S-duality and new rank 1 SCFTs

by

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Argyres-Seiberg Duality (PCA & NS, 0711.0054)

$\mathfrak{g}\,[d_i]$ w/ $\boldsymbol{r} \simeq \widetilde{\mathfrak{g}}\left[\tilde{d}_i\right]$ w/ $(\widetilde{\boldsymbol{r}} \oplus \mathsf{SCFT}\,[d, \mathfrak{h}])$

- **LHS, we have a gauge group, $\mathfrak{g}$, with half-hypermultiplets in representations $\boldsymbol{r}$.**
- **RHS, we have a rank 1 superconformal fixed point with the mass dimension of the Coulomb branch vev d and flavor symmetry group $\mathfrak{h}$. We gauge a subgroup, $\tilde{\mathfrak{g}}$, of $\mathfrak{h}$ and add half-hypermultiplets charged in representations $\tilde{\boldsymbol{r}}$ of $\tilde{\mathfrak{g}}$.**

The way to think of $N = 2$ superconformal fixed points is that the dimension of the Coulomb branch vev is equivalent to the gauge group in lagrangian theories.

Criteria for Duality (PCA & JRW, 0712.2028)

$\mathfrak{g}\left[d_i\right]$ w/ $\boldsymbol{r} \simeq \widetilde{\mathfrak{g}}\left[\tilde{d}_i\right]$ w/ $\left(\widetilde{\boldsymbol{r}} \oplus \mathsf{SCFT}\left[d, \mathfrak{h}\right]\right)$

1. **The rank of the gauge group and spectrum of the dimensions of Coulomb branch vevs.**
   - $\{d_i\} = \{\tilde{d}_i\} \cup \{d\}$
2. **The flavor symmetry algebras.**
   - $f = \tilde{f} \oplus H$
3. **The contribution to the beta function from weakly gauging the flavor symmetry.**
   - $\mathsf{T}\left(\boldsymbol{r}\right) = \mathsf{T}\left(\widetilde{\boldsymbol{r}}\right) + k_{\mathfrak{h}} \cdot \mathsf{I}_{f \hookrightarrow \mathfrak{h}}$
4. **The number of marginal couplings.**
   - $2 \cdot \mathsf{T}\left(\widetilde{\mathfrak{g}}\right) = \mathsf{T}\left(\widetilde{\boldsymbol{r}}\right) + k_{\mathfrak{h}} \cdot \mathsf{I}_{\widetilde{\mathfrak{g}} \hookrightarrow \mathfrak{h}}$
5. **The contribution to the $\mathsf{U}\left(1\right)_R$ (and subsequently the c conformal anomaly) symmetry central charge.**
   - $(3/2) \cdot k_R = 24 \cdot c = 4 \cdot \left(|\mathfrak{g}| - |\widetilde{\mathfrak{g}}|\right) + \left(|\boldsymbol{r}| - |\widetilde{\boldsymbol{r}}|\right)$
6. **The contribution to the a conformal anomaly.**
   - $48 \cdot a = 10 \cdot \left(|\mathfrak{g}| - |\widetilde{\mathfrak{g}}|\right) + \left(|\boldsymbol{r}| - |\widetilde{\boldsymbol{r}}|\right)$
7. **The existence of a global $\mathbb{Z}_2$ obstruction to gauging the flavor symmetry.**

$$4 \cdot (2 \cdot a - c) = |\mathfrak{g}|$$

- **In lagrangian theories, the a and c anomalies can be computed by t' Hooft anomaly matching.**
- **When looking at $N = 2$ superconformal gauge theories we find an interesting relationship amongst a and c.**
- $4 \cdot (2a - c) = |\mathfrak{g}| = \sum_i (2d_i - 1)$**.**
- **Now looking at criteria (1), (5), and (6):** $4 \cdot (2a - c) = (2d - 1)$
- **Recent work by Shapere and Tachikawa (AS & YT, 0804.1957) provides a proof that this formula is true for a large class of theories.**

$\mathbb{Z}_2$ obstruction - an example

G_2 **w/** $8 \cdot \mathbf{7} \simeq \mathsf{SU}(2)$ **w/** $(\mathbf{2} \oplus \mathsf{SCFT}[6, \mathsf{Sp}(5)])$

- **Since the 7 of** G_2 **is a real representation the flavor symmetry group is** $\mathsf{Sp}(4)$**. If we try to gauge this** $\mathsf{Sp}(4)$ **we get a** $\mathbb{Z}_2$ **obstruction since there are** 7 **half-hypermultiplets in the 8, which is pseudoreal, of** $\mathsf{Sp}(4)$**.**

- **The embedding on the RHS is** $\mathsf{SU}(2) \oplus \mathsf{Sp}(4) \subset \mathsf{Sp}(5)$ **with** $\mathsf{I}_{\mathsf{SU}(2) \hookrightarrow \mathsf{Sp}(5)} = \mathsf{I}_{\mathsf{Sp}(4) \hookrightarrow \mathsf{Sp}(5)} = 1$**.**

- **The** $\mathsf{Sp}(5)$ **must have a** $\mathbb{Z}_2$ **obstruction to cancel the anomaly coming from the half-hypermultiplet in the 2 of** $\mathsf{SU}(2)$**.**

- **Therefore, the** $\mathsf{Sp}(4)$**, on the RHS, has a** $\mathbb{Z}_2$ **obstruction matching the LHS.**

The details of this was first worked out by Witten, An $\mathsf{SU}(2)$ **Anomaly, Phys.Lett.B117:324-328,1982.**

Examples of Duality

	$\mathfrak{g}$	$\boldsymbol{r}$	$\tilde{\mathfrak{g}}$	$\tilde{\boldsymbol{r}}$	SCFT $[d:\mathfrak{h}]$
1	$\mathsf{Sp}(3)$	$\mathbf{14}\oplus 11\cdot\mathbf{6}$	$\mathsf{Sp}(2)$		$[6:\mathsf{E}_8]$
2	$\mathsf{SU}(6)$	$\mathbf{20}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}\oplus 5\cdot\mathbf{6}\oplus 5\cdot\bar{\mathbf{6}}$	$\mathsf{SU}(5)$	$\mathbf{5}\oplus\bar{\mathbf{5}}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}$	$[6:\mathsf{E}_8]$
3	$\mathsf{SO}(12)$	$3\cdot\mathbf{32}\oplus\mathbf{32}'\oplus 4\cdot\mathbf{12}$	$\mathsf{SO}(11)$	$3\cdot\mathbf{32}$	$[6:\mathsf{E}_8]$
4	G_2	$8\cdot\mathbf{7}$	$\mathsf{SU}(2)$	$\mathbf{2}$	$[6:\mathsf{Sp}(5)]$
5	$\mathsf{SO}(7)$	$4\cdot\mathbf{8}\oplus 6\cdot\mathbf{7}$	$\mathsf{Sp}(2)$	$5\cdot\mathbf{4}$	$[6:\mathsf{Sp}(5)]$
6	$\mathsf{SU}(6)$	$\mathbf{21}\oplus\overline{\mathbf{21}}\oplus\mathbf{20}\oplus\mathbf{6}\oplus\bar{\mathbf{6}}$	$\mathsf{SU}(5)$	$\mathbf{10}\oplus\overline{\mathbf{10}}$	$[6:\mathsf{Sp}(5)]$
7	$\mathsf{Sp}(2)$	$12\cdot\mathbf{4}$	$\mathsf{SU}(2)$		$[4:\mathsf{E}_7]$
8	$\mathsf{SU}(4)$	$2\cdot\mathbf{6}\oplus 6\cdot\mathbf{4}\oplus 6\cdot\bar{\mathbf{4}}$	$\mathsf{SU}(3)$	$2\cdot\mathbf{3}\oplus 2\cdot\bar{\mathbf{3}}$	$[4:\mathsf{E}_7]$
9	$\mathsf{SO}(7)$	$6\cdot\mathbf{8}\oplus 4\cdot\mathbf{7}$	G_2	$4\cdot\mathbf{7}$	$[4:\mathsf{E}_7]$
10	$\mathsf{SO}(8)$	$6\cdot\mathbf{8}\oplus 4\cdot\mathbf{8}'\oplus 2\cdot\mathbf{8}$”	$\mathsf{SO}(7)$	$6\cdot\mathbf{8}$	$[4:\mathsf{E}_7]$
11	$\mathsf{SO}(8)$	$6\cdot\mathbf{8}\oplus 6\cdot\mathbf{8}'$	G_2		$[4:\mathsf{E}_7]\oplus[4:\mathsf{E}_7]$
12	$\mathsf{Sp}(2)$	$6\cdot\mathbf{5}$	$\mathsf{SU}(2)$		$[4:\mathsf{Sp}(3)\oplus\mathsf{SU}(2)]$
13	$\mathsf{Sp}(2)$	$4\cdot\mathbf{4}\oplus 4\cdot\mathbf{5}$	$\mathsf{SU}(2)$	$3\cdot\mathbf{2}$	$[4:\mathsf{Sp}(3)\oplus\mathsf{SU}(2)]$
14	$\mathsf{SU}(4)$	$\mathbf{10}\oplus\overline{\mathbf{10}}\oplus 2\cdot\mathbf{4}\oplus 2\cdot\bar{\mathbf{4}}$	$\mathsf{SU}(3)$	$\mathbf{3}\oplus\bar{\mathbf{3}}$	$[4:\mathsf{Sp}(3)\oplus\mathsf{SU}(2)]$
15	$\mathsf{SU}(3)$	$6\cdot\mathbf{3}\oplus 6\cdot\bar{\mathbf{3}}$	$\mathsf{SU}(2)$	$2\cdot\mathbf{2}$	$[3:\mathsf{E}_6]$
16	$\mathsf{SU}(4)$	$4\cdot\mathbf{6}\oplus 4\cdot\mathbf{4}\oplus 4\cdot\bar{\mathbf{4}}$	$\mathsf{Sp}(2)$	$6\cdot\mathbf{4}$	$[3:\mathsf{E}_6]$
17	$\mathsf{SU}(3)$	$\mathbf{3}\oplus\bar{\mathbf{3}}\oplus\mathbf{6}\oplus\bar{\mathbf{6}}$	$\mathsf{SU}(2)$	$n\cdot\mathbf{2}$	$[3:\mathfrak{h}]$

Results: New SCFT's

d	$\mathfrak{h}$	$k_{\mathfrak{h}}$	$24 \cdot c$	$48 \cdot a$	$\mathbb{Z}_2$
6	E_8	12	124	190	**no**
6	$\mathsf{Sp}(5)$	7	98	164	**yes**
4	E_7	8	76	118	**no**
4	$\mathsf{Sp}(3) \oplus \mathsf{SU}(2)$	$5 \oplus 8$	58	100	**yes⊕no**
3	E_6	6	52	82	**no**
3	$\mathfrak{h}$	$\frac{8-n}{\mathsf{I}_{\mathsf{SU}(2) \hookrightarrow \mathfrak{h}}}$	$38 - 2n$	$68 - 2n$	**?**

- **The central charges of the E_6, E_7, and E_8 flavor symmetry groups were confirmed by an F-theory calculation by Aharony and Tachikawa, 0711.4532.**

Seiberg-Witten Theory

- **The physics is encoded by:**
  - **the Seiberg-Witten curve:** $y^2 = x^3 + f\,(u, m_i)\,x + g\,(u, m_i)$
  - **the Seiberg-Witten** 1**-form:** λ_{SW}
- **From** $N = 2$ **susy,** $M^2 \geq |\mathsf{Z}|^2$
- $\mathsf{U}(1)$ **charges of a physical state are defined by the homology class of cycle,** γ**.**
- **The central charge of the state associated to** γ **is** $\mathsf{Z} = \oint_\gamma \lambda_{SW}$**.**
- λ_{SW} **satisfies** $\partial_u \lambda_{SW} = \frac{\mathrm{d}x}{y} + \partial_x\,(*)\,\mathrm{d}x$**.**
- **The singularities are at:** $\Delta = 4 \cdot f^3 - 27 \cdot g^2 = 0$**.**
  - **Physically, the singularities correspond to a breakdown of the low-energy description. This occurs when charged states become massless.**

Central Charges and Curves

Recently, Shapere and Tachikawa have used a topological twisted version of these theories to relate properties of the Seiberg-Witten curve to numerical values of the anomalies and central charges. These results allow us to get a handle on:

- **the number of neutral hypermultiplets**
- **the number of singularities of the Seiberg-Witten curve.**

In the twisted theory the measure of the path integral involves functions holomorphic in the moduli.

1. **The scaling behaviour of these functions encodes the R-anomaly of the states that are becoming massless at a singularity in moduli space.**
2. $\int [\mathrm{d}u]\,[\mathrm{d}q]\,\mathsf{A}^{\chi}\mathsf{B}^{\sigma}\mathsf{C}^{n}e^{-\mathsf{S}_{low-energy}}$
   (a) $[\mathrm{d}u]$ **&** $[\mathrm{d}q]$ **represent vector multiplets and neutral hypermultiplets massless on moduli space.**
   (b) χ **and** σ **are the Euler characteristic and the signature of the** 4**-manifold.**
   (c) $\mathsf{A}^2 = det\left[\frac{\partial u_i}{\partial a_j}\right]$
   (d) $\mathsf{B}^8 = Radical\,[\triangle]$

Results for 1D Coulomb branches

The normalization of R-charges is: $R(\sharp) = 2 \cdot D(\sharp)$**.**
The central charges are then determined to be:

1. $48 \cdot a = 12 \cdot R(\mathsf{A}) + 8 \cdot R(\mathsf{B}) + 10 \cdot \mathrm{r} + 2 \cdot \mathrm{h}$
2. $24 \cdot c = 8 \cdot R(\mathsf{B}) + 4 \cdot \mathrm{r} + 2 \cdot \mathrm{h}$
   (a) r **is the complex dimension of the Coulomb branch.**
   (b) h **is the number of massless neutral hypermultiplets on moduli space.**

In the case we are interested in (1 **dimensional Coulomb branches) the R-charges are:**

- $R(\mathsf{A}) = d - 1$
- $R(\mathsf{B}) = \frac{1}{4} \cdot Z \cdot d$
  - Z **is the number of singularities of the S-W curve.**

Notice:

- $4 \cdot (2 \cdot a - c) = 2 \cdot R(\mathsf{A}) + \mathrm{r} = 2 \cdot (d-1) + 1 = 2 \cdot d - 1$

Reproducing the relation between a **and** c **is comforting. In the case** $\mathrm{r} = 1$ **we find the following two relations:**

1. $24 \cdot c = 2 \cdot Z \cdot d + 4 + 2 \cdot \mathrm{h}$
2. $k_{\mathfrak{h}} = 2 \cdot d - \mathrm{h}$

Results, Revisited

d	$\mathfrak{h}$	Z	$2 \cdot \mathrm{h}$	**rep.**
6	E_8	10	0	**N/A**
6	$\mathsf{Sp}(5)$	7	10	$\mathbf{10(s)}$
4	E_7	9	0	**N/A**
4	$\mathsf{Sp}(3) \oplus \mathsf{SU}(2)$	6	$(6, 0)$	$\mathbf{6} \oplus \mathbf{1(s)}$
3	E_6	8	0	**N/A**
3	$\mathsf{SU}(3)$	4	6	$\mathbf{3} \oplus \bar{\mathbf{3}}\mathbf{(c)}$

- **Two candidate Seiberg-Witten curves with the** E_6 **singularity and flavor symmetry** $\mathsf{SU}(3)$ **have been constructed.**
- **The neutral hypermultiplets account for the** $\mathbb{Z}_2$ **obstruction.**

Future Directions

- **Use the information about the number of singularities along with other techniques to compute:**
  - **Seiberg-Witten curves for the new mass deformations.**
  - **Seiberg-Witten 1-forms for the new mass deformations.**
- **Attempt a classification of all mass deformations of $N = 2$ superconformal theories with a 1D Coulomb branch.**
- **Attempt, using these new techniques, to extend (complete) our classification of $N = 2$ superconformal theories with a 2D Coulomb branch.**