EXAM 1

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

TOTAL: __________

SOLUTION KEY

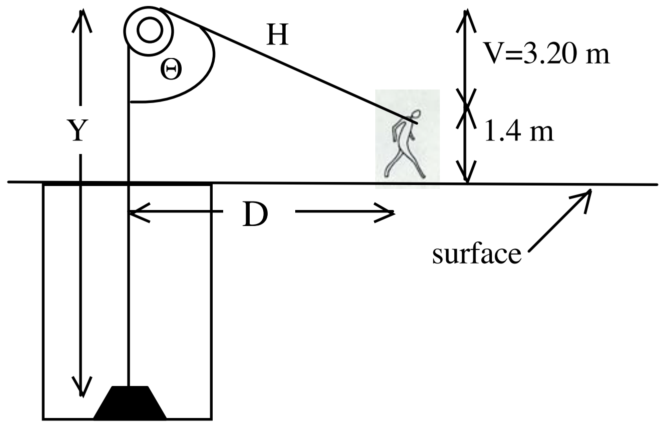
Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80\ \text{m/s}^2$

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

PROBLEM 1

A person of height 1.4 m pulls on a massless rope of fixed length that rises to a frictionless pulley at an angle of Θ from vertical and then drops vertically a distance Y to a mass M=5.55 kg that is in a hole, as shown. The pulley is 3.20 m above the height at which the person is holding the rope. At time t=0 the mass is 10.0 m below the surface and the angle Θ=51.4°.



a. What is D, the horizontal distance between the person and the vertical part of the rope, at time t=0? *(5 pts.)*

V=H cos Θ and D = H sin Θ, so D = V tan Θ.

So D = (3.20 m) tan (51.4°) =

4.01 m

b. The person now walks away so that the mass rises at a constant speed of v=2.3 m/s. Find the tension in the rope just after the person starts walking. *(5 pts.)*

Speed constant, so
$F = Mg = (5.55\ \text{kg})(9.80\ \text{m s}^{-2}) =$

54.4 N

c. The person stops walking when the mass is at ground level. Find the horizontal component of the distance from the mass to the person at this point. *(5 pts.)*

The distance V is still 3.20 m, but now H=H(t=0)+10.0m = D(t=0)/sin Θ(t=0) + 10.0 m = 4.01m/sin(51.4°) + 10.0 m = 15.13 m. The distance D satisfies $D^2=H^2-V^2$, so

$$D = \sqrt{H^2 - V^2} = \sqrt{(15.13m)^2 - (3.20m)^2} =$$

14.8 m

d. After stopping, the person lets go of the rope and the mass drops 10.0 m down to the bottom of the hole. How long does it take for the mass to fall to the bottom of the hole? *(5 pts.)*

t determined via $D=gt^2/2$, or $t=(2D/g)^{1/2}$ =

1.43 s

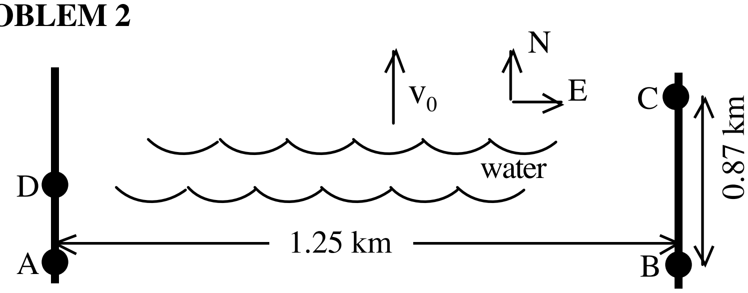
e. After stopping, the person lets go of the rope and the mass drops 10.0 m down to the bottom of the hole. What is the speed at which the mass hits the bottom ? *(5 pts.)*

$v = g(2D/g)^{1/2} = (2Dg)^{1/2} = ((2)(10.0\ \text{m})(9.80\ \text{m s}^{-2}))^{1/2} =$

14.0 m/s

PROBLEM 2

A swimmer who swims at a speed of V=0.75 km/hr in still water swims in a river that is 1.25 km across with a current with velocity v_0=0.33km/hr pointing north.



a. What direction should the swimmer point in order to start out at point A and end up directly across the river at point B? Please express your answer in degrees N,S,E or W of N,S,E or W. *(5 pts.)*

Southward component velocity must be equal and opposite to velocity of current, so $V \sin \Theta = v_0$, or $\Theta = \sin^{-1}(v_0/V) = \sin^{-1}(0.33/0.75) =$

26° S of E

b. How long, in hours, does it take for the swimmer to get from point A to point B? *(5 pts.)*

time = distance/(V cos Θ) = (1.25 km)/(0.75 km/hr × cos 26°) =

1.9 hr

c. The swimmer walks along the shore 0.87 km at 5.28 km/hr north along the shore to point C, and then swims back to the west shore at point D by pointing at an angle Φ =36° south of west. What is the distance from point D to point A? *(5 pts.)*

W component of velocity has magnitude V cos Φ = (0.75 km/hr)(cos 36°)
= 0.61 km/hr, so time elapsed when west bank is reached is
(1.25 km)/(0.61 km/hr) = 2.06 hr.
Magnitude of NS component of velocity is $-V \sin\Phi + v_0$
= (-0.75 km/hr)(sin 36°) + 0.33 km/hr
= -0.11 km/hr, so swimmer hits shore
(0.87 km)-(0.11 km/hr)(2.06 hr) = 0.64 km N of A

0.64 km

d. The swimmer then walks along the shore from point D to point A at a speed of 5.28 km/hr. How far did the swimmer travel altogether (both in the water and on shore)? *(5 pts.)*

Total distance traveled is 1.25 km + 0.87 km + $\sqrt{(1.25)^2 + (0.23)^2}\,km$ + 0.64 km =

4.0 km

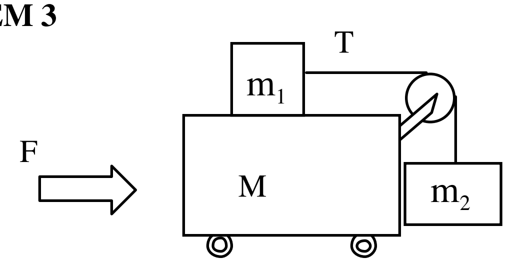
e. What is the swimmer's average speed for the entire trip, in the frame of an observer on shore? *(5 pts.)*

time elapsed = 1.85 hr + (0.87 km)/(5.28 km/hr) + 2.06 hr + (0.64 km)/(5.28 km/hr)
= (1.85 + 0.16 + 2.06 + 0.12) hr = 4.2 hr, so average velocity = distance/time
= (4.0 km)/(4.2 hr) =

0.96 km/hr

PROBLEM 3

A cart of mass M=2.5 kg with frictionless wheels has a frictionless pulley mounted on it, as shown. There are two blocks with masses m_1=0.53 kg and m_2= 1.44 kg connected by a rope, as shown. The coefficient of friction of block m_1 with the cart is μ_1=0.33, while the coefficient of friction of block m_2 with the cart is zero. In parts d and e, a horizontal force F is applied to the cart.



a. Find the magnitude and direction (left or right) of the friction force on block m_1 when the cart is stationary. *(5 pts.)*

Magnitude of friction force is bounded above by $\mu m_1 g$ = (0.33)(0.53 kg)(9.80 ms^{-2}) = 1.7 N. It attains this value, because it is not enough to keep m_1 from sliding.
Friction force is in a direction that opposes force T from rope, so it points to the left.

1.7 N to left

b. What is the tension in the string when the cart is stationary (F=0)? *(5 pts.)*

Force on m_2 = $m_2 g$-T; horizontal force on m_1 = T-$\mu m_1 g$. Both blocks have the same acceleration, so a_2 =g-T/m_2 = a_1 = T/m_1-μg, and

$$T=g(1+\mu)/(1/m_1+1/m_2) = \frac{(9.8ms^{-2})(1+0.33)}{\frac{1}{0.53kg}+\frac{1}{1.44kg}} =$$

5.0 N

c. What is the acceleration of block m_1 when F=0? *(5 pts.)*

a_1 = T/m_1-μg = (5.0 N)/(0.53 kg) – (0.33)(9.8 ms^{-2}) =9.434-3.234 =

6.2 ms^{-2}

d. What is the vertical component of the acceleration of m_2 when the acceleration of the cart is 6.2 m/s^2? *(5 pts.)*

$m_1 a_1$ = T+f_1, where T is tension and f_1 is friction force (positive points to right). (1)
Vertical acceleration a_2 is $m_2 a_2 = m_2 g - T \Rightarrow T = m_2(g-a_2)$ (positive points down). (2)
Because m_1 and m_2 are connected by the rope, $a_1 = a_2+A$, where A is the cart's acceleration. (3)
Substituting for T from (2) and a_1 from (3) into (1), yields $m_1(a_2+A) = m_2(g-a_2)+f_1$,
so $a_2 = (m_2 g-m_1 A+f_1)/(m_1+m_2)$.
Because $m_2 g-m_1 A$=(1.44 kg)(9.8 ms^{-2})-0.53 kg)(6.2 ms^{-2})=10.8 N >0, the friction force f_1<0. The maximum friction force is $\mu_1 m_1 g$=(0.33)(0.53 kg)(9.8 ms^{-2}) = 1.71 N, which, since it is less than $m_2 g-m_1 A$, is the value attained. Therefore,
$a_2 =[m_2 g-m_1 A-\mu_1 m_1 g]/(m_1+m_2)$=(14.1N–3.29N–1.71N)/(1.44kg+0.53kg)=

4.62 ms^{-2}

e. Now suppose the coefficient of friction μ=0, and a force F is applied as shown that causes the cart to accelerate at 1.83 m/s^2. What is the vertical component of the acceleration of m_2? *(5 pts.)*

From (d), since f_1=0, $a_2 = (m_2 g-m_1 A)/(m_1+m_2)$ =
(14.1 N – (0.53 kg)(1.83 ms^{-2}))/(1.44 kg + 0.53 kg) =

6.67 ms^{-2}

PROBLEM 4

To prepare with his battle with Goliath, David is preparing a sling shot. He finds that the fastest that he can revolve a particular stone in a circular horizontal orbit of radius 1.34 m is 5.33 revolutions per second. Assume that the speed along the orbit is uniform.

a. What is the speed of the stone in this circular orbit? *(5 pts.)*

In one period T, stone goes a distance 2πr, so speed = 2πr/T = (2)(3.1416)(1.34 m)(5.33 s^{-1}) =

44.9 m/s

b. Find the magnitude of the acceleration of the stone in the circular orbit. *(5 pts.)*

$a=v^2/r = (44.9\ m/s)^2/(1.34\ m) =$

1500 m/s^2

c. The stone of parts a and b comes out of the sling moving horizontally when it is 0.984 m above the ground. Find the time elapsed between when the stone comes out of the sling and when the stone hits the ground. (The ground is flat, and air resistance can be neglected). *(5 pts.)*

$y = y_0 - v_{y0}t - gt^2/2$, so t is solution of $gt^2/2 + v_{y0}t - (y_0-y) = 0$. Now $v_{y0}=0$, so

$$t = \frac{\sqrt{2g(y-y_0)}}{g} = \sqrt{\frac{2(y-y_0)}{g}}.\ \text{So}\ t = \sqrt{\frac{2(0.984m)}{9.80ms^{-2}}} =$$

0.448 s

d. Find the speed at which the stone in part c hits the ground. *(5 pts.)*

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{x0}^2 + (gt)^2} =$$

$$\sqrt{(44.9m/s)^2 + \left((9.8ms^{-2})(0.448s)\right)^2} =$$

45.1 m/s

e. Find the horizontal distance between the point where the stone comes out of the sling and the point where it hits the ground. *(5 pts.)*

$x-x_0 = v_{x0}\ t = (44.9\ m/s)\ (0.448\ s) =$

2.0.1 m