S-duality and new rank 1 SCFTs

by

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In collaboration with

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Argyres-Seiberg Duality (PCA & NS, 0711.0054)

\[ g [d_i] \rightarrow \tilde{g} [\tilde{d}_i] \rightarrow (\tilde{r} \oplus \text{SCFT} [d, \mathfrak{h}]) \]

- LHS, we have a gauge group, \( g \), with half-hypermultiplets in representations \( r \).
- RHS, we have a rank 1 superconformal fixed point with the mass dimension of the Coulomb branch vev \( d \) and flavor symmetry group \( \mathfrak{h} \). We gauge a subgroup, \( \tilde{g} \), of \( \mathfrak{h} \) and add half-hypermultiplets charged in representations \( \tilde{r} \) of \( \tilde{g} \).

The way to think of \( N = 2 \) superconformal fixed points is that the dimension of the Coulomb branch vev is equivalent to the gauge group in lagrangian theories.
Criteria for Duality \textit{(PCA & JRW, 0712.2028)}

\[ g[d_i] \text{ w/ } r \simeq \tilde{g}[\tilde{d}_i] \text{ w/ } (\tilde{r} \oplus \text{SCFT}[d, \tilde{h}]) \]

1. The rank of the gauge group and spectrum of the dimensions of Coulomb branch vevs.
   \[ \{d_i\} = \{	ilde{d}_i\} \cup \{d\} \]

2. The flavor symmetry algebras.
   \[ f = \tilde{f} \oplus H \]

3. The contribution to the beta function from weakly gauging the flavor symmetry.
   \[ T(r) = T(\tilde{r}) + k_{\tilde{h}} \cdot |f \rightarrow h| \]

4. The number of marginal couplings.
   \[ 2 \cdot T(\tilde{g}) = T(\tilde{r}) + k_{\tilde{h}} \cdot |\tilde{g} \rightarrow \tilde{h}| \]

5. The contribution to the \( U(1)_R \) (and subsequently the \( c \) conformal anomaly) symmetry central charge.
   \[ (3/2) \cdot k_R = 24 \cdot c = 4 \cdot (|g| - |\tilde{g}|) + (|r| - |\tilde{r}|) \]

6. The contribution to the \( a \) conformal anomaly.
   \[ 48 \cdot a = 10 \cdot (|g| - |\tilde{g}|) + (|r| - |\tilde{r}|) \]

7. The existence of a global \( \mathbb{Z}_2 \) obstruction to gauging the flavor symmetry.
\[ 4 \cdot (2 \cdot a - c) = \left| g \right| \]

- In lagrangian theories, the \( a \) and \( c \) anomalies can be computed by t’Hooft anomaly matching.
- When looking at \( N = 2 \) superconformal gauge theories we find an interesting relationship amongst \( a \) and \( c \).
- \( 4 \cdot (2a - c) = |g| = \sum_i (2d_i - 1) \).
- Now looking at criteria (1), (5), and (6): \( 4 \cdot (2a - c) = (2d - 1) \)
- Recent work by Shapere and Tachikawa (AS & YT, 0804.1957) provides a proof that this formula is true for a large class of theories.
**\(\mathbb{Z}_2\) obstruction - an example**

\(G_2 \cong SU(2) \oplus SCFT[6, Sp(5)]\) 

- Since the 7 of \(G_2\) is a real representation the flavor symmetry group is \(Sp(4)\). If we try to gauge this \(Sp(4)\) we get a \(\mathbb{Z}_2\) obstruction since there are 7 half-hypermultiplets in the 8, which is pseudoreal, of \(Sp(4)\).

- The embedding on the RHS is \(SU(2) \oplus Sp(4) \subset Sp(5)\) with \(l_{SU(2) \hookrightarrow Sp(5)} = l_{Sp(4) \hookrightarrow Sp(5)} = 1\).

- The \(Sp(5)\) must have a \(\mathbb{Z}_2\) obstruction to cancel the anomaly coming from the half-hypermultiplet in the 2 of \(SU(2)\).

- Therefore, the \(Sp(4)\), on the RHS, has a \(\mathbb{Z}_2\) obstruction matching the LHS.

The details of this was first worked out by Witten, An \(SU(2)\) Anomaly, Phys.Lett.B117:324-328, 1982.
Examples of Duality

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>Sp(3)</td>
<td>14 ⊕ 11 ∙ 6</td>
<td>Sp(2)</td>
<td>[6 : E₆]</td>
</tr>
<tr>
<td>2</td>
<td>SU(6)</td>
<td>20 ⊕ 15 ⊕ 15 ∙ 5 ∙ 6 ∙ 5 ∙ 6</td>
<td>SU(5)</td>
<td>5 ⊕ 5 ⊕ 10 ⊕ 10</td>
</tr>
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<td>3</td>
<td>SO(12)</td>
<td>3 ∙ 32 ⊕ 32' ⊕ 4 ∙ 12</td>
<td>SO(11)</td>
<td>3 ∙ 32</td>
</tr>
<tr>
<td>4</td>
<td>G₂</td>
<td>8 ∙ 7</td>
<td>SU(2)</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>SO(7)</td>
<td>4 ∙ 8 ⊕ 6 ∙ 7</td>
<td>Sp(2)</td>
<td>5 ∙ 4</td>
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<tr>
<td>6</td>
<td>SU(6)</td>
<td>21 ∙ 21 ⊕ 20 ⊕ 6 ∙ 6</td>
<td>SU(5)</td>
<td>10 ⊕ 10</td>
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<td>7</td>
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<td>12 ∙ 4</td>
<td>SU(2)</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>SU(4)</td>
<td>2 ∙ 6 ⊕ 6 ∙ 4 ⊕ 6 ∙ 4</td>
<td>SU(3)</td>
<td>2 ∙ 3 ⊕ 2 ∙ 3</td>
</tr>
<tr>
<td>9</td>
<td>SO(7)</td>
<td>6 ∙ 8 ⊕ 4 ∙ 7</td>
<td>G₂</td>
<td>4 ∙ 7</td>
</tr>
<tr>
<td>10</td>
<td>SO(8)</td>
<td>6 ∙ 8 ⊕ 4 ∙ 8' ⊕ 2 ∙ 8''</td>
<td>SO(7)</td>
<td>6 ∙ 8</td>
</tr>
<tr>
<td>11</td>
<td>SO(8)</td>
<td>6 ∙ 8 ⊕ 6 ∙ 8'</td>
<td>G₂</td>
<td>[4 : E₇] ⊕ [4 : E₇]</td>
</tr>
<tr>
<td>12</td>
<td>Sp(2)</td>
<td>6 ∙ 5</td>
<td>SU(2)</td>
<td>2</td>
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<tr>
<td>13</td>
<td>Sp(2)</td>
<td>4 ∙ 4 ⊕ 4 ∙ 5</td>
<td>SU(2)</td>
<td>3 ∙ 2</td>
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<tr>
<td>14</td>
<td>SU(4)</td>
<td>10 ⊕ 10 ⊕ 2 ∙ 4 ⊕ 2 ∙ 4</td>
<td>SU(3)</td>
<td>3 ⊕ 3</td>
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<td>SU(2)</td>
<td>2 ∙ 2</td>
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<tr>
<td>16</td>
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<td>6 ∙ 4</td>
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<td>17</td>
<td>SU(3)</td>
<td>3 ⊕ 3 ∙ 6 ⊕ 6</td>
<td>SU(2)</td>
<td>n ∙ 2</td>
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Results: New SCFT’s

<table>
<thead>
<tr>
<th>$d$</th>
<th>$\mathfrak{h}$</th>
<th>$k_{\mathfrak{h}}$</th>
<th>$24 \cdot c$</th>
<th>$48 \cdot a$</th>
<th>$\mathbb{Z}_2$</th>
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</thead>
<tbody>
<tr>
<td>6</td>
<td>$E_8$</td>
<td>12</td>
<td>124</td>
<td>190</td>
<td>no</td>
</tr>
<tr>
<td>6</td>
<td>$Sp(5)$</td>
<td>7</td>
<td>98</td>
<td>164</td>
<td>yes</td>
</tr>
<tr>
<td>4</td>
<td>$E_7$</td>
<td>8</td>
<td>76</td>
<td>118</td>
<td>no</td>
</tr>
<tr>
<td>4</td>
<td>$Sp(3) \oplus SU(2)$</td>
<td>$5 \oplus 8$</td>
<td>58</td>
<td>100</td>
<td>yes$\oplus$no</td>
</tr>
<tr>
<td>3</td>
<td>$E_6$</td>
<td>6</td>
<td>52</td>
<td>82</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>$\mathfrak{h}$</td>
<td>$\frac{8-n}{I_{SU(2)} \rightarrow \mathfrak{h}}$</td>
<td>$38 - 2n$</td>
<td>$68 - 2n$</td>
<td>?</td>
</tr>
</tbody>
</table>

- The central charges of the $E_6$, $E_7$, and $E_8$ flavor symmetry groups were confirmed by an F-theory calculation by Aharony and Tachikawa, 0711.4532.
Seiberg-Witten Theory

- The physics is encoded by:
  - the Seiberg-Witten curve: $y^2 = x^3 + f(u,m_i) x + g(u,m_i)$
  - the Seiberg-Witten 1-form: $\lambda_{SW}$

- From $N = 2$ susy, $M^2 \geq |Z|^2$

- $U(1)$ charges of a physical state are defined by the homology class of cycle, $\gamma$.

- The central charge of the state associated to $\gamma$ is $Z = \oint_\gamma \lambda_{SW}$.

- $\lambda_{SW}$ satisfies $\partial_u \lambda_{SW} = \frac{dx}{y} + \partial_x (\ast) dx$.

- The singularities are at: $\Delta = 4 \cdot f^3 - 27 \cdot g^2 = 0$.
  - Physically, the singularities correspond to a breakdown of the low-energy description. This occurs when charged states become massless.
Central Charges and Curves

Recently, Shapere and Tachikawa have used a topological twisted version of these theories to relate properties of the Seiberg-Witten curve to numerical values of the anomalies and central charges. These results allow us to get a handle on:

- the number of neutral hypermultiplets
- the number of singularities of the Seiberg-Witten curve.

In the twisted theory the measure of the path integral involves functions holomorphic in the moduli.

1. The scaling behaviour of these functions encodes the R-anomaly of the states that are becoming massless at a singularity in moduli space.

2. \( \int [du] [dq] A^\chi B^\sigma C^n e^{-S_{\text{low-energy}}} \)

   (a) \([du]\) \& \([dq]\) represent vector multiplets and neutral hypermultiplets massless on moduli space.

   (b) \(\chi\) and \(\sigma\) are the Euler characteristic and the signature of the 4-manifold.

   (c) \(A^2 = \det \left[ \frac{\partial u_i}{\partial a_j} \right] \)

   (d) \(B^8 = \text{Radical} [\Delta] \)
Results for 1D Coulomb branches

The normalization of R-charges is: \( R(\#) = 2 \cdot D(\#) \).

The central charges are then determined to be:

1. \( 48 \cdot a = 12 \cdot R(A) + 8 \cdot R(B) + 10 \cdot r + 2 \cdot h \)
2. \( 24 \cdot c = 8 \cdot R(B) + 4 \cdot r + 2 \cdot h \)

(a) \( r \) is the complex dimension of the Coulomb branch.
(b) \( h \) is the number of massless neutral hypermultiplets on moduli space.

In the case we are interested in (1 dimensional Coulomb branches) the R-charges are:

- \( R(A) = d - 1 \)
- \( R(B) = \frac{1}{4} \cdot Z \cdot d \)

- \( Z \) is the number of singularities of the S-W curve.

Notice:

- \( 4 \cdot (2 \cdot a - c) = 2 \cdot R(A) + r = 2 \cdot (d - 1) + 1 = 2 \cdot d - 1 \)

Reproducing the relation between \( a \) and \( c \) is comforting. In the case \( r = 1 \) we find the following two relations:

1. \( 24 \cdot c = 2 \cdot Z \cdot d + 4 + 2 \cdot h \)
2. \( k_\emptyset = 2 \cdot d - h \)
Two candidate Seiberg-Witten curves with the $E_6$ singularity and flavor symmetry $SU(3)$ have been constructed.

The neutral hypermultiplets account for the $\mathbb{Z}_2$ obstruction.
Future Directions

• Use the information about the number of singularities along with other techniques to compute:
  
  - Seiberg-Witten curves for the new mass deformations.
  - Seiberg-Witten 1-forms for the new mass deformations.

• Attempt a classification of all mass deformations of $N = 2$ superconformal theories with a 1D Coulomb branch.

• Attempt, using these new techniques, to extend (complete) our classification of $N = 2$ superconformal theories with a 2D Coulomb branch.