The Cube Paradox

Consider a cube, oriented as so:

and table, oriented parallel to AB

Now let the cube move with velocity \(-v\hat{y}\)

According to relativity, it now looks like:

It is clear that point A hits the table before point B. If \(h_0 = CD\) at \(t = 0\), then \(t_A = \frac{h_0 - L}{v}\) is when A hits the table.

and \(t_B = \frac{h_0 - \frac{1}{2}v^2}{v}\) is when B hits.

so \(t_A - t_B = \frac{L}{v} (\frac{1}{v^2} - 1) < 0\)
Now, go to the frame where the table is moving. The table appears tilted.

\[ y' = -\frac{x'}{\beta} + v t' \]

In this case, B hits before A!

\[ t'_B = \frac{h_0 - L}{v} \quad t'_A = \frac{h_0 - \frac{L}{V}}{v} \]

\[ \therefore t'_A - t'_B = \frac{L}{V} (1 - \frac{1}{\gamma}) > 0 \]

How can we resolve this paradox? The key is that the two events (A striking table or B striking table) are not causally connected; i.e., light cannot move fast enough between B and A to tell A what happened to B or vice versa. Thus, for example, the speed at which A hits the table is guaranteed to be the same in either frame.