EXAM 3

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: _____

Problem 2: _____

Problem 3: _____

Problem 4: _____

TOTAL: _____

SOLUTION KEY

Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80$ m/s^2

Atmospheric pressure at sea level = 1.01 x 10^5 Pa = 1.01 x 10^5 N/m^2

Gravitational constant: G = 6.67 x 10^{-11} N·m^2/kg^2

ρ(water) = 1.00 x 10^3 kg/m^3 = 1.00 g/cm^3

1 Liter = 10^{-3} m^3

PROBLEM 1

A 0.100 kg object hangs from a long vertical massless spring. When the object is pulled down 0.10 m below its equilibrium position and released, it oscillates with a period of 1.8 sec.

a. What is force constant of the spring? *(5 pts.)*

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = (2\pi)^2 \frac{m}{T^2} = \frac{4\pi^2(0.100\text{kg})}{(1.80\text{s})^2} =$$

$$\boxed{1.2 \text{ N/m}}$$

b. What is the object's speed as it passes through the equilibrium position? *(5 pts.)*

$$x = 0 \Rightarrow \tfrac{1}{2}mv^2 = \tfrac{1}{2}kA^2 \Rightarrow v = A\sqrt{\frac{k}{m}} = A\cdot\frac{2\pi}{T} = \frac{2\pi(0.100\text{m})}{1.80\text{s}} = 0.349\,\text{m/s}$$

$$\boxed{0.35 \text{ m/s}}$$

c. What is its acceleration when it is 0.050 m above the equilibrium position? *(5 pts.)*

$$ma = -kx \Rightarrow a = -\frac{k}{m}x = -\left(\frac{2\pi}{T}\right)^2(-0.050\text{m}) = \left(\frac{2\pi}{1.80\text{s}}\right)^2(0.050\text{m}) = 0.609\,\text{m/s}^2$$

(positive x is down)

$$\boxed{0.61 \text{ m/s}^2}$$

d. When it is moving upward, how much time is required to move from a point 0.050 m below the equilibrium position to a position 0.050 m above the equilibrium position? *(5 pts.)*

$$x = A\sin\omega t = A\sin\left(\frac{2\pi t}{T}\right) \Rightarrow 0.050\text{m} = 0.100\sin\left(\frac{2\pi t}{T}\right) \Rightarrow \frac{2\pi t}{T} = \sin^{-1}(0.50) \Rightarrow \frac{2\pi t}{T} = \frac{\pi}{6} \Rightarrow$$

$$t(0 \rightarrow +0.50\text{m}) = \frac{T}{12} = \frac{1.80\text{s}}{12} = 0.150\text{s} \Rightarrow t(-0.50\text{m} \rightarrow +0.50\text{m}) = 0.150\text{s} \times 2 =$$

$$\boxed{0.30 \text{ s}}$$

e. When motion is stopped and the object is removed from the spring, by how much does the spring shorten? *(5 pts.)*

$$\sum F_x = ma_x \Rightarrow mg - kd = 0 \Rightarrow d = \left(\frac{m}{k}\right)g = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{1.80\text{s}}{2\pi}\right)^2 9.80\frac{m}{s^2} = 0.804\text{m}$$

$$\boxed{0.80 \text{ m}}$$

PROBLEM 2

A satellite of mass 4320 kg is in circular orbit 1550 km above the surface of the earth ($M_E = 5.98$ x 10^{24} kg, $R_E = 6.37$ x 10^6 m).

a. What is the speed of the satellite in its orbit? *(5 pts.)*

$$\frac{GM_E m}{r_0^2} = \frac{mv_0^2}{r_0} \Rightarrow v_0 = \sqrt{\frac{GM_E}{r_0}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2)(5.98 \times 10^{24}\,\text{kg})}{(6.37 + 1.55) \times 10^6\,\text{m}}} = 7,097\,\text{m/s}$$

$$\boxed{7.10 \text{x} 10^3 \text{ m/s}}$$

b. What is the period of the satellite's orbit (in terms of hours and minutes)? *(5 pts.)*

$$T = \frac{2\pi r_0}{v} = \frac{2\pi(6.37 + 1.55) \times 10^6\,\text{m}}{7,097\,\text{m/s}} = 7,012\text{s} =$$

$$\boxed{1 \text{ hr } 57 \text{ min}}$$

c. What is total angular momentum of the satellite about the center of the earth? *(5 pts.)*

$L = mv_0 r_0 = (4,320 \text{ kg})(7,097 \text{ m/s})(6.37+1.55)\text{x}10^6\text{m} =$

$$\boxed{2.43 \text{x} 10^{13} \text{kgm}^2/\text{s}}$$

d. What is total energy of the satellite in its orbit? *(5 pts.)*

$$E = \frac{1}{2}mv_0^2 - \frac{GM_E m}{r_0} = -\frac{1}{2}\frac{GM_E m}{r_0} = -\frac{1}{2}\frac{(6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2)(5.98 \times 10^{24}\,\text{kg})(4320\text{kg})}{(6.37 + 1.55) \times 10^6\,\text{m}} =$$

$$\boxed{-1.09 \text{x} 10^{11} \text{J}}$$

e. From its position in orbit, what speed must the satellite reach to escape from the earth? *(5 pts.)*

$$\frac{1}{2}mv_{esc}^2 = \frac{GM_E m}{r_0} \Rightarrow v_{esc} = \sqrt{\frac{2GM_E}{r_0}} = \sqrt{\frac{2(6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2)(5.98 \times 10^{24}\,\text{kg})}{(6.37 + 1.55) \times 10^6\,\text{m}}} =$$

$$\boxed{1.00 \text{x} 10^4 \text{m/s}}$$

PROBLEM 3

A full water storage tank is an upright cylinder of height 20.0 m and radius 10.0 m that is open to the air. The bottom of the tank is 20.0 m above the ground at sea level. A pipe of 16 cm *diameter* runs vertically down from the tank and goes 1.0 m underground (taken to its center) before turning horizontal.
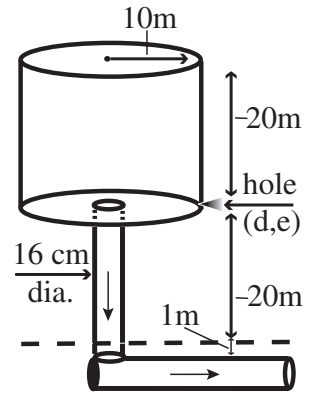
a. Find the water pressure at the bottom of the tank. *(5 pts.)*

$P_1 = P_a + \rho gh = (1.01 \times 10^5 \text{ Pa}) + (1.00 \times 10^3 \text{kg/m}^3)(9.80\text{m/s}^2)(20.0\text{m}) =$

$$\boxed{3.0 \times 10^5 \text{ Pa}}$$

b. What is the water pressure in center of the horizontal pipe with no water flow? *(5 pts.)*

$P_2 = P_a + \rho gh = (1.01 \times 10^5 \text{ Pa}) + (1.00 \times 10^3 \text{kg/m}^3)(9.80\text{m/s}^2)(41.0\text{m}) =$

$$\boxed{5.0 \times 10^5 \text{ Pa}}$$

c. Water starts to flow out the horizontal pipe at 100.0 L/s. How fast does the water level at the top of the tank drop? *(5 pts.)*

$V_0 A_0 = V_1 A_1 \Rightarrow (100.0\text{L/s})(10^{-3} \text{ m}^3/\text{L}) = V_0 \pi (10.0\text{m})^2 \Rightarrow V_0 = \dfrac{(100.0\text{L/s})(10^{-3} \text{ m}^3/\text{L})}{\pi(10.0\text{m})^2} =$

$$\boxed{3.2 \times 10^{-4} \text{ m/s}}$$

d. A hole 8.00 mm in diameter is put into the side of the tank at the bottom. How fast does the water shoot out of this new hole? *(5 pts.)*

$P_a + \frac{1}{2}\rho v_0^2 + \rho gh_0 = P_3 + \frac{1}{2}\rho v_3^2 + \rho gh_3, \; P_3 = P_a \Rightarrow \cancel{P_a} + 0 + \rho gh_0 = \cancel{P_a} + \frac{1}{2}\rho v_3^2 + \rho gh_3 \Rightarrow$

$v_3 = \sqrt{2g(h_0 - h_3)} = \sqrt{2(9.8\text{m/s}^2)(20.0\text{m})} =$

$$\boxed{20 \text{ m/s}}$$

e. How many L/s flows out of this new hole? *(5 pts.)*

$\dfrac{dV}{dt} = v_3 A_3 = (20\text{m/s})\pi(4.0 \times 10^{-3}\text{m})^2 = 1.0 \times 10^{-3} \text{ m}^3/\text{s} =$

$$\boxed{1.0 \text{ L/s}}$$

PROBLEM 4

Two uniform boards are hinged together at one end to form a right angle as shown. One board, of length 3.0 m has a weight of magnitude 120 N, while the other board of length 4.0 m has a weight of magnitude 160 N. The boards rest on a frictionless floor and a massless horizontal cable connects the boards at a height 1.0 m off the floor.
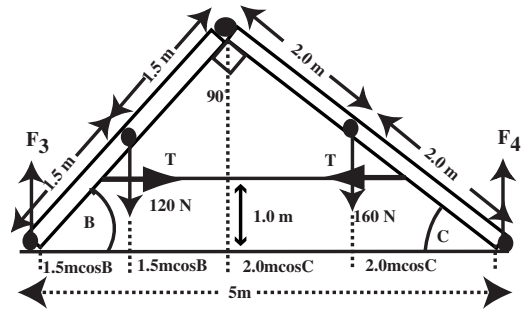
a. Find the magnitude of the force of the floor on the 4.0 m board by computing the torque around the point of contact of the 3.0 m board with the floor. *(5 pts.)*

τ around B: $(1.5m)(cosB)(120N) + [(3.0m)cosB+(2.0m)cosC](160N) – (5.0m)F_4 = 0 \Rightarrow$
$(5.0m)F_4 = (1.5m)(0.6)(120N) + [(3.0m)(0.6)+(2.0m)(0.8)](160N) \Rightarrow$
$F_4 = (108Nm + 544Nm)/(5.0m) =$

$$\boxed{130 \text{ N}}$$

b. Find the magnitude of the force of the floor on the 3.0 m board. *(5 pts.)*

$\Sigma F_y = 0 = 120 \text{ N} + 160 \text{ N} = 280 \text{ N} = F_3 + F_4 \Rightarrow F_3 = 280 \text{ N} - F_4 = 280 \text{ N} – 130 \text{ N} =$

$$\boxed{150 \text{ N}}$$

c. Find the magnitude of tension in the cable by computing torque about the hinge. *(5 pts.)*

τ around hinge: $(2.0m)(cos37°)(160N) + [(4.0m)(sin37°) – 1.0m]T - (4.0m)(cos37°)F_3 = 0$
$\Rightarrow (1.6m)(160N) + (1.4m)T = (3.2m)(130N) \Rightarrow T = (416Nm – 256Nm)/(1.4m) = 114N$

$$\boxed{110 \text{ N}}$$

d. Find the magnitude and angle (*in degrees* with respect to the horizontal) of the force on the hinge. *(5 pts.)*

$F_y = 160N – F_4 = 160N – 130N = 30 \text{ N}, \quad F_x = T = 114 \text{ N}$

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(114N)^2 + (30N)^2} = 118N, \quad \theta = tan^{-1}\left(\frac{F_x}{F_y}\right) = tan^{-1}\left(\frac{30N}{114N}\right) = 15°$

$$\boxed{|F|=120 \text{ N} \quad \theta=15°}$$

e. If the cable has a Young's modulus of 2.0 x 10^{11} and a diameter of 3.0 mm, find the fractional increase in length over that with no tension. *(5 pts – partial credit will be given for answers in terms of the Tension T if part c was not solved)*

$Y = \dfrac{F/A}{\Delta L/L_0} \Rightarrow \dfrac{\Delta L}{L_0} = \dfrac{F/A}{Y} = \dfrac{F/\pi r^2}{Y} = \dfrac{114N/\pi(1.5\times10^{-3}m)^2}{2.0\times10^{11}} = 8.06\times10^{-5}$

$$\boxed{8.1\text{x}10^{-5}}$$