Invariant Mass

Suppose we have a number of particles with energies \( E_i \) and momenta \( \vec{p}_i \) in a particular reference frame. The invariant mass is defined via

\[
m^2_{\text{inv}} c^4 = \left( \sum_i E_i \right)^2 - \left( \sum_i p_{ix} c \right)^2 - \left( \sum_i p_{iy} c \right)^2 - \left( \sum_i p_{iz} c \right)^2
\]

Now we go to a different reference frame moving with respect to the first with velocity \( v \hat{x} \). The Lorentz transformations give in the new frame \( p'_{iy} = p_{iy} \) and \( p'_{iz} = p_{iz} \) so the y and z parts of the invariant mass formula are the same in both frames. The energy and \( p_x \) terms become in the new frame

\[
\left( \sum_i E_i \right)^2 - \left( \sum_i p_{ix} c \right)^2 = \left( \sum_i \gamma (E'_i - vp'_{ix}) \right)^2 - \left( \sum_i \gamma (cp'_{ix} - \frac{v}{c} E'_i) \right)^2
\]

\[
= \gamma^2 \left( \sum_i E'_i \right)^2 + v^2 \gamma^2 \left( \sum_i p'_{ix} c \right)^2 - 2\gamma^2 v \sum_{ij} E'_i p'_j
\]

\[
- \gamma^2 \left( \sum_i p'_{ix} c \right)^2 - v^2 \gamma^2 \left( \sum_i E'_i \right)^2 + 2\gamma^2 v \sum_{ij} E'_i p'_j
\]

\[
= \left( \gamma^2 (1 - v^2/c^2) \right) \left( \sum_i E'_i \right)^2 - \left( \sum_i p'_{ix} c \right)^2
\]

We therefore conclude that the invariant mass of a system of particles is independent of the reference frame, as suggested by the name "invariant" mass.