September 25, 2002 Physics 201

EXAM 1

Print your name and section <u>**clearly**</u> **on all** <u>**five**</u> **pages.** (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

SOLUTION KEY

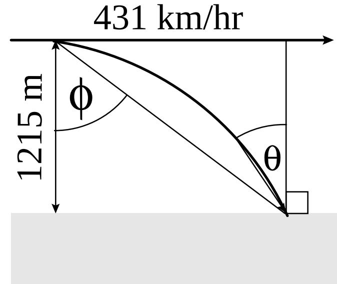
TOTAL: __________

Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80 \text{ m/s}^2$

For $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

PROBLEM 1



A plane carrying a rescue capsule is flying at a constant height of 1215 m at a speed of 431 km/hr toward a point directly over a person struggling in the water. (Ignore air resistance.)

a. How many seconds before the plane passes over the person must the rescue capsule be released *(5 pts.)*

$$y - y_0 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \Rightarrow -1215\text{m} = 0 - \frac{1}{2}(9.8\,\text{m/s}^2)t^2 \Rightarrow t = \sqrt{\frac{2(1215\text{m})}{9.8\,\text{m/s}^2}} =$$

15.7 s

b. What is *horizontal* distance (in kilometers) between the plane and the person when the capsule is released? *(5 pts.)*

$x\text{-}x_0 = (v_0\cos\theta_0)t = (431\text{km/hr})(\cos 0°)(15.75\text{s})(1\text{hr}/3600\text{s})=$

1.89 km

c. At what line of sight angle, ϕ (in degrees), towards the person should the pilot be at when he releases the capsule? *(5 pts)*

$x_0 = 0 \Rightarrow x = 1885 \text{ m} \Rightarrow \phi = \tan^{-1}(x/h) = \tan^{-1}(1885 \text{ m}/1215 \text{ m}) =$

57.2°

d. With what speed (in m/s) does the capsule hit the water? *(5 pts.)*

$v_x = v_0 = (431 \text{ km/hr})(1000 \text{ m/km})(1\text{hr}/3600 \text{ s}) = 119.7 \text{ m/s}$
$v_y = v_0\sin\theta_0 + (1/2)gt^2 = 0 + (9.8\text{m/s}^2)(15.75\text{s}) = 154.4 \text{ m/s}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(119.7\,\text{m/s})^2 + (154.4\,\text{m/s})^2} =$$

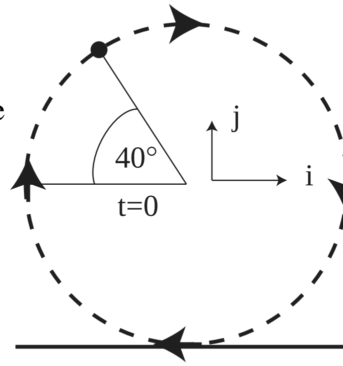
195m/s

e. At what angle, θ (in degrees), with respect to the vertical, does the capsule hit the water? *(5 pts.)*

$$\theta = \tan^{-1}\left(\frac{v_x}{v_y}\right) = \tan^{-1}\left(\frac{119.7\,\text{m/s}}{154.4\,\text{m/s}}\right) =$$

37.8°

PROBLEM 2



A ball connected to a string of length 1.2 m and negligible mass is swinging clockwise in a vertical circular path a constant speed yielding 0.93 revolutions per second. The ball just brushes the ground at the bottom of the circle. At time t = 0, the string breaks and the ball flies away when the string is forming an angle $\theta = 40°$ with the horizontal as shown in the figure.

a. Find the magnitude of the acceleration of the ball for t < 0. *(5 pts.)*

$v = (2\pi r)(0.93 \text{ rev/s}) = 7.0 \text{ m/s}$, $a = v^2/r = (7.0\text{m/s})^2/(1.2\text{m}) =$

41 m/s²

b. Find the height of the ball above the ground at t = 0. *(5 pts.)*

$h = r + r\sin\theta = 1.2 \text{ m} + (1.2 \text{ m})\sin(40°) =$

2.0 m

c. Express the velocity of the ball at t = 0 in terms of i and j components ($\mathbf{v} = a\mathbf{i} + b\mathbf{j}$). *(5 pts.)*

$\mathbf{v} = v\sin\theta\mathbf{i} + v\cos\theta\mathbf{j} = (7.0\text{m/s})\sin(40°)\mathbf{i} + (7.0\text{m/s})\cos(40°)\mathbf{j} =$

(4.5**i**+5.4**j**)m/s

d. Find the length of time after the string breaks after which the ball strikes the ground. *(5 pts.)*

$$-h = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - v_{0y}t - h = 0 \Rightarrow$$

$$t = \frac{v_{0y} \pm \sqrt{v_{0y}^2 + 2gh}}{g} = \frac{5.4\,\text{m/s} \pm \sqrt{(5.4\,\text{m/s})^2 + 2(9.8\,\text{m/s}^2)(7.0\text{m})}}{9.8\,\text{m/s}^2} =$$

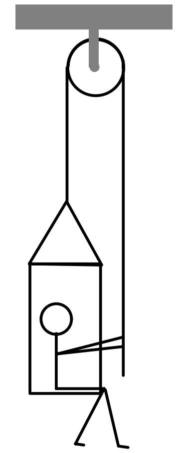
1.4 s

e. Find the horizontal distance the ball travels after the string breaks. *(5 pts.)*

$x - x_0 = v_{0x}t = (v\sin\theta)t = (7.0\text{m/s})\sin(40°)(1.4\text{s}) =$

6.3 m

PROBLEM 3



A woman sits in a chair that dangles from a massless rope, which runs over a massless frictionless pulley and back down into the woman's hand. The combined mass of the chair and woman is 95.0 kg.

a. With what force must the woman pull on the rope for her to rise at a constant speed *(5 pts)*?

$2T - mg = ma = 0 \Rightarrow T = mg/2 = (95.0\ kg)(9.8 m/s^2)/2 = 465.5\ N$

466 N

b. What is the downward force on the pulley in part a) *(5 pts)*?

$F_p = 2T = 2(465.5\ N) =$

931 N

c. With what force must the woman pull the rope to rise with an acceleration of 1.30 m/s^2 (5 pts)?

$2T - mg = ma = 0 \Rightarrow T = m(g+a)/2 = (95.0\ kg)(9.8 ms^{-2} + 1.30\ ms^{-2})/2 = 527.3\ N$

527 N

d. If the rope is now held by a man on the ground, with what force must he pull on the rope to cause the woman to rise the same as in part c) *(5 pts)*?

$T - mg = ma = 0 \Rightarrow T = m(g+a) = (95.0\ kg)(9.8 ms^{-2} + 1.30\ ms^{-2}) = 1054.5\ N$

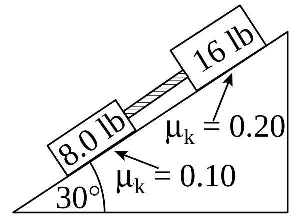
1050 N

e. What is the downward force on the pulley in part d) *(5 pts)* ?

$F_p = 2T = 2(1054.5\ N) = 2109\ N$

2110 N

PROBLEM 4



A 8.0 lb and a 16 lb block are connected by a massless rope and slide down a 30° inclined plane (with the 8.0 lb block lower down the plane). The coefficient of kinetic friction between the 8.0 lb block and the plane is 0.10 and between the 16 lb block and the plane is 0.20. (g = 32 ft/s^2).

a. If the blocks were not connected, find the acceleration (in ft/s^2) of the 8.0 lb block. *(5 pts.)*

$a_8 = g\sin\theta - \mu_{K8} g\cos\theta = (32\text{ ft/s}^2)[(\sin 30°) - (0.10)(\cos 30°) =$

13 ft/s^2

b. If the blocks were not connected, find the acceleration (in ft/s^2) of the 16 lb block. *(5 pts.)*

$a_{16} = g\sin\theta - \mu_{K16} g\cos\theta = (32\text{ ft/s}^2)[(\sin 30°) - (0.20)(\cos 30°) =$

10 ft/s^2

c. Use the results of parts a) and b) to indicate whether the rope when connected is loose or taut and explain why. *(5 pts.)*

Since $a_8 > a_{16}$

taut

d. Find the acceleration (in ft/s^2) of the two blocks when they are connected. *(5 pts.)*

$m_8 a + m_{16} a = (m_8 g\sin\theta - \mu_{K8} m_8 g\cos\theta - T) + (m_{16} g\sin\theta - \mu_{K16} m_{16} g\cos\theta + T) =$
$(m_8 + m_{16}) g\sin\theta - (\mu_{K8} m_8 + \mu_{K16} m_{16}) g\cos\theta \Rightarrow a = g\sin\theta - g\cos\theta(\mu_{K8} m_8 + \mu_{K16} m_{16})/(m_8 + m_{16})$

$$32\frac{\text{ft}}{\text{s}^2}(\sin 30°) - 32\frac{\text{ft}}{\text{s}^2}(\cos 30°)\left(\frac{(0.10)(8.0\text{ lb}) - (0.20)(16\text{ lb})}{8.0\text{ lb} + 16.0\text{ lb}}\right) = 15\frac{\text{ft}}{\text{s}^2} - 4.6\frac{\text{ft}}{\text{s}^2} = 11.4\frac{\text{ft}}{\text{s}^2}$$

11 ft/s^2

e. Find the tension in the rope (in lbs) when it is connected *(5 pts.)*

$T = m_8 g\sin\theta - \mu_{K8} g\cos\theta - m_8 a =$
$(8.0\text{ lb})(\sin 30°) - 0.10(8.0\text{ lb})(\cos 30°) - (8.0\text{ lb}/32\text{ft/s}^2)(11.4\text{ft/s}^2) =$

0.46 lb