P12.36 $B = -\frac{\Delta P}{\frac{\Delta V}{V_i}} = -\frac{\Delta P V_i}{\Delta V}$

(a) $\Delta V = -\frac{\Delta P V_i}{B} = -\frac{\left(1.13\times10^{8}\ \text{N/m}^2\right)1\ \text{m}^3}{0.21\times10^{10}\ \text{N/m}^2} = \boxed{-0.053\,8\ \text{m}^3}$

(b) The quantity of water with mass 1.03×10^{3} kg occupies volume at the bottom $1\ \text{m}^3 - 0.053\,8\ \text{m}^3 = 0.946\ \text{m}^3$. So its density is $\frac{1.03\times10^{3}\ \text{kg}}{0.946\ \text{m}^3} = \boxed{1.09\times10^{3}\ \text{kg/m}^3}$.

(c) With only a 5% volume change in this extreme case, liquid water is indeed nearly incompressible.

***P12.37** Part of the load force extends the cable and part compresses the column by the same distance $\Delta \ell$:

$$F = \frac{Y_A A_A \Delta\ell}{\ell_A} + \frac{Y_s A_s \Delta\ell}{\ell_s}$$

$$\Delta\ell = \frac{F}{\frac{Y_A A_A}{\ell_A} + \frac{Y_s A_s}{\ell_s}} = \frac{8\,500\ \text{N}}{\frac{7\times10^{10}\pi\left(0.162\,4^2 - 0.1614^2\right)}{4(3.25)} + \frac{20\times10^{10}\pi(0.0127)^2}{4(5.75)}}$$

$$= \boxed{8.60\times10^{-4}\ \text{m}}$$

Chapter Fifteen: Oscillatory Motion

SOLUTIONS TO PROBLEMS

P15.1 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use: $x = \frac{1}{2}gt^2$.

The time for the ball to hit the ground is $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00\ \text{m})}{9.80\ \text{m/s}^2}} = 0.909\ \text{s}$

This equals one-half the period, so $T = 2(0.909\ \text{s}) = \boxed{1.82\ \text{s}}$.

(c) $\boxed{\text{No}}$. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

P15.4 (a) The spring constant of this spring is

$$k = \frac{F}{x} = \frac{0.45\ \text{kg}\ 9.8\ \text{m/s}^2}{0.35\ \text{m}} = 12.6\ \text{N/m}$$

we take the x-axis pointing downward, so $\phi = 0$

$$x = A\cos\omega t = 18.0\ \text{cm}\cos\sqrt{\frac{12.6\ \text{kg}}{0.45\ \text{kg}\cdot\text{s}^2}}\,84.4\ \text{s} = 18.0\ \text{cm}\cos 446.6\ \text{rad} = \boxed{15.8\ \text{cm}}$$

(d) Now $446.6\ \text{rad} = 71\times 2\pi + 0.497\ \text{rad}$. In each cycle the object moves $4(18) = 72\ \text{cm}$, so it has moved $71(72\ \text{cm}) + (18 - 15.8)\ \text{cm} = \boxed{51.1\ \text{m}}$.

(b) By the same steps, $k = \dfrac{0.44\ \text{kg}\ 9.8\ \text{m/s}^2}{0.355\ \text{m}} = 12.1\ \text{N/m}$

$$x = A\cos\sqrt{\frac{k}{m}}t = 18.0\ \text{cm}\cos\sqrt{\frac{12.1}{0.44}}\,84.4 = 18.0\ \text{cm}\ \cos 443.5\ \text{rad} = \boxed{-15.9\ \text{cm}}$$

(e) $443.5\ \text{rad} = 70(2\pi) + 3.62\ \text{rad}$

$$\text{Distance moved } = 70(72\ \text{cm}) + 18 + 15.9\ \text{cm} = \boxed{50.7\ \text{m}}$$

(c) The answers to (d) and (e) are not very different given the difference in the data about the two vibrating systems. But when we ask about details of the future, the imprecision in our knowledge about the present makes it impossible to make precise predictions. The two oscillations start out in phase but get completely out of phase.

P15.19 (a) $E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(35.0\ \text{N/m})(4.00\times 10^{-2}\ \text{m})^2 = \boxed{28.0\ \text{mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2}$

.

$$|v| = \sqrt{\frac{35.0}{50.0\times 10^{-3}}}\sqrt{(4.00\times 10^{-2})^2 - (1.00\times 10^{-2})^2} = \boxed{1.02\ \text{m/s}}$$

(c) $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2 - \dfrac{1}{2}kx^2 = \dfrac{1}{2}(35.0)\left[(4.00\times 10^{-2})^2 - (3.00\times 10^{-2})^2\right] = \boxed{12.2\ \text{mJ}}$

(d) $\dfrac{1}{2}kx^2 = E - \dfrac{1}{2}mv^2 = \boxed{15.8\ \text{mJ}}$

P15.20 (a) $$k = \frac{|F|}{x} = \frac{20.0\text{ N}}{0.200\text{ m}} = \boxed{100\text{ N/m}}$$

(b) $$\omega = \sqrt{\frac{k}{m}} = \sqrt{50.0}\text{ rad/s} \quad \text{so} \quad f = \frac{\omega}{2\pi} = \boxed{1.13\text{ Hz}}$$

(c) $$v_{\max} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41\text{ m/s}} \text{ at } x = 0$$

(d) $$a_{\max} = \omega^2 A = 50.0(0.200) = \boxed{10.0\text{ m/s}^2} \text{ at } x = \pm A$$

(e) $$E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.200)^2 = \boxed{2.00\text{ J}}$$

(f) $$|v| = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\frac{8}{9}(0.200)^2} = \boxed{1.33\text{ m/s}}$$

(g) $$|a| = \omega^2 x = 50.0\left(\frac{0.200}{3}\right) = \boxed{3.33\text{ m/s}^2}$$

P15.31 Using the simple harmonic motion model:

$$A = r\theta = 1\text{ m } 15°\frac{\pi}{180°} = 0.262\text{ m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\text{ m/s}^2}{1\text{ m}}} = 3.13\text{ rad/s}$$

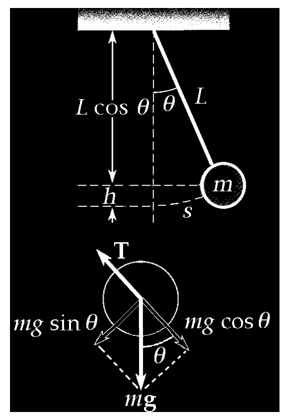

FIG. P15.31

(a) $$v_{\max} = A\omega = 0.262\text{ m } 3.13/\text{s} = \boxed{0.820\text{ m/s}}$$

(b) $$a_{\max} = A\omega^2 = 0.262\text{ m}(3.13/\text{s})^2 = 2.57\text{ m/s}^2$$

$$a_{\tan} = r\alpha \qquad \alpha = \frac{a_{\tan}}{r} = \frac{2.57\text{ m/s}^2}{1\text{ m}} = \boxed{2.57\text{ rad/s}^2}$$

(c) $$F = ma = 0.25\text{ kg } 2.57\text{ m/s}^2 = \boxed{0.641\text{ N}}$$

More precisely,

(a) $$mgh = \frac{1}{2}mv^2 \quad \text{and} \quad h = L(1 - \cos\theta)$$

$$\therefore v_{\max} = \sqrt{2gL(1 - \cos\theta)} = \boxed{0.817\text{ m/s}}$$

(b) $$I\alpha = mgL\sin\theta$$

$$\alpha_{\max} = \frac{mgL\sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54\text{ rad/s}^2}$$

(c) $$F_{\max} = mg\sin\theta_i = 0.250(9.80)(\sin 15.0°) = \boxed{0.634\text{ N}}$$

P15.41 $\theta_i = 15.0°$ $\quad\quad\quad \theta(t = 1\,000) = 5.50°$

$$x = Ae^{-bt/2m} \qquad \frac{x_{1000}}{x_i} = \frac{Ae^{-bt/2m}}{A} = \frac{5.50}{15.0} = e^{-b(1000)/2m}$$

$$\ln\left(\frac{5.50}{15.0}\right) = -1.00 = \frac{-b(1\,000)}{2m}$$

$$\therefore \frac{b}{2m} = \boxed{1.00 \times 10^{-3}\ \text{s}^{-1}}$$

P15.53 $\left(\dfrac{d^2x}{dt^2}\right)_{\max} = A\omega^2$

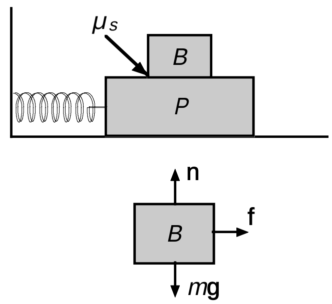

FIG. P15.53

$$f_{\max} = \mu_s n = \mu_s mg = mA\omega^2$$

$$A = \frac{\mu_s g}{\omega^2} = \boxed{6.62\ \text{cm}}$$