Strings

and

(Conjectured) Meson Melting

In collaberation with

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Outline

- Setup of the problem
- Stationary solutions
- Question raised by light-heavy meson solution
- Normal mode analysis
- Numerical results and problems
- Conclusion

AdS/CFT Correspondence [Maldacena 9802]

- AdS/CFT posits an equivalence between $\mathcal{N}=4$ SYM and type IIB string theory in an AdS $\times$ S^5 background.
- The temperature of the SYM corresponds to the radius of the black hole horizon.
- Quarks in the SYM correspond to the addition of a D7-brane in the string theory. [Karch, Katz]

AdS				**$\mathcal{N}=4$ SYM**
Radius of AdS and S^5	L			
String coupling	$4\pi g_s$	:	$g_{\rm YM}^2$	Yang-Mills coupling
String length scale	$(L/\ell_s)^4$	:	$g_{\rm YM}^2 N_c$	't Hooft coupling
Black hole horizon	r_h/π	:	T	Temperature
Radius of brane	$\ell_s^2 L^2 (r_q - r_h)$	:	$2\pi M_q$	Quark rest mass

Equations of Motion

Limiting attention to a 3D slice in the AdS space with coordinates (t, r, x) in the guage $\sigma = r$, $\tau = t$, we have the metric:

$$ds^2 = L^2\left(\frac{dr^2}{h(r)} - h(r)dt^2 + r^2\delta_{ij}dx^i dx^j\right)$$

where

$$h(r) = r^2\left[1 - \left(\frac{r_h}{r}\right)^4\right].$$

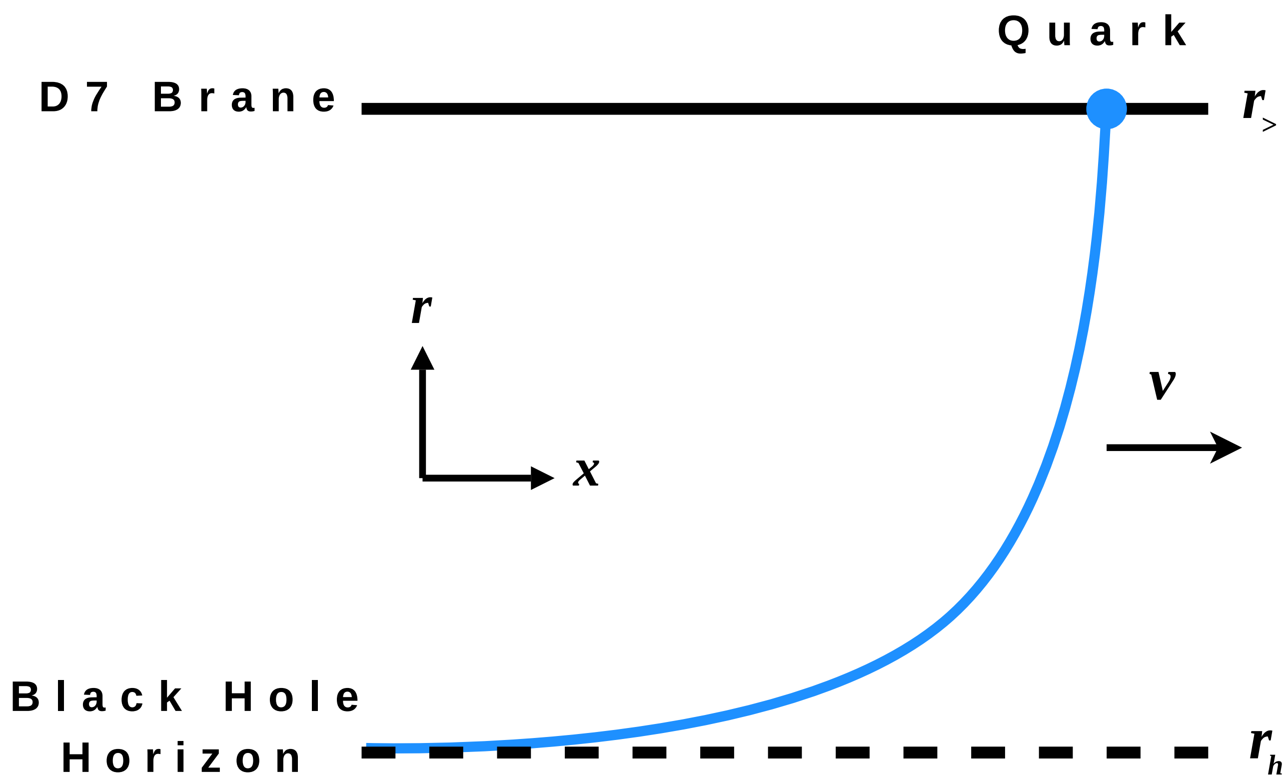
The Nambu-Goto action $S = -T_0\int d^2\sigma\sqrt{-g}$ gives the equations of motion

$$\frac{\partial}{\partial r}\left(h r^2 \frac{\partial_r x}{\sqrt{-g}}\right) - \frac{r^2}{h}\frac{\partial}{\partial t}\left(\frac{\partial_t x}{\sqrt{-g}}\right) = 0$$

which are equivalent to the conservation of energy and momentum on the string world sheet.

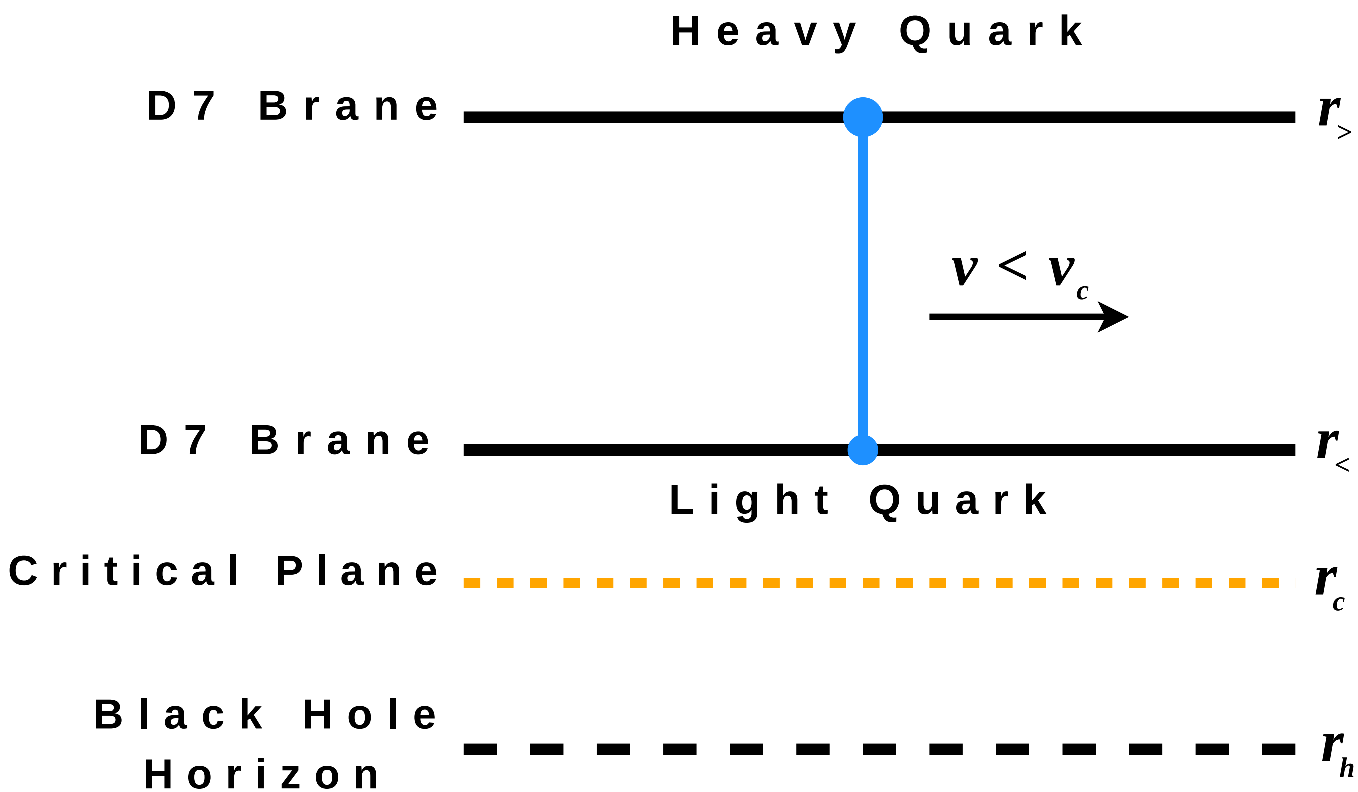
Dragged String

[Hertzog, Karch, Kovtun, Kozacz, Yaffe 0605]



String is forced by means of an electromagnetic field on the D7-brane. Energy flows down the string toward the horizon and the quark moves at a constant terminal velocity.

Light-Heavy Meson [Hertzog et al. 0605]



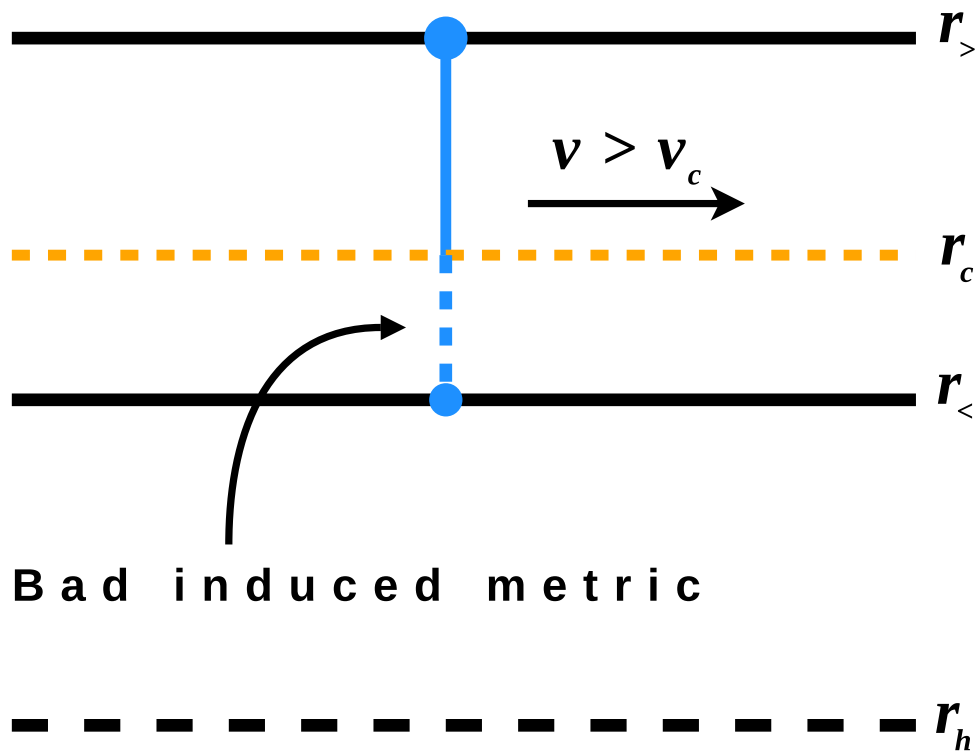
- Meson feels no resistance as it moves through the plasma.
- Only valid if the light brane is above $r_c = \frac{r_h}{(1-v^2)^{1/4}}$ (The critical plane).

Bad Light-Heavy Meson Solution

If $v > v_c \equiv \sqrt{1-(r_h/r_<)^4}$ then the critical plane is above the bottom brane.

Below the critical plane, the induced metric swiches sign and the energy becomes imaginary.

Note that in this solution the light quark is moving faster than the local velocity of light.



Question

What keeps us from applying a small electromagnetic field on the top brane to adiabatically accelerate the good light-heavy meson solution into the bad solution?

Three posibilities I can think of:

- A light-heavy string moving with velocity $v = v_c$ has infinite energy so we can never reach the velocity v_c.
- There is no gap in the spectrum of the string, so as we approach $v = v_c$ we can't prevent the inputted energy from cascading into the higher energy modes of the string.
- The quark on the top brane continues to accelerate, but the quark on the bottom brane falls behind it, moving at a velocity less than v_c. Energy inputted goes into stretching the string.

Vertical string has finite energy at $v = v_c$

The energy density in our gauge is given by:

$$\pi_t^0 \;=\; \frac{T_0 L^4}{\sqrt{-g}}\Big[-1-(r^4-r_h^4)(\partial_r x)^2\Big],$$

$$-g \;=\; L^4\Bigg[1-\frac{1}{1-(r_h/r)^4}(\partial_t x)^2+(r^4-r_h^4)(\partial_r x)^2\Bigg].$$

For a vertical string moving with velocity $v_c = (1-r_</r_h)^{-1/4}$ we have

$$\pi_t^0 = -T_0 L^2\left(1-\frac{1-(r_h/r_<)^4}{1-(r_h/r)^4}\right)^{-1/2} \propto \frac{1}{\sqrt{r-r_<}} + \mathcal{O}\big(\sqrt{r-r_<}\big).$$

Thus we see that the energy density diverges at $r = r_<$, but not strongly enough to make the energy of the string infinite.

Normal Modes

To do a normal mode analysis of our solution we linearize our equations of motion around the moving vertical string solution. To do this we plug

$$x(r,t) = vt + \varepsilon\,\xi(r)e^{i\omega t}$$

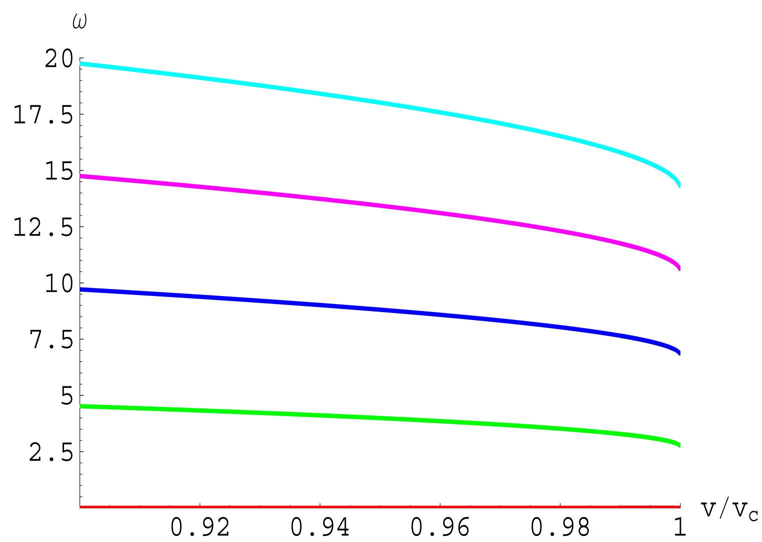
into our equations of motion and take the term linear in ε giving us

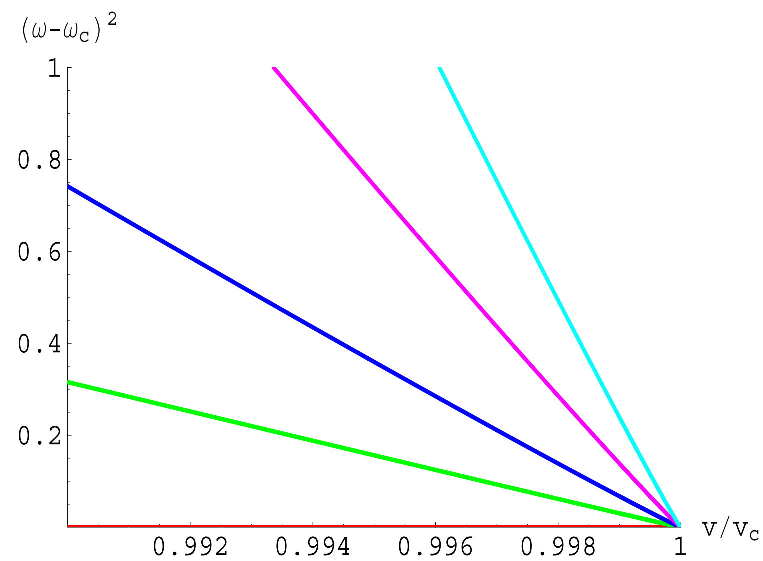
$$r^4\omega^2\xi - 2r^3[2 + v^2 + 2r^4(v^2 - 1)]\xi' - (r^4 - 1)(1 + r^4(v^2 - 1)]\xi'' \ = \ 0$$

where we have taken $T_0 = L = r_h = 1$. This can be solved by setting Neumann boundary conditions at the bottom brane and then shooting a given solution to the top brane. One then checks whether the solution intersects the top brane perpendicularly.

Results of Shooting Method

Performing the shooting method for two branes at 1.1 and 1.5, we obtain the following plots:



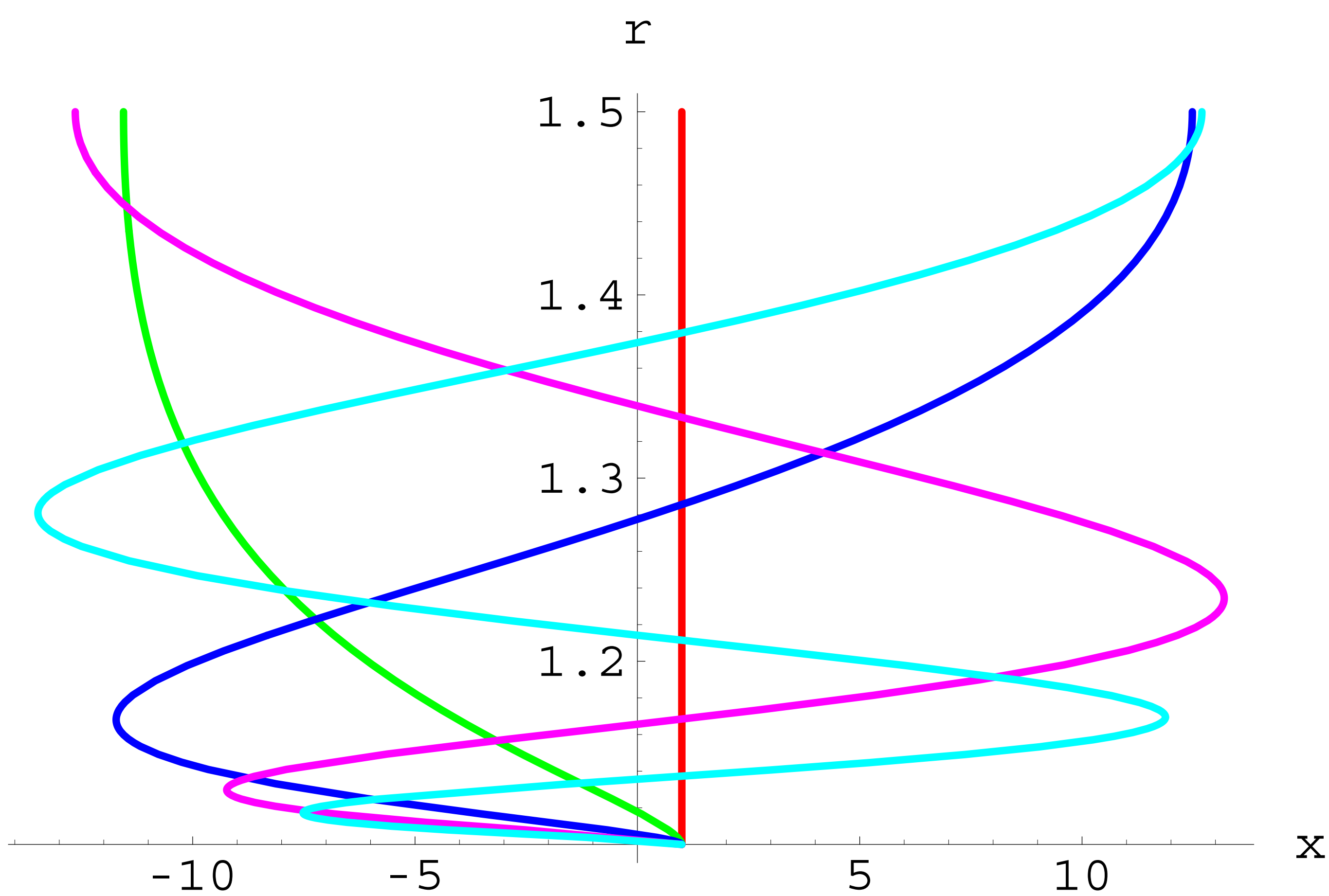


- The left plot shows the frequencies for the first five modes as a function of v/v_c. Notice that the gap between the zeroth frequency and the first frequency does not go to zero as $v \to v_c$.

- The right plot of $(\omega - \omega_c)^2$ which is proportional to $-v/v_c$.

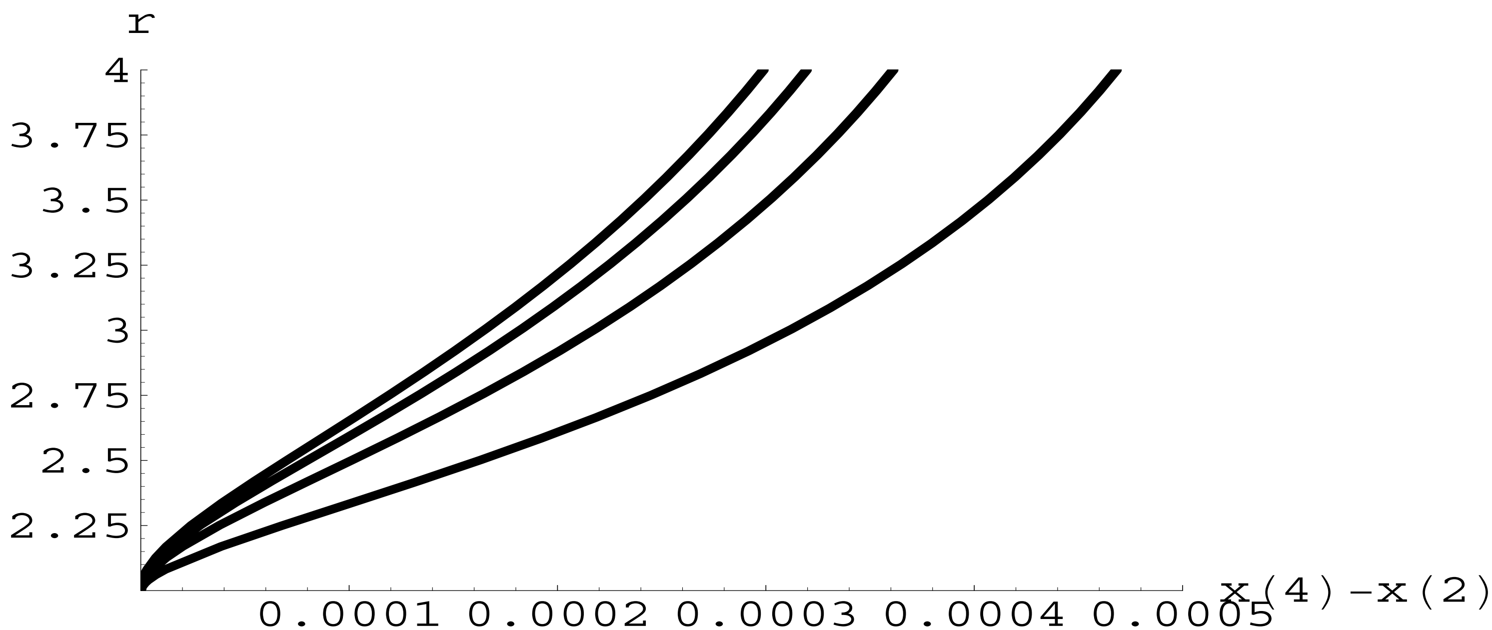
Plot of First Five Modes

Below is a plot of the functions $\xi(r)$ for the first five modes for $v/v_c = .999$



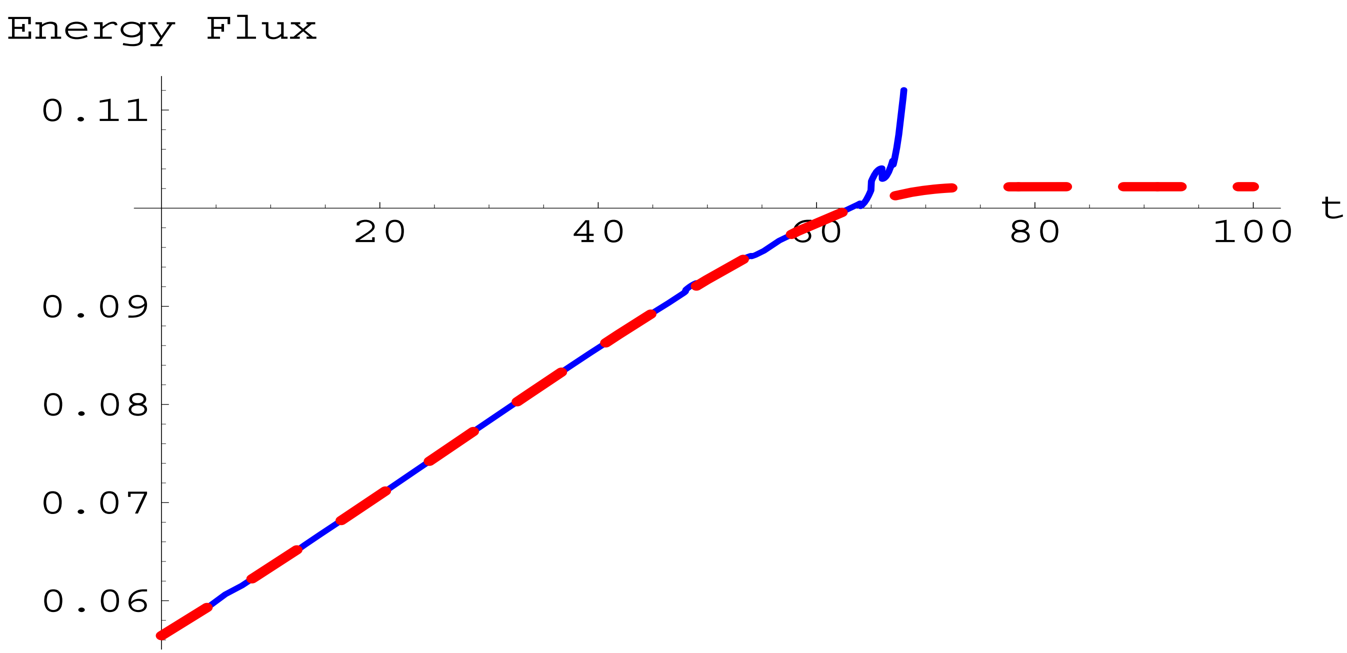
Numerical Integration

The following plots are for a string stretched between branes at AdS radii 2 and 4, at times 0, 20, 40, 60:



The horizontal axis is $x(4) - x(2)$. One can see that the top quark does start accelerating away from the bottom quark. The solution continues to evolve, but can't be trusted from times after $t \approx 64$.

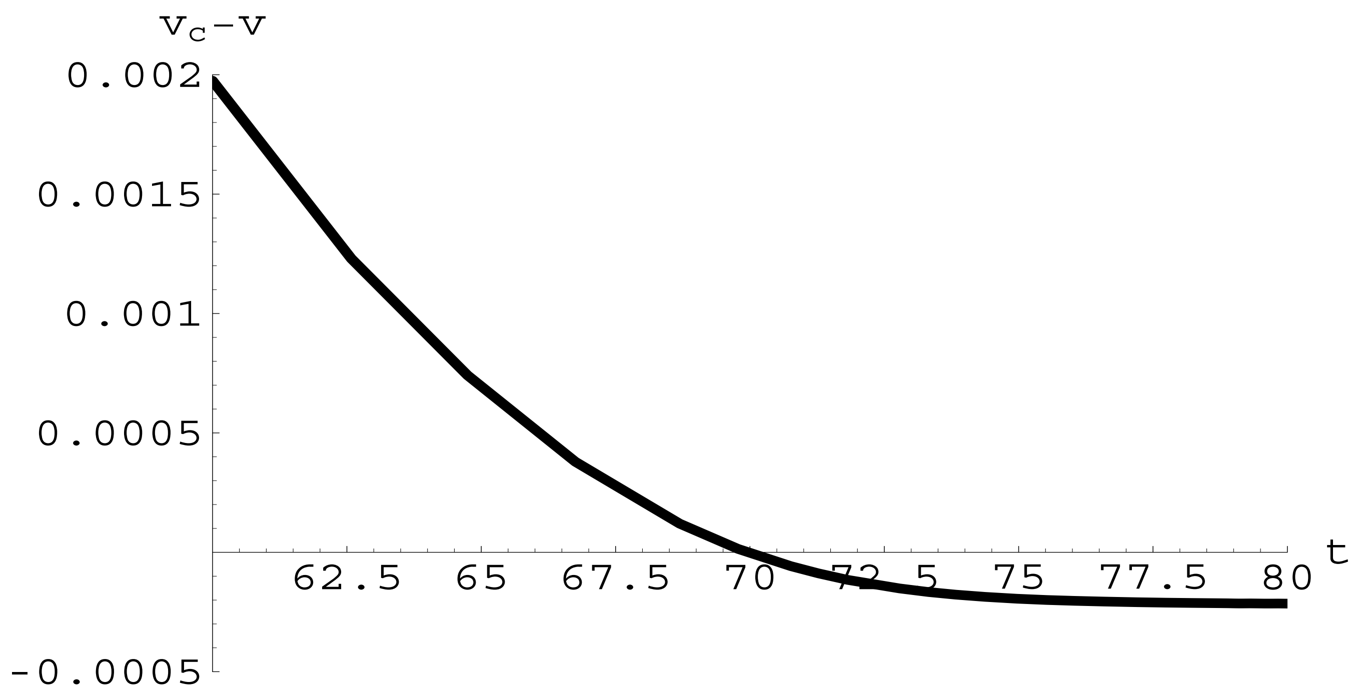
Numerical Integration Breaks Down



- The red line is the energy flux entering string from top brane.
- The blue line is the derivative of energy with respect to time.
- The two diverge at $t \approx 64$ so the solution can't be trusted after that time.

Cause of Numerical Problem

Below is a plot of $v_c - v$. Note that it becomes negative around the same time as the energy and energy flux diverge. If this is the cause of our troubles perhaps a dynamic method of changing the coordinate system during the evolution could solve our numerical problems.



Conclusion

- Initial numerical results indicate that the light quark does fall behind the heavy quark.
- More advanced numerical techniques are needed to determine if the light quark comes to rest or continues to follow in the wake of the heavy quark.
- Techniques need to increase resolution dynamically to keep numerical errors from allowing the bottom quark to exceed the local speed of light.

Meason Melting?