P11.35 (a) $L_i = mvl \qquad \sum \tau_{\text{ext}} = 0$, so $L_f = L_i = \boxed{mvl}$

$L_f = (m + M)v_f l$

$$v_f = \left(\frac{m}{m+M}\right)v$$

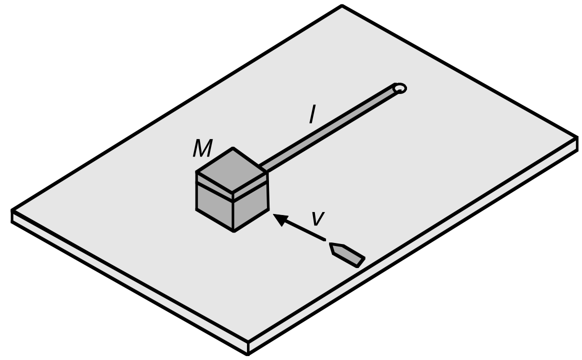

FIG. P11.35

(b) $K_i = \frac{1}{2}mv^2$

$$K_f = \frac{1}{2}(M+m)v_f^2$$

$v_f = \left(\frac{m}{M+m}\right)v \Rightarrow$ velocity of the bullet and block

$$\text{Fraction of } K \text{ lost} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2}{M+m}v^2}{\frac{1}{2}mv^2} = \boxed{\frac{M}{M+m}}$$

P11.36 For one of the crew,

$$\sum F_r = ma_r: \; n = \frac{mv^2}{r} = m\omega_i^2 r$$

We require $n = mg$, so $\omega_i = \sqrt{\frac{g}{r}}$

Now, $I_i\omega_i = I_f\omega_f$

$$\left[5.00 \times 10^8 \text{ kg} \cdot \text{m}^2 + 150 \times 65.0 \text{ kg} \times (100 \text{ m})^2\right]\sqrt{\frac{g}{r}} = \left[5.00 \times 10^8 \text{ kg} \cdot \text{m}^2 + 50 \times 65.0 \text{ kg}(100 \text{ m})^2\right]\omega_f$$

$$\left(\frac{5.98 \times 10^8}{5.32 \times 10^8}\right)\sqrt{\frac{g}{r}} = \omega_f = 1.12\sqrt{\frac{g}{r}}$$

Now, $|a_r| = \omega_f^2 r = 1.26g = \boxed{12.3 \text{ m/s}^2}$

P11.46 (a) The radial coordinate of the sliding mass is $r(t) = (0.012\,5 \text{ m/s})t$. Its angular momentum is

$$L = mr^2\omega = (1.20 \text{ kg})(2.50 \text{ rev/s})(2\pi \text{ rad/rev})(0.012\,5 \text{ m/s})^2 t^2$$

or $\quad L = (2.95 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}^3)t^2$

The drive motor must supply torque equal to the rate of change of this angular momentum:

$$\tau = \frac{dL}{dt} = (2.95 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}^3)(2t) = \boxed{(0.005\,89 \text{ W})t}$$

(b) $\tau_f = (0.005\,89 \text{ W})(440 \text{ s}) = \boxed{2.59 \text{ N} \cdot \text{m}}$

(c) $P = \tau\omega = (0.005\,89 \text{ W})t(5\pi \text{ rad/s}) = \boxed{(0.092\,5 \text{ W/s})t}$

(d) $\mathcal{P}_f = (0.092\,5\ \text{W/s})(440\ \text{s}) = \boxed{40.7\ \text{W}}$

(e) $T = m\dfrac{v^2}{r} = mr\omega^2 = (1.20\ \text{kg})(0.012\,5\ \text{m/s})t(5\pi\ \text{rad/s})^2 = \boxed{(3.70\ \text{N/s})t}$

(f) $W = \int_0^{440\ \text{s}} \mathcal{P}\,dt = \int_0^{440\ \text{s}} (0.092\,5\ \text{W/s})t\,dt = \frac{1}{2}(0.092\,5\ \text{J/s}^2)(440\ \text{s})^2 = \boxed{8.96\ \text{kJ}}$

(g) The power the brake injects into the sliding block through the string is

$$\mathcal{P}_b = \mathbf{F}\cdot\mathbf{v} = Tv\cos 180° = -(3.70\ \text{N/s})t(0.012\,5\ \text{m/s}) = -(0.046\,3\ \text{W/s})t = \frac{dW_b}{dt}$$

$$W_b = \int_0^{440\ \text{s}} \mathcal{P}_b\,dt = -\int_0^{440\ \text{s}} (0.046\,3\ \text{W/s})t\,dt$$

$$= -\frac{1}{2}(0.046\,3\ \text{W/s})(440\ \text{s})^2 = \boxed{-4.48\ \text{kJ}}$$

(h) $\sum W = W + W_b = 8.96\ \text{kJ} - 4.48\ \text{kJ} = \boxed{4.48\ \text{kJ}}$

Just half of the work required to increase the angular momentum goes into rotational kinetic energy. The other half becomes internal energy in the brake.

P11.50 (a) Angular momentum is conserved:

$$\frac{mv_i d}{2} = \left(\frac{1}{12}Md^2 + m\left(\frac{d}{2}\right)^2\right)\omega$$

$$\omega = \boxed{\frac{6mv_i}{Md + 3md}}$$

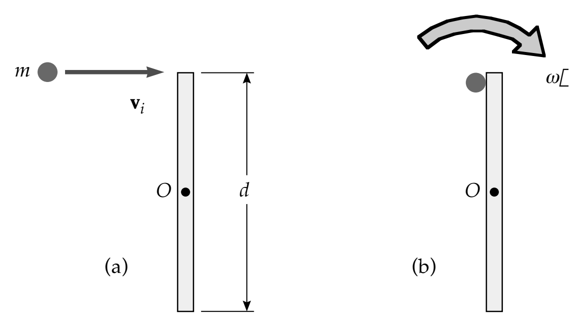

FIG. P11.50

(b) The original energy is $\frac{1}{2}mv_i^2$.

The final energy is

Continued on next page

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{12}Md^2 + \frac{md^2}{4}\right)\frac{36m^2v_i^2}{(Md+3md)^2} = \frac{3m^2v_i^2 d}{2(Md+3md)}.$$

The loss of energy is

$$\frac{1}{2}mv_i^2 - \frac{3m^2v_i^2 d}{2(Md+3md)} = \frac{mMv_i^2 d}{2(Md+3md)}$$

and the fractional loss of energy is

$$\frac{mMv_i^2 d2}{2(Md+3md)mv_i^2} = \boxed{\frac{M}{M+3m}}.$$

P11.54 For the cube to tip over, the center of mass (CM) must rise so that it is over the axis of rotation AB. To do this, the CM must be raised a distance of $a\left(\sqrt{2}-1\right)$.

$$\therefore Mga\left(\sqrt{2}-1\right)=\frac{1}{2}I_{\text{cube}}\omega^2$$

From conservation of angular momentum,

$$\frac{4a}{3}mv=\left(\frac{8Ma^2}{3}\right)\omega$$

$$\omega=\frac{mv}{2Ma}$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\frac{m^2v^2}{4M^2a^2}=Mga\left(\sqrt{2}-1\right)$$

$$v=\boxed{\frac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}}$$

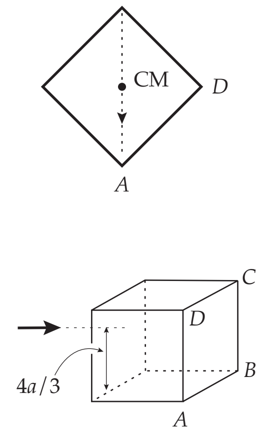

FIG. P11.54

Chapter Twelve: Static Equilibrium and Elasticity

SOLUTIONS TO PROBLEMS

P12.1 To hold the bat in equilibrium, the player must exert both a force and a torque on the bat to make

$$\sum F_x=\sum F_y=0 \text{ and } \sum\tau=0$$

$\sum F_y=0\Rightarrow F-10.0\text{ N}=0$, or the player must exert a net upward force of $F=\boxed{10.0\text{ N}}$

To satisfy the second condition of equilibrium, the player must exert an applied torque τ_a to make $\sum\tau=\tau_a-(0.600\text{ m})(10.0\text{ N})=0$. Thus, the required torque is

$$\tau_a=+6.00\text{ N}\cdot\text{m or }\boxed{6.00\text{ N}\cdot\text{m counterclockwise}}$$

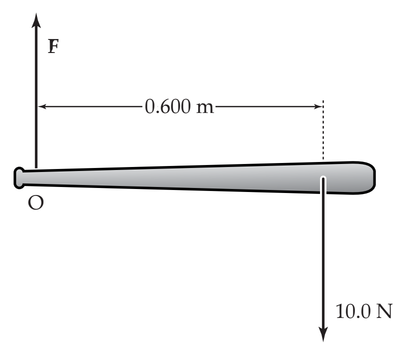

FIG. P12.1

P12.6 Let σ represent the mass-per-face area. A vertical strip at position x, with width dx and height $\frac{(x-3.00)^2}{9}$ has mass

$$dm = \frac{\sigma(x-3.00)^2\,dx}{9}.$$

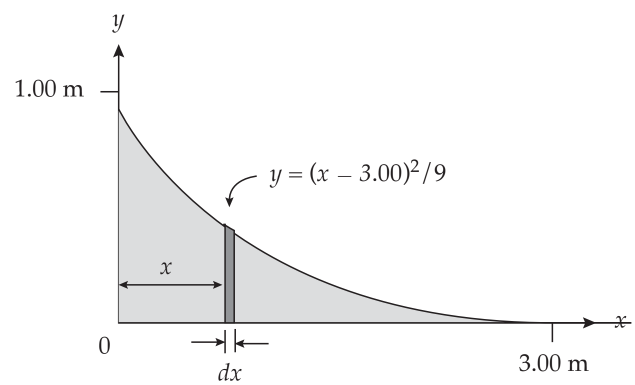

FIG. P12.6

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \frac{\sigma(x-3)^2\,dx}{9}$$

$$M = \left(\frac{\sigma}{9}\right)\int_0^{3.00}\left(x^2-6x+9\right)dx$$

$$M = \left(\frac{\sigma}{9}\right)\left[\frac{x^3}{3}-\frac{6x^2}{2}+9x\right]_0^{3.00} = \sigma$$

The x-coordinate of the center of gravity is

$$x_{\mathrm{CG}} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma}\int_0^{3.00}\sigma x(x-3)^2\,dx = \frac{\sigma}{9\sigma}\int_0^{3.00}\left(x^3-6x^2+9x\right)dx = \frac{1}{9}\left[\frac{x^4}{4}-\frac{6x^3}{3}+\frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75\text{ m}}{9.00} = \boxed{0.750\text{ m}}$$

P12.19 (a) $T_e \sin 42.0° = 20.0\text{ N}$ $\boxed{T_e = 29.9\text{ N}}$

(b) $T_e \cos 42.0° = T_m$ $\boxed{T_m = 22.2\text{ N}}$

P12.25 To find U, measure distances and forces from point A. Then, balancing torques,

$$(0.750)U = 29.4(2.25) \qquad \boxed{U = 88.2\text{ N}}$$

To find D, measure distances and forces from point B. Then, balancing torques,

$$(0.750)D = (1.50)(29.4) \qquad \boxed{D = 58.8\text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$.

P12.42 Call the normal forces A and B. They make angles α and β with the vertical.

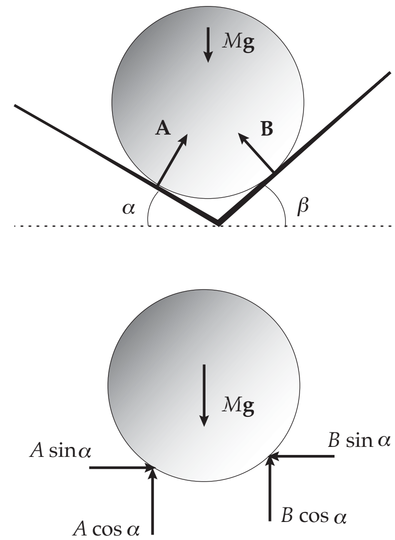

FIG. P12.42

$$\sum F_x = 0: \quad A\sin\alpha - B\sin\beta = 0$$
$$\sum F_y = 0: \quad A\cos\alpha - Mg + B\cos\beta = 0$$

Substitute $B = \dfrac{A\sin\alpha}{\sin\beta}$

$$A\cos\alpha + A\cos\beta\frac{\sin\alpha}{\sin\beta} = Mg$$
$$A(\cos\alpha\sin\beta + \sin\alpha\cos\beta) = Mg\sin\beta$$
$$A = \boxed{Mg\frac{\sin\beta}{\sin(\alpha+\beta)}}$$
$$B = \boxed{Mg\frac{\sin\alpha}{\sin(\alpha+\beta)}}$$

P12.46 $\sum \tau_{\text{point }0} = 0$ gives

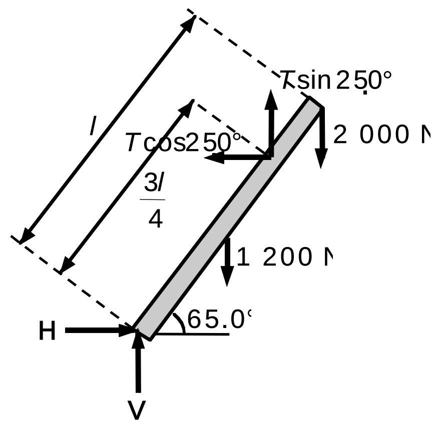

FIG. P12.46

$$(T\cos 25.0°)\left(\frac{3l}{4}\sin 65.0°\right) + (T\sin 25.0°)\left(\frac{3l}{4}\cos 65.0°\right)$$
$$= (2\ 000\ \text{N})(l\cos 65.0°) + (1\ 200\ \text{N})\left(\frac{l}{2}\cos 65.0°\right)$$

From which, $T = 1\ 465\ \text{N} = \boxed{1.46\ \text{kN}}$

From $\sum F_x = 0$,

$$H = T\cos 25.0° = 1\ 328\ \text{N}(\text{toward right}) = \boxed{1.33\ \text{kN}}$$

From $\sum F_y = 0$,

$$V = 3\ 200\ \text{N} - T\sin 25.0° = 2\ 581\ \text{N}(\text{upward}) = \boxed{2.58\ \text{kN}}$$