November 18, 2002 Physics 201

EXAM 3

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

SOLUTION KEY

TOTAL: __________

Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80\ \text{m/s}^2$

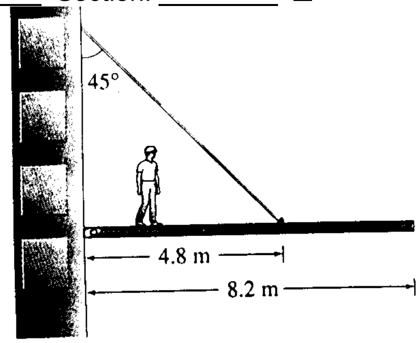
Gravitational constant: $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

$\rho(\text{water}) = 1.00 \times 10^3\ \text{kg/m}^3 = 1.00\ \text{g/cm}^3$

$1\ \text{Liter} = 10^{-3}\ \text{m}^3$

PROBLEM 1

A 580.0 kg uniform horizontal beam 8.2 m long is attached by a frictionless pivot to a wall. A steel cable (Young's modulus = 2.0 x 10^{11} N/m^2) of cross sectional area 1.6 cm^2 makes an angle of 45° with the wall, supports the beam at point 4.8 m from the wall. The cable will stretch elastically until it breaks suddenly when tension exceeds 8000.0 N. At the start, no one is on the beam.



a. What is tension in the cable? *(5 pts.)*

Let d = distance on beam to cable support, L = beam length, θ = cable angle, T = cable tension, M= beam mass:

$\Sigma\tau = 0 = -(T\sin\theta)d + Mg(1/2)L \Rightarrow T = (1/2)MgL/d\sin\theta =$

$$\frac{(1/2)(580.0\text{kg})(9.80\,\text{m/s}^2)(8.2\text{m})}{(4.8\text{m})(\sin(45°))} = 6.87\times10^3\text{N}$$

$6.9\text{x}10^3$N

b. What is the change in length of the cable due to tension? *(5 pts.)*

$$Y = \frac{F/A}{\Delta L/L_0} \Rightarrow \Delta L = \frac{F\cdot L_0}{Y\cdot A} = \frac{(6.87\times10^3\text{N})(4.8\text{m}/\sin 45°)}{(2.0\times10^{11}\,\text{N/m}^2)(1.6\times10^{-4}\,\text{m}^2)} = 1.46\times10^{-3}\text{m}$$

1.5mm

c. What is the horizontal component of the force on the pivot? *(5 pts.)*

$\Sigma F_x = F_H - T\cos\theta = 0 \Rightarrow F_H = T\cos\theta = (6.87\text{x}10^3)\cos(45°) = 4.85\text{x}10^3$ N

$4.9\text{x}10^3$ N

d. What is the vertical component of the force on the pivot? *(5 pts.)*

$\Sigma F_y = F_V + T\sin\theta - Mg = 0 \Rightarrow F_V = Mg - T\sin\theta =$
$(580.0 \text{ kg})(9.8 \text{ m/s}^2) - (6.87\text{x}10^3)\sin(45°) = 826$ N

$8.3\text{x}10^2$ N

e. How far along the beam can a 95 kg man walk before the cable breaks? *(5 pts.)*

Let m = mass of the man:

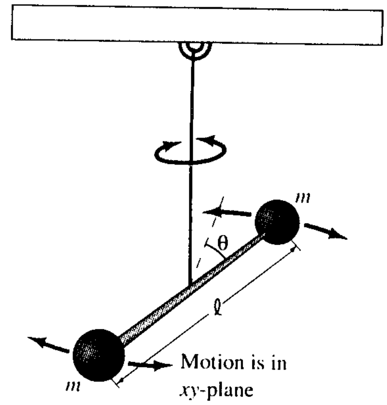
$\Sigma\tau = 0 = mgx - (T\sin\theta)d + Mg(1/2)L \Rightarrow$

$$x_{max} = \frac{(T_{max}\sin\theta)d - \frac{1}{2}MgL}{mg} = \frac{(8.00\times10^3\text{N})\sin(45°)(4.8\text{m}) - \frac{1}{2}(580\text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(8.2\text{m})}{(95\text{kg})(9.8\,\text{m/s}^2)} = 4.13\text{m}$$

4.1m

PROBLEM 2

A torsion pendulum with torsion constant $\kappa = 1.0 \times 10^4$ g·cm^2/s^2 consists of a dumbbell composed of two equal masses m = 50.0 g separated by a massless rod of length l = 20.0 cm suspended from its center by a wire that resists being twisted by an angle θ.



a. What is the period of oscillation? *(5 pts.)*

$$T = 2\pi\sqrt{\frac{I}{\kappa}} = 2\pi\sqrt{\frac{2m(\ell/2)^2}{\kappa}} = 2\pi\sqrt{\frac{m\ell^2}{2\kappa}} =$$

$$2\pi\sqrt{\frac{(50.0\text{g})(20\text{cm})^2}{2(1.0\times10^4\ \text{gcm}^2/\text{s}^2)}} = 6.28\text{s}$$

6.3 s

b. The wire is twisted θ = 0.050 rad and released. What is the total energy? *(5 pts.)*

$E = (1/2)\kappa\theta^2 = (1/2)(10^4\ \text{gcm}^2/\text{s}^2)(10^{-3}\ \text{kg/g})(10^{-2}\ \text{m/cm})^2(.050\ \text{rad})^2 = 1.25 \times 10^{-6}$ J

1.3×10^{-6} J

c. What is the maximum linear velocity of either mass after the release in part b? *(5 pts.)*

$$E = \tfrac{1}{2}I\omega_{max}^2 \Rightarrow \omega_{max}^2 = \sqrt{\frac{2E}{I}} = \sqrt{\frac{2E}{2M\left(\frac{1}{2}\ell\right)^2}} = \sqrt{\frac{4E}{M\ell^2}} =$$

$$\sqrt{\frac{4(1.25\times10^{-6}\ \text{J})}{(0.050\text{kg})(0.20\text{m})^2}} = 0.050\ \text{rad/s} \Rightarrow v = \omega r = (0.050\ \text{rad/s})(0.10\text{m}) = 5.0\times10^{-3}\ \text{m/s}$$

5.0×10^{-3} m/s

d. Find the total system maximum angular momentum after the release in part b. *(5 pts.)*

$L = 2m\omega r^2 = 2(0.050\ \text{kg})(0.050\ \text{rad/s})(0.10\ \text{m})^2 = 5.0 \times 10^{-5}$ kgm/s

5.0×10^{-5} kgm/s

e. Find the maximum angular acceleration of either mass after the release in part b. *(5 pts.)*

$$\alpha_{max} = -\frac{\kappa}{I}\theta_{max} = \frac{(1.00\times10^4\ \text{gcm}^2/\text{s}^2)(0.050\text{rad})}{2(50.0\text{g})(10.0\text{cm}^2)} = 0.050\ \text{rad/s}^2$$

0.050 rad/s^2

PROBLEM 3

A satellite of mass 500.0 kg is in circular orbit 1000.0 km above the surface of the earth. (M_E = 6.0 x 10^{24} kg, r_E = 6370 km).

a. What is the orbital (linear) speed of the satellite? *(5 pts.)*

$$\frac{GMm}{r_0^2} = \frac{mv_0^2}{r_0} \Rightarrow v_0 = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{\left(6.67\times10^{-11}\,Nm^2/kg^2\right)\left(6.0\times10^{24}\,kg\right)}{7.37\times10^{6}m}} = 7.37\times10^{3}\,m/s$$

$7.4x10^3$ m/s

b. What is the angular momentum of the satellite? *(5 pts.)*

$L = mv_0r_0$ = (500.0 kg)(7.37x10^3 m/s)(7.37x10^6 m) = 2.72x10^{13} kgm^2/s

$2.7x10^{13}$ kgm^2/s

c. What is the total energy of the satellite? *(5 pts.)*

$$E = K + U = \tfrac{1}{2}mv^2 - \frac{GMm}{r} =$$

$$\tfrac{1}{2}\left(500.0kg\right)\left(7.37\times10^{3}\,m/s\right)^2 - \frac{\left(6.67\times10^{-11}Nm^2kg^{-2}\right)\left(6.0\times10^{24}\,kg\right)\left(500.0\,kg\right)}{7.37\times10^{6}m} = -1.36\times10^{10}\,J$$

$-1.4x10^{10}$ J

d. What speed would the satellite need at this orbit to escape from the earth? *(5 pts.)*

$$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2\left(6.67\times10^{-11}Nm^2kg^{-2}\right)\left(6.0\times10^{24}\,kg\right)}{7.37\times10^{6}m}} = 10{,}421m/s$$

$1.0x10^4$ m/s

e. A rocket engine on the satellite fires directly towards the center of the earth until the satellite is 1500.0 km above the earth's surface. Find the new orbital (linear) speed of the satellite at the instant when the engine shuts off. *(5 pts.)*

Force = radial ⇒ No Torque ⇒ Angular Momentum is conserved

$$mv_0r_0 = mv_1r_1 \Rightarrow v_1 = v_0\frac{r_0}{r_1} = \frac{\left(7.37\times10^{3}\,m/s\right)\left(7.37\times10^{6}m\right)}{\left(7.87\times10^{6}m\right)} = 6.90\times10^{3}\,m/s$$

$6.9x10^3$ m/s

PROBLEM 4

A small garden fountain shoots a vertical jet of water at 0.10 liters/sec to a height of 0.50 m.
The density of water is 1.00 x 10^3 kg/m^3.

a. What is the speed of the water when it emerges from the fountain outlet? *(5 pts.)*

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 \Rightarrow p_{atm} + \tfrac{1}{2}\rho v_1^2 + 0 = p_{atm} + 0 + \rho g h_2 \Rightarrow$$
$$v_1^2 = 2gh_2 = 2\left(9.8\,\mathrm{m/s^2}\right)\left(0.50\mathrm{m}\right) = 3.1\mathrm{m/s}$$

3.1 m/s

b. What is the radius of the outlet out of which the water passes? *(5 pts.)*

$$\frac{dV}{dt} = v_1 A_1 \Rightarrow \left(0.10\,\ell/\mathrm{s}\right)\left(1.0\times10^{-3}\,\mathrm{m^3}/\ell\right) = \left(3.1\mathrm{m/s}\right)\left(\pi R_1^2\right) \Rightarrow R_1 = \sqrt{\frac{1.0\times10^{-4}\,\mathrm{m^3/s}}{\pi\cdot\left(3.1\mathrm{m/s}\right)}} = 3.2\times10^{-3}\mathrm{m}$$

3.2mm

c. What is the gauge pressure in Pascals just below the water outlet? *(5 pts.)*

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 \Rightarrow p_{atm} + 0 + \left(1.00\times10^3\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(0.50\mathrm{m}\right) = p_2 + 0 + 0 \Rightarrow$$
$$p_2 - p_{atm} = 4.9\times10^3\mathrm{Pa}$$

$4.9\mathrm{x}10^3$ Pa

d. At a height of 0.25 m, what is the speed of the water jet ? *(5 pts.)*

$$p_3 + \tfrac{1}{2}\rho v_3^2 + \rho g h_3 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 \Rightarrow p_{atm} + \tfrac{1}{2}\rho v_3^2 + \rho g h_3 = p_{atm} + 0 + \rho g h_2 \Rightarrow$$
$$v_3^2 = 2g\left(h_2 - h_3\right) = 2\left(9.8\,\mathrm{m/s^2}\right)\left(0.50\mathrm{m} - 0.25\mathrm{m}\right) = 2.2\,\mathrm{m/s}$$

2.2 m/s

e. A rubber duck of average density 0.68 g/cm^3 floats in the fountain. What percentage of it is submerged *(5 pts.)*

$$\beta = \rho_{H_2O} V_{disp} g = mg = \rho_{duck} V_{duck} g \Rightarrow \frac{V_{disp}}{V_{duck}} = \frac{\rho_{duck}}{\rho_{H_2O}} = \frac{0.68\,\mathrm{g/cm^3}}{1.00\,\mathrm{g/cm^3}} = .68$$

68%