FINAL EXAM

Print your name and section clearly on all nine pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 8.5" x 11" sheet and no other references. The exam lasts exactly two hours.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

Problem 5: __________

Problem 6: __________

Problem 7: __________

Problem 8: __________

TOTAL: __________

SOLUTION KEY

Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80$ m/s^2
Gravitational Constant: G =6.67 x 10^{-11} Nm^2/kg^2
1 calorie = 4.186 Joules
1 atm = 1.013 x 10^5 Pa
Universal Gas Constant: R = 8.314 J/(mol·K)
Stefan-Boltzmann Constant: σ = 5.669 x 10^{-8} W/m^{2}K^4
Avogadro's Number: N_A=6.022 x 10^{23} molecules/mole
Boltzmann's Constant: k_b=1.38 x 10^{-23} J/K

PROBLEM 1

4.00 moles of N_2 (an ideal *diatomic* gas with $\gamma = 1.40$ and molecular weight 28.0 g/mole) undergo a temperature increase from 300.0 K to 360.0 K at constant pressure.

a. How much heat in Joules was added to the gas? *(5 pts.)*

$$\gamma = \frac{C_P}{C_V} = \frac{7}{5}, C_P = C_V + R = \frac{5}{7}C_P + R \Rightarrow \frac{2}{7}C_P = R \Rightarrow C_P = \frac{7}{2}R = \frac{7}{2}\left(8.31\frac{J}{K}\right) = 29.1\frac{J}{K}$$

$$Q = nC_P\Delta T = (4.00\text{moles})(29.1\,J/K)(360K - 300K) = 6{,}980J$$

6,980 J

b. By how much did the internal energy of the gas increase? *(5 pts.)*

$$C_V = \frac{5}{7}C_P = \frac{5}{7}\left(\frac{7}{2}R\right) = \frac{5}{2}R, \quad \Delta U = nC_V\Delta T = (4.00\text{moles})\left(\frac{5}{2}\left(8.31\frac{J}{K}\right)\right)(360K - 300K) = 4{,}986J$$

4,990 J

c. How much work was done by the gas? *(5 pts.)*

$W = Q - \Delta U = 6980\ J - 4986\ J = 1994\ J$

1,990 J

d. By how much did the translational kinetic energy of the gas increase? *(5 pts.)*

$\Delta K = (3/2)nR\Delta T = (3/2)(4.00\text{ moles})(8.31\ J/K)(360K - 300K) = 2{,}992\ J$

2,990 J

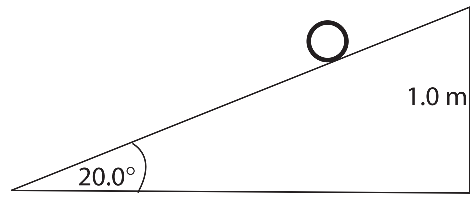
e. What is the rms velocity of the N_2 molecules at 360 K? *(5 pts.)*

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3((8.31J/K))(360K)}{0.028\,kg/mole}} = 566\,m/s$$

566 m/s

PROBLEM 2

A 3.0 kg uniform thin circular ring ($I=MR^2$) is released from rest at the top of a 1.0 m high ramp inclined at an angle of 20.0° and rolls down the ramp without slipping.



a. Find the total kinetic energy of the ring when it reaches the bottom of the ramp. *(5 pts.)*

$K = mgh = (3.0\ \text{kg})(9.8\ \text{m/s}^2)(1.0\ \text{m}) = 29.4\ \text{J}$

29 J

b. What percentage of this energy is in translation of the center of mass? *(5 pts.)*

$$f = \frac{mv^2/2}{mv^2/2 + I\omega^2/2} = \frac{v^2}{v^2 + r^2\omega^2} = \frac{v^2}{v^2 + v^2} = \frac{1}{2}$$

50%

c. Find the speed of the center of mass when the ring reaches the bottom part of the ramp. *(5 pts.)*

$$\frac{1}{2}\text{mv}^2 = \frac{1}{2}\text{mgh} \Rightarrow \text{v} = \sqrt{\text{gh}} = \sqrt{(9.8\,\text{m/s}^2)(1.0\text{m})} = 3.13\text{m/s}$$

3.1 m/s

d. What is the magnitude of the acceleration of the center of mass when the ring reaches the bottom part of the ramp? *(5 pts.)*

$$\text{v}^2 = 2\text{as} = 2\text{a}\frac{\text{h}}{\sin\theta} \Rightarrow \text{a} = \frac{\text{v}^2\sin\theta}{2\text{h}} = \frac{(3.13\text{m/s})(\sin 20°)}{2(1.0\text{m})} = 1.68\,\text{m/s}^2$$

1.7 m/s²

e. How much time does it take for the ring to reach the bottom of the ramp? *(5 pts.)*

$$\text{v} = \text{at} \Rightarrow \text{t} = \frac{\text{v}}{\text{a}} = \frac{3.13\text{m/s}}{1.68\,\text{m/s}^2} = 1.86\ \text{s}$$

1.9 s

PROBLEM 3

A fire hose is attached to a hydrant outlet at ground level. The outlet has an area of 0.12 m^2, and provides 4.0×10^5 Pa of water pressure with a water speed of 5.0 m/s. The mass density of water is 1000.0 kg/m^3. Assuming an idea flow:

a. Find the volume flow rate in m^3/s of the water exiting the hydrant. *(5 pts.)*

Flow rate = (area)(speed) = (0.12 m^2)(5.0 m/s) =

0.60 $m^3\ s^{-1}$

b. The hydrant outlet is attached to a hose with a nozzle with an area of 0.088 m^2 at the same height as the hydrant outlet. What is the speed of the water as it exits the nozzle? *(5 pts.)*

Mass conservation ⇒ (0.088 m^2)(speed) = 0.60 m^3/s, so water speed = (0.60 m^3/s)/(0.088 m^2) =

6.8 m/s

c. In part b, what is the gauge pressure just inside the end of the nozzle? *(5 pts.)*

Bernoulli's equation $\Rightarrow P+\frac{1}{2}\rho v^2+\rho gh = \text{constant}$. Since h same, $P_1+\frac{1}{2}\rho v_1^2 = P_2+\frac{1}{2}\rho v_2^2$ and

$P_2 = P_1+\frac{1}{2}\rho\left(v_1^2-v_2^2\right)$ = $(4.0\times10^5$ Pa)-(1/2)(1000 kg m^{-3})((5.0 m/s)^2-(6.8 m/s)^2)=

4.1×10^5 Pa

d. As the firefighter holding the hose from part b climbs a ladder, the speed of the water coming out of the hose decreases. What is the maximum height that the nozzle can be at which water still flows? *(5 pts.)*

Bernoulli's equation $\Rightarrow P+\frac{1}{2}\rho v^2+\rho gh = \text{constant}$, and at maximum height the water exits nozzle at atmospheric pressure and zero velocity. So

$$h=\left(P_1+\frac{1}{2}\rho v_1^2-P_3-\frac{1}{2}\rho v_3^2\right)/(\rho g)=\left(P_1+\frac{1}{2}\rho v_1^2-P_{atm}\right)/(\rho g)$$

$$=\left(4.0\times10^5 Pa+(0.5)(1.0\times10^3 kg/m^3)(5.0m/s)^2-1.013\times10^5 Pa\right)/\left((1.0\times10^3 kg/m^3)(9.8m/s^2)\right)=$$

32 m

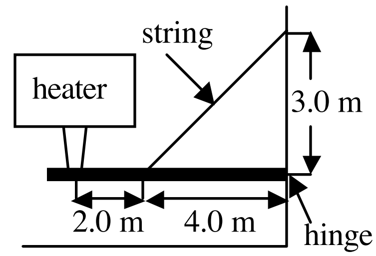
e. Another firefighter holds a different hose with water exiting the nozzle horizontally at a height of 12.5 m above the ground and speed of 5.5 m/s. Ignoring air resistance, how far from the point on the ground directly below the nozzle does the water hit the ground? *(5 pts.)*

Vertical distance d=$gt^2/2$, so water hits ground after $t=\sqrt{2d/g}=\sqrt{2(12.5m)/(9.8m/s^2)}=1.60s$.
In 1.60s the horizontal distance moved is (1.60s)(5.5m/s)=

8.8 m

PROBLEM 4

A pine log cabin has a wall area of 60.0 m^2 and walls 0.14 m thick. The ceiling and floor are well-insulated, so you may assume that all heat is lost through the walls. The thermal conductivity of pine is $k_p = 0.21\ Wm^{-1}K^{-1}$. A heater with mass 5.5 kg is resting on a rigid massless shelf supported by a hinge and a massless string, as shown. When set on medium, the heater supplies 1800.0 W continuously to the cabin interior.



a. What is the tension in the string? *(5 pts.)*

Torques about hinge sum to zero: Mg(6.0 m) = (3/5)T (4.0 m), and

$$T = \frac{6\cdot 5}{12}Mg = \frac{5}{2}Mg = \frac{5}{2}(5.5\text{kg})\left(9.8\,\text{m/s}^2\right) = 134.8\text{N}$$

140 N

b. What is the magnitude of the force on the hinge? *(5 pts.)*

Forces on shelf sum to zero, so

$F_x = (4/5)T = (4/5)(134.8\ N) = 107.8\ N$

$F_y = -Mg + (3/5)T = -Mg + (3/5)(5/2)Mg = (1/2)Mg =(1/2)(5.5\ kg)(9.8 m/s^2)= 27.0\ N$

$|F| = (F_x^2+F_y^2)^{1/2} = ((107.8\ N)^2 + (27.0\ N)^2)^{1/2}= 111.1\ N$

110 N

c. If the outside temperature is 0.0° C, what is the temperature inside the cabin? *(5 pts.)*

Heat flow across wall $\wp = dQ/dt = k_pA|dT/dx| = (0.21\ Wm^{-1}K^{-1})(60.0\ m^2)(T_{in}-0.0°C)/(0.14\ m)$, so

$T_{in} = 0.0°C + \wp\Delta x_{pine}/k_pA = (1800.0\ W)(0.14\ m)/(0.21\ Wm^{-1}K^{-1})(60.0\ m^2) =$

20° C

d. The interior walls of the cabin are sprayed with a layer of insulating foam 0.02 m thick, with thermal conductivity κ_f=0.015 $Wm^{-1}K^{-1}$. If the outside temperature is 0.0° C and the heater is set on low (1000.0 W continuously), what is the temperature at the foam/wood interface? *(5 pts.)*

Heat flow in pine $\wp_{pine} = k_pA(T_{interface}-T_{outside})$ equals heat flow in insulation, which in turn equals power supplied by heater. So

$T_{interface} = 0.0°\ C + \wp_{pine}\Delta x_{pine}/(k_pA) = (1000.0\ W)(0.14\ m)/((0.21\ Wm^{-1}K^{-1})(60.0\ m^2)) =$

11° C

e. In part d, what is the temperature inside the cabin? *(5 pts.)*

Heat flow in insulation $\wp_{ins} = k_fA(T_{inside}-T_{interface})$, so

$T_{inside} = T_{interface} + \wp_{ins}\Delta x_{ins}/k_fA = 11.1°\ C + (1000.0\ W)(0.02\ m)/((0.015\ Wm^{-1}K^{-1})(60.0\ m^2)) =$

33° C

PROBLEM 5

A person adds 0.0900 kg of ice at 0.00° C to an insulated mug containing tea at 50.0° C. The heat of fusion for ice is L_{ICE} = 333. kJ/kg, and the specific heat of water (and tea) is c_w = 4.20 $kJkg^{-1}K^{-1}$. The mass density of ice ρ_I is 0.931 gm/cm^3, while the mass density of water ρ_W is 1.00 gm/cm^3.

a. What percent of the ice is above the surface of the water?

Archimedes principle ⇒ submerged volume V_s satisfies $\rho_W g V_s = \rho_I g V$, where V is volume of ice cube. So fraction above surface = $1 - V_s/V = 1 - \rho_I/\rho_W = 0.069$.

6.9%

b. Find the smallest amount of grams of tea at 50.0 degrees C that will just melt all the ice, leaving only liquid at 0.00 degrees C. *(5 pts.)*

Let m_{TEA} be the mass of tea that just melts the ice, so that the final temperature is 0°C. The heat lost by tea is $\Delta Q_{TEA} = m_{TEA} c_W (T_{HOT} - T_{COLD})$, while the heat gained by melted ice is $\Delta Q_{ICE} = m_{ICE} L_{ICE}$. Since $\Delta Q_{TEA} = \Delta Q_{ICE}$, one obtains

$m_{TEA} = m_{ICE} L_{ICE} / (c_W (T_{HOT} - T_{COLD}))$ = (0.0900 kg)(333 $kJkg^{-1}$)/((4.20 $kJkg^{-1}K^{-1}$)(50.0 K)=

0.140 kg

c. If the mug has 0.400 kg of tea before the 0.0900 kg of ice is added, what is the final temperature T_f, in degrees C, of the mixture when it reaches equilibrium? *(5 pts.)*

For final temperature of T_f, the heat lost by M_{TEA}= 0.400 kg of tea is $M_{TEA}\, c_W\, (T_{HOT} - T_f)$. The heat gained by ice is the latent heat of melting $m_{ICE} L_{ICE}$ plus the heat required to heat the melted ice, $c_W(T_f - 0^oC)$. The heat lost by the tea equals the heat gained by the ice, so

$$T_F = \frac{M_{TEA} c_W T_{HOT} - m_{ICE} L_{ICE}}{(M_{TEA} + m_{ICE}) c_W} = \frac{(0.400kg)(4.20kJkg^{-1}C^{-1})(50.0C) - (0.0900kg)(333kJ/kg)}{(0.400kg + 0.0900kg)(4.20kJkg^{-1}C^{-1})} =$$

26.3 ºC

d. A straw with thermal expansion coefficient of 3.200×10^{-4} K^{-1} is a hollow cylinder of inner diameter 0.530 cm when sitting in tea at 50.00 degrees C. What is the inner diameter of the straw when the liquid is cooled down to 5.00 degrees C? *(5 pts.)*

During thermal expansion or contraction, all dimensions shrink proportionally. Upon cooling from 50.0° C to 5.0° C, the inner diameter is reduced by a factor $(3.2 \times 10^{-4}\ ^oC^{-1})(45.0^oC) = 0.0144$, and so it is (0.530 cm)(1-0.0144)=

0.522 cm

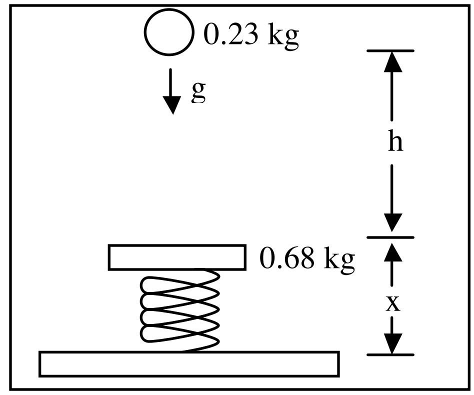
e. A person moves the 0.490 kg of cool tea and the 0.100 kg mug containing it from from a tabletop 1.00 m above the floor to a shelf 1.90 m above the floor. How much work was done by the gravitational force on the mug of tea during this move? *(5 pts.)*

Work done by gravity is –mgh = -(0.590 kg)(9.80 ms^{-2})(0.90 m)=

-5.2 J

PROBLEM 6

A ball of mass m=0.23 kg starts from rest and after falling a distance h accelerated by gravity, it is moving at a speed of 8.5 m/s when it hits and sticks to a platform of mass M=0.68 kg that is in mechanical equilibrium, supported by a massless, frictionless spring with spring constant k=55.5N/m.



a. What is the distance h? *(5 pts.)*

$$h = \frac{v^2}{2g} = \frac{(8.5m/s)^2}{(2)(9.8m/s^2)} = 3.69m$$

3.7 m

b. What is the velocity of the ball and platform at the instant just after the ball collides and sticks to the platform? *(5 pts.)*

Conservation of momentum ⇒

$$mv_i = (m+M)v_f \Rightarrow v_f = \frac{mv_i}{(m+M)} = \frac{(0.23kg)(8.5m/s)}{(0.23kg+0.68kg)} = 2.148 \text{ m/s}$$

2.1 m/s

c. Now suppose that at the instant of impact of the ball with the platform, the platform is a distance x=0.44 m above the floor. The spring then compresses, decreasing x until it hits a minimum value, and the platform then undergoes oscillatory motion. What is the period of the oscillation? *(5 pts.)*

period $T = 2\pi/\omega$ with $\omega = \sqrt{k/(m+M)}$, so
$T = 2\pi\sqrt{(m+M)/k} = (2)(3.142)\sqrt{(0.91\text{kg})/(55.5\text{N/m})} = 0.8046$ s

0.80 s

d. What is the potential energy of the spring at the instant of impact in part c? *(5 pts.)*

$U_{spring} = \frac{1}{2}k(x-x_0)^2$, where x_0 is the position of the platform when the spring is unstretched. x_0 satisfies $k(x_0-x)=Mg$, so $x_0=x+Mg/k=(0.44m)+(0.68kg)(9.8ms^2)/(55.5N/m)=0.56m$.

So $U_{spring} = \frac{1}{2}k(x-x_0)^2 = \frac{1}{2}(55.5N/m)(0.44m-0.56m)^2 =$

0.40 J

e. After a very long time the ball-platform-spring system comes to rest. What is the value of x at which this occurs? *(5 pts.)*

In mechanical equilibrium, $(m+M)g=k(x_0-x_{eq})$, where x_0 is the position of the platform when the spring is unstretched. The value of x_0 is found by requiring $k(x_0-x)=Mg$, so
$x_0=x+Mg/k=0.44m+(0.68kg)(9.8ms2)/(55.5N/m)=0.56m$.
So $x_{eq}=x_0-(m+M)g/k = (0.56m) -(0.23kg+0.68kg)(9.8ms^2)/(55.5N/m) =$

0.40 m

PROBLEM 7

A planet has mass $M=7.0\times10^{21}$ kg, uniform mass density, and radius $R=2.0\times10^{6}$ m.

a. What is the acceleration of gravity on the planet's surface? *(5 pts.)*

$g=GM/R^2=(6.67\times10^{-11}Nm^2/kg^2)(7.0\times10^{21}kg)/(2.0\times10^{6}m)^2=$

0.12 ms^{-2}

b. What is the average mass density of the planet? (5 pts.)

$\rho=M/(4\pi R^3/3)=(7.0\times10^{21}kg)/(4\pi(2.0\times10^{6}m)^3/3)=$

210 kg m^{-3}

c. A mass of 8.0 kg is in a circular orbit around the planet at an altitude $R= 2.0\times10^{6}$ m above the surface. What is its kinetic energy? *(5 pts.)*

Mass times acceleration of the mass is $mv^2/(2R)$, which equals gravitational force $GmM/(2R)^2$. So kinetic energy $mv^2/2=GmM/(4R)=(6.67\times10^{-11}Nm^2/kg^2)(8.0kg)(7.0\times10^{21}kg)/(4\times2.0\times10^{6}m)=$

4.7×10^{5} J

d. A small object hangs motionless with respect to the planet at a very large (infinite) distance from the planet. A very slight (negligible) nudge towards the planet causes it to be captured by the planet's gravity. The object eventually crashes onto the planet's surface. What is its speed when it lands? *(5 pts.)*

Initially, kinetic energy and potential energy are both zero. When object crashes into planet's surface, potential energy is $U=-GMm/R$, and since total mechanical energy is conserved,

$\frac{1}{2}mv^2 = GMm/R$, so

$v = \sqrt{2GM/R} = \sqrt{2(6.67\times10^{-11}Nm^2kg^{-2})(7.0\times10^{21}kg)/(2.0\times10^{6}m)} =$

680 ms^{-1}

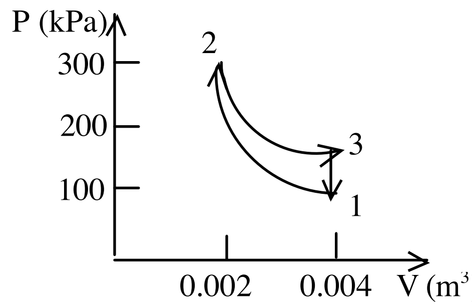
e. A person on the planet's surface fashions a simple pendulum from a (nearly) massless 2.0 m length of string and a lead weight. What is the pendulum period for small oscillations about the vertical? *(5 pts)*

Angular frequency and period $T=2\pi/\omega$, so $T = 2\pi\sqrt{L/g} = (2)(3.14)\sqrt{(2.0m)/(0.12ms^{-2})} =$

26 s

PROBLEM 8

An engine operates in the following cycle, using a monatomic ideal gas as its working substance: (1→2): The gas starts in initial state (P_1, V_1, T_1)=(100.0 kPa, 0.004 m^3, 300.0 K) and is compressed adiabatically to (P_2, V_2, T_2)=(317kPa, 0.002m^3, 475.5K) by a piston. (2→3): The gas expands isothermally to (P_3, V_3, T_3)= (158.5 kPa, 0.004 m^3, 475.5 K), doing work on the piston. (3→1): The gas then cools at constant volume back to the state (P_1, V_1, T_1). The P-V diagram for the process is shown in the figure.



a. What is the change in entropy of the gas along the (2→3) leg? *(5 pts.)*

For isothermal expansion, the change in entropy is $\Delta S=nR\ln(V_f/V_i)$. Since PV=nRT, nR=PV/T=(100 kPa)(0.004 m^3)/(300 K)=1.33 J/K, and

ΔS=(1.33 J/K)ln(0.004/0.002)=(1.33)(0.693) J/K =

0.92 J/K

b. What is Q_{23}, the amount of heat absorbed by the engine in the (2→3) leg? *(5 pts.)*

Heat absorbed in isothermal leg is $Q_{23} = T\Delta S$ = (475.5 K)(0.92 J/K) = 438 J

440 J

c. What is Q_{31}, the amount of heat ejected by the engine in the (3→1) leg? *(5 pts.)*

Heat ejected in constant volume leg

$Q_{31} = n\, c_V \Delta T = 3/2(nR)(T_3 - T_1) = (3/2)(1.33\ JK^{-1})(475.5K-300K)$=

350 J

d. Using the First Law of Thermodynamics and the heats from parts b and c, calculate the work done by the gas per (1→2→3→1) cycle. *(5 pts.)*

Since dE=ΔQ +ΔW (ΔW=work done on gas) and dE=0 upon going around a cycle,
total work done by gas W is

$W=Q_{12}+Q_{23}+Q_{31}$ = 0 + 438J – 350 J =

88 J

e. Find the efficiency ε of this engine. *(5 pts.)*

Heat supplied to engine is Q_{23}, so

$\varepsilon = W/Q_{23}$ = 88 J/438 J = 0.201

20%