Name__________________________________

Physics 248  
Spring 2002  
Final Exam  
May 13, 2002

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Total /100
1. (4 pts) Consider the following arrangement of slits and a screen set up in the path of a plane wave.

What is the minimum value of $H$ such that the wave arriving at point $P$ from the upper slit is exactly out of phase with that arriving from the lower one?

$$H = \text{_____________ wavelengths}$$

2. (6 pts) The Earth is 1 AU from the Sun, takes 1 year to make an orbit, and has a temperature of roughly 300 K. Mars is about 1.52 AU from the Sun. What is the period of its orbit in Earth years and its approximate temperature (assuming it absorbs and emits like a black body)?

The period is _______ Earth years, and the temperature about ________ K
An electron of mass 9.1 \times 10^{-31} \text{ kg} is in a circular orbit around a much more massive (and therefore effectively stationary) carbon nucleus ("H, He, Li, Be, B, C, . . ."). The radius of the orbit is 5.0 \times 10^{-8} \text{ meters}. (This distance is large enough that we can ignore quantum mechanics and treat the motion with Newtonian mechanics. Also, the motion is taking place in a vacuum tank shielded from external electric and magnetic fields.) (k = 9 \times 10^9 \text{ N m}^2/\text{C}^2, e = 1.6 \times 10^{-19} \text{ C}, h = 6.63 \times 10^{-34} \text{ J s}, and c = 3 \times 10^8 \text{ m/s}.)

a. (3 pts) The force of attraction between the two has magnitude \__________\text{N}.

b. (4 pts) The speed of the electron in this orbit is \__________\text{m/s}. Does this justify ignoring relativity? ______

c. (4 pts) Assuming \Delta p \approx p, what is the minimum uncertainty in the location of an electron with this speed, divided by the radius of the orbit? \Delta x/r = \__________ Does this help justify our neglect of quantum mechanics? _____

d. (4 pts) The orbital angular momentum quantum number of the electron in this orbit is approximately \ell \approx \___________. Does this help justify our Newtonian approximation? _____

e. (1 pt) Does this have anything to do with Bohr's Correspondence Principle? ______
4. Two very long concentric nested metal cylinders are arranged as shown in the figure. The inner cylinder has a total net charge per unit length of $+\lambda$, while the outer one is grounded.

Circle the little Roman numeral in front of the best choices below and fill in blanks if chosen.

a. (2 pts) The charge on the outer cylinder is
   i. zero
   ii. spread over its inner and outer surfaces with $-\lambda$ on the inner and $+\lambda$ on the outer (per unit length)
   iii. spread over its inner and outer surfaces with $-\lambda$ on the inner and zero on the outer (per unit length)
   iv. dependent on where the grounding wire is attached to the outer cylinder.

b. (2 pts) The electric field between the two cylinders ($R_2 > r > R_1$) is
   i. zero
   ii. the same as that of a point charge located at the center, with a charge of _________.
   iii. the same as that of a line charge located at the center, with a charge per unit length of _________.
   iv. cannot be calculated from the given information.

c. (2 pts) The electric field inside the inner cylinder ($r < R_1$) is
   i. zero
   ii. the same as that of a point charge located at the center, with a charge of _________.
   iii. the same as that of a line charge located at the center, with a charge per unit length of _________.
   iv. cannot be calculated from the given information.

d. (8 pts) Showing your calculation, starting from the electric field between the cylinders, find the electric potential (voltage) difference between the cylinders, and the capacitance per unit length of these cylinders.

$$V(R_1)-V(R_2) = \text{____________________}$$

$$C/\text{length} = \text{____________________}$$
The wavefunction for a particle of mass \( m \) is

\[
\psi(x) = A x e^{-x/a} \quad \text{for } x \geq 0, \quad \text{and } \psi(x) = 0 \quad \text{for } x \leq 0
\]

where \( a \) is a constant length and \( A \) a constant coefficient.

A. [2 pts.] Sketch \( \psi(x) \).

B. [6 pts.] Normalize this wavefunction, finding the value of \( A \) such that the probability of finding the particle between \( x = 0 \) and \( \infty \) is one.  (See integral table below.)

Integral Table: CRC Standard Math Tables, 26th edition, p. 342, #661:

\[
\int_{0}^{\infty} x^n e^{-bx} \, dx = \frac{n!}{b^{n+1}} \quad (b > 0, \text{n positive integer})
\]
In the circuit shown above, assume you know R, L, C, and ω, and that the charge on the capacitor is known to be

\[ Q(t) = Q_0 \sin(\omega t) \]

where \( Q_0 \) is also known.

A. (10 pts) Your job is to find each of the quantities requested below, putting your reasons on the lines indicated. Your answers may contain \( R, L, C, Q_0, \omega, \cos(\omega t), \) and \( \sin(\omega t) \).

\[ Q(t) = \quad Q_0 \sin(\omega t) \quad \text{[Example]} \quad \text{(put reasons here ↓)} \]

\[ I(t) = \quad \text{_____________________________} \quad \text{_____________________________} \]

\[ V_R(t) = \quad \text{_____________________________} \quad \text{_____________________________} \]

\[ V_L(t) = \quad \text{_____________________________} \quad \text{_____________________________} \]

\[ V_C(t) = \quad \text{_____________________________} \quad \text{_____________________________} \]

\[ V_g(t) = \quad \text{_____________________________} \quad \text{_____________________________} \]

B. (6 pts) What are the maximum values of each of the following quantities? Your answers may contain \( R, L, C, Q_0, \) and \( \omega \).

\[ (I)_{\text{max}} = \quad \text{__________} \quad (V_R)_{\text{max}} = \quad \text{__________} \]

\[ (V_L)_{\text{max}} = \quad \text{__________} \quad (V_C)_{\text{max}} = \quad \text{__________} \]

\[ (V_g)_{\text{max}} = \quad \text{_____________________________} \]

C. (2 pts) What is

\[ Z(\omega) = (V_g)_{\text{max}}/(I)_{\text{max}} = \quad \text{_____________________________} \]

D. (2 pts) Finally, what is the ratio of \( (V_L)_{\text{max}}/(V_g)_{\text{max}} \) if \( \omega^2 = 1/(LC) \) and \( R = 10^{-3} \sqrt{L/C} \)?

\[ (V_L)_{\text{max}}/(V_g)_{\text{max}} = \quad \text{_____________________________} \]
7. In the questions below, "indicate" means to draw it on the figure.
   A. (2pts) Sketch the magnetic field lines in the vicinity of the bar magnet shown in Figure A. Don't forget to indicate their directions.

   \[ \text{Fig. A} \]

   B. (8 pts) Indicate the direction of the magnetic force on the positively charged particle shown moving through the magnetic field in Figure B. If \( B = 0.3 \, \text{T} \), \( v = 2 \, \text{m/s} \), \( Q = +1.6 \times 10^{-19} \, \text{C} \), and \( m = 9.1 \times 10^{-31} \, \text{kg} \), find the magnitude of this force, the radius of curvature of the path, the time to make one orbit, and give this particle its proper physics name (hint: it's not an electron).

   \[ F = \quad \text{N} \]
   \[ r = \quad \text{meters} \]
   \[ T(\text{orbit}) = \quad \text{seconds} \]
   Particle Name = ________________

8. A flat coil of wire with average radius \( r = 0.1 \, \text{m} \) and 250 turns carries a current of \( I = 50 \, \text{Amps} \) in the direction shown.

   \[ \text{Fig. B} \]

   A. (2pts) Sketch the magnetic field produced by the loop.
   B. (3pts) What is the magnitude of the magnetic field at point \( P \), a distance 0.1 m above the center of the coil?

   \[ B = \quad \text{Teslas} \]

   C. (3pts) What is the total magnetic moment of this 250 turn coil?

   \[ \mu = \quad \text{A m}^2 \]
9. The fine structure splitting of the 2P_{3/2} and 2P_{1/2} levels in hydrogen is 4.5 x 10^{-5} eV. From this, you are to estimate the effective magnetic field \( B \), in parts A-D below, that the 2p electron experiences. Assume that the effective field \( B \) is along the \( z \)-direction.

A. (2 pts) What is the energy shift \( U \) of a magnetic moment \( \mu \) in a magnetic field \( B \)? (Remember, these are vectors.)

B. (2 pts) What is the energy difference between spin-up and spin-down electrons, in terms of \( \mu_z \) and \( B \)?

C. (2 pts) What is the magnetic moment of the electron in terms of the Bohr magneton \( \mu_B \)?

D. (2 pts) Use the above information to solve for the magnitude of the effective field \( B \) that the electron experiences.
10. (6 pts.) You are Harry Potter's side-kick in his next big movie. The two of you enter the east door of a square room, and you want to leave through a locked door on the west side of the room. In order to unlock the door, you must pass a logic test devised by Professor McGonagall.

Coming out of a hole on the south side of the room is a beam of electrons, all of which are in the spin-up eigenstate. The electrons strike the north wall of the room, which is sensitive to the spin eigenstate of the electrons. If even one electron strikes the north wall in a spin-down eigenstate, the west door will unlock.

On the floor in the middle of the room are two pairs of Stern-Gerlach magnets that you and Harry can each pick up. How do you and Harry hold the two pairs of S-G magnets so that some of the electrons hit the north wall in a spin-down eigenstate, unlocking the door, and allowing Harry (who knows no physics) to steal all the credit and save the world from a terrible fate? Keep in mind, you are asked to put at least one electron in a spin-down eigenstate, not just for that electron to have a finite probability of being spin-down.

It may be convenient to call west the +x direction, south the +y direction, and up the +z direction, and it wouldn’t be a bad idea to explain why your answer is correct.

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1 This is not quite as simple as it sounds, but for now we take it as given.