(\alpha)} M^\dagger_{(\alpha)} M_{(\alpha)} \phi_{(\alpha)} + \frac{e_0^2}{2}\sum_{a=1}^{n-d}\left(\sum_\alpha Q^a_{(\alpha)}\phi^\dagger_{(\alpha)}\phi_{(\alpha)} - r^a\right)^2$$

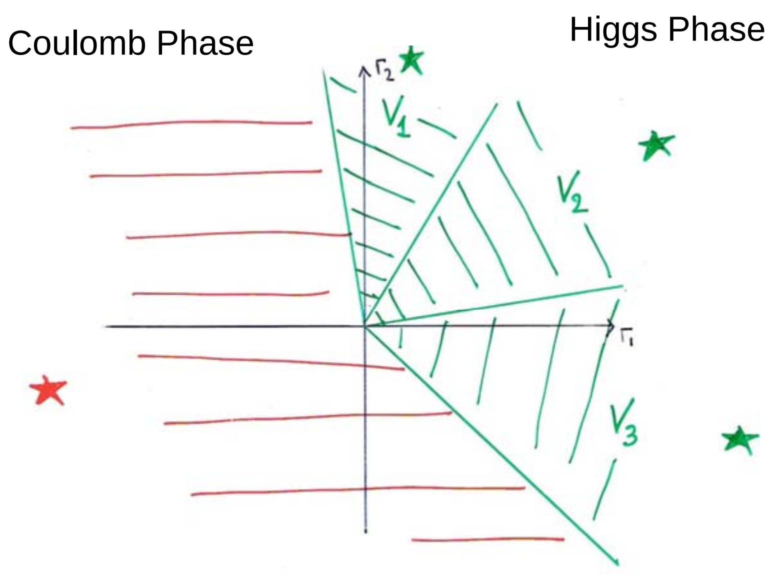
- Look for vacua…

# Vacua of (Compact) (2,2) GLSMs

- Higgs Vacua: $\langle \sigma \rangle = 0$
  - Quantum description is in terms of gauge instantons
- Coulomb Vacua: $\langle \phi \rangle = 0$
  - Quantum description by an effective potential (1-loop exact)

- Computations are independent of phase (analytic continuation)

- (0,2) Deformations do not destroy this picture



# Coulomb Vacua

- On the (2,2) locus for compact GLSMs:
  - Coulomb vacua described by an effective superpotential $\widetilde{W}_{\text{eff}}$.
  - In (0,2)-language:

$$\mathcal{L}_{\text{eff}} = \int d\theta^+ \Upsilon_a \widetilde{J}^a + \text{h.c.} \quad \text{with} \quad \widetilde{J}^a = \frac{\partial \widetilde{W}_{\text{eff}}}{\partial \sigma_a}$$

- For (0,2) deformations, picture qualitatively the same with

$$\widetilde{J}^a = \log\left[\Pi_\alpha \left(\det M_{(\alpha)}\right)^{Q^a_{(\alpha)}} / q^a\right]$$

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# Coulomb Branch: Correlators for (2,2)

- In the A-model *Melnikov-Plesser* showed that:

$$\langle \sigma^{a_1}(x_1)\ldots\sigma^{a_k}(x_k)\rangle = \sum_{\sigma^*}\sigma^{a_1}\ldots\sigma^{a_k}\left[\det \mathrm{Hess}\,\widetilde{W}_{\mathrm{eff}}(\sigma)\,\Pi_i(Q_i^b\sigma_b)\right]^{-1}$$

  - $\widetilde{W}_{\mathrm{eff}}$ is 1-loop effective potential
  - $\det \mathrm{Hess}\,\widetilde{W}_{\mathrm{eff}}$ from integrating out $(\Sigma_a, \Upsilon_a)$ multiplets
  - $\Pi_i(Q_i^b\sigma_b)$ from the zero modes of $(\Gamma^i, \Phi^i)$multiplets
  - Sum is over $\sigma$ vacua: solutions of $d\widetilde{W}_{\mathrm{eff}} = 0$
  - In the limit of large worldsheet, localization is arbitrarily good

- We generalize this to (0,2) correlators

# Coulomb Branch: Correlators for (0,2)

- For the Half-twisted Model:

$$\langle \sigma^{a_1}(x_1)\ldots\sigma^{a_k}(x_k)\rangle = \sum_{\sigma^*}\sigma^{a_1}\ldots\sigma^{a_k}\left[\det(J_{a,b})\Pi_\alpha \det M_{(\alpha)}\right]^{-1}$$

  - $J_a$ is the 1-loop effective potential
  - $\det(J_{a,b})$ from integrating out $(\Sigma_a, \Upsilon_a)$ multiplets
  - $\Pi_\alpha \det M_{(\alpha)}$ from the zero modes of $(\Gamma^i, \Phi^i)$ multiplets
  - Sum over $\sigma$ vacua: $J_a = 0$
  - In the limit of large worldsheet, localization is *still* arbitrarily good (*Melnikov-Sethi 2007*)
- Using this formula let's now apply it to a series of examples…

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Time to stay awake!

# Examples & Applications

- Compute (0,2) correlators for a variety examples:
  1. $\mathbb{P}^1 \times \mathbb{P}^1$ (exactly matches *Guffin-Katz* -> Our formula works!*)*
  2. $\mathbb{P}^4_{1,1,2,2,2}$ (resolved weighted projective space)
  3. Compact Conformal Example (i.e. Yukawa Couplings!)

# Examples: $\mathbb{P}^4_{1,1,2,2,2}$

- Well studied example (eg. *Candelas et al* (1993), *Morrison-Plesser* (1995))
- The GLSM has $n = 6$ matter fields and $n - d = 2$

$$Q = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & -2 \end{pmatrix}$$

- 13 possible bundle deformations
- We consider the following subset of deformations:

$$M = \begin{pmatrix} \sigma_2 + \epsilon_1\sigma_1 & \epsilon_2\sigma_1 \\ \epsilon_3\sigma_1 & \sigma_2 \end{pmatrix}$$

- Apply our formula

$$\langle \sigma^{a_1}(x_1) \ldots \sigma^{a_k}(x_k) \rangle = \sum_{\sigma^*} \sigma^{a_1} \ldots \sigma^{a_k} \left[ \det(J_{a,b}) \Pi_\alpha \det M_{(\alpha)} \right]^{-1}$$

to give

(…suspense…)

# Examples: $\mathbb{P}^4_{1,1,2,2,2}$

- Results:

$$\begin{aligned}
\langle \sigma_1^4 \rangle &= \frac{2}{D_1}, \\
\langle \sigma_1^3 \sigma_2 \rangle &= \frac{1}{D_1}, \\
\langle \sigma_1^2 \sigma_2^2 \rangle &= \frac{\epsilon_1 - 2\epsilon_3\epsilon_3 + 2q_2}{D_1 D_2}, \\
\langle \sigma_1 \sigma_2^3 \rangle &= \frac{\epsilon_1^2 + \epsilon_2\epsilon_3(1-2\epsilon_1) + (6\epsilon_1 - 12\epsilon_2\epsilon_3 + 1)q_2 + 4q_2^2}{D_1 D_2^2},
\end{aligned}$$

$$\begin{aligned}
D_1 &= 1 + 2\epsilon_1 - 4\epsilon_2\epsilon_3, \\
D_2 &= 4q_2 - 1.
\end{aligned} \qquad q_2 = e^{-2\pi r_2 + i\theta_2}$$

- Interesting singularity structure:
  - $D_2 = 0$ Kähler singularity. Familiar from (2,2)
  - $D_1 = 0$ Bundle singularity. Visible even when q -> 0 (large radius limit)
    - Corresponds to a massless $\sigma$ field
    - In (0,2) parameter space -> find a new branch (mixed Coulomb-Higgs phase)
    - Example of new structures present in the Heterotic bundle moduli space

# Examples: Compact Conformal Example

- Construct compact CY hypersurface in $\mathbb{P}^4_{1,1,2,2,2}$
- Construct ambient toric variety by adding an new field $\Phi^0$ giving

$$Q = \begin{pmatrix} -4 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & -2 \end{pmatrix}$$

- Hypersurface defined using a superpotential W. On the (2,2) locus W is:

$$W = \Phi_0 P(\Phi_1, \ldots, \Phi_6), \quad P = (\Phi_1^8 + \Phi_2^8)\Phi_6^4 + \Phi_3^4 + \Phi_4^4 + \Phi_5^4$$

- Deform from (2,2) with same bundle deformations as for $\mathbb{P}^4_{1,1,2,2,2}$
- $\sum_i Q_i^a = 0$ implies:
  - $U(1)_{L,R}$ is non-anomalous -> IR NLSM is a CFT
  - There is are no Coulomb vacua! Does this mean that all is lost?...

No….not quite

# Examples: Compact Conformal Example – Singularities

- Still have 1-loop effective potential. Can determine the locus of points where SCFT is singular:

$$(1-2^8 q_1)^2 - 2^{18} q_1^2 q_2 = 0 \longrightarrow (1-2^8 q_1)^2 - 2^{18} q_1^2 q_2 + 2\epsilon_1(1-2^8 q_1) - 4\epsilon_2\epsilon_3 = 0$$

(2,2) (0,2)

  - Kähler and bundle moduli mixing -> treated on the same footing
  - Large volume limit q -> 0 -- still can get bundle moduli singularities

- What about the correlators (i.e. Yukawa couplings)?

# Examples: Compact Conformal – Correlators

- Quantum Restriction Formula:
  - Constructed by *Morrison-Plesser* for (2,2) A-model theories.
  - Relates correlators on the hypersurface to ambient toric space via:

$$\langle\langle \sigma_1^a \sigma_2^b \rangle\rangle_X = \langle \sigma_1^a \sigma_2^b \tfrac{-K}{1-K} \rangle_{\mathbb{P}^4} \quad \text{with} \quad -K = \textstyle\sum_{i>0} Q_i^a \sigma_a = 4\sigma_1$$

- We try to apply an analogous formula for Half-twisted theories:

$$\langle\langle \sigma_1^a \sigma_2^b \rangle\rangle_X = \langle \sigma_1^a \sigma_2^b \tfrac{-K}{1-K} \rangle_{\mathbb{P}^4} \quad \text{with} \quad -K = \textstyle\sum_{i>0} Q_i^a \sigma_a = 4\sigma_1$$

Computed in $\mathbb{P}^4_{1,1,2,2,2}$ with (0,2) deformations

# Examples: Compact Conformal – Correlators

- The results:

$$\langle\langle \sigma_1^3 \rangle\rangle = \frac{8}{D_\epsilon}, \qquad \langle\langle \sigma_1^2 \sigma_2 \rangle\rangle = \frac{4(1-2^8 q_1)}{D_\epsilon},$$

$$\langle\langle \sigma_1 \sigma_2^2 \rangle\rangle = \frac{4(2^{10} q_1 q_2 - 2q_2 + 2^8 \epsilon_1 q_1 + 2\epsilon_2\epsilon_3 - \epsilon_1)}{(1-4q_2) D_\epsilon},$$

$$\begin{aligned}\langle\langle \sigma_2^3 \rangle\rangle = 4 \big[ & q_2(1 + 4q_2 - 2^8 q_1 - 3072 q_1 q_2) + \epsilon_1^2 (1 - 2^8 q_1) \\ & + 2\epsilon_1(-2^{10} q_1 q_2 + 3q_2 - \epsilon_2\epsilon_3) \\ & + \epsilon_2\epsilon_3(-2^8 q_1 + 2^{10} q_2 q_1 + 1 - 12 q_2) \big] / (1-4q_2)^2 D_\epsilon .\end{aligned}$$

$$\text{where} \quad D_\epsilon = (1-2^8 q_1)^2 - 2^{18} q_1^2 q_2 + 2\epsilon_1(1-2^8 q_1) - 4\epsilon_2\epsilon_3 = 0$$

- Singularity structure *consistent with effective superpotential ->* good evidence for our proposal
- “Yukawa” couplings exhibit interesting bundle moduli dependence
- Not quite the whole story: additional (0,2) deformations of the hypersurface neglected (*work in progress with I. Melnikov & S. Sethi*)

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***Almost There!!!***

# Conclusions & Future Work

- *Summary*
  - Extension from (2,2) to (0,2) theories is easy!
  - We have computed correlators of compact toric GLSMs
  - We have made progress in understanding (0,2) Yukawa couplings
  - Our methods are easily extended to more involved theories.
- *Future Work*
  - What is the generalization of Quantum Restriction Formula?
  - What is the generalization of special geometry? What is the generalization of Mirror symmetry?
  - Generalize to (0,2) theories without a (2,2) locus

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