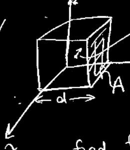
ch 21 Kinetic theory of gases

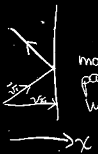
Assumptions:

1) large # of molecules
   large separation between molecules
2) molecules obey Newton's laws
   as a whole they move randomly
   "molecular chaos"
3) molecules interact via only short-range forces during elastic collisions
4) molecules make elastic collisions with walls
5) all molecules in gas are identical



Calculate pressure.

find $\frac{\text{force}}{\text{area}}$ exerted by molecules on box



momentum change of i^{th} particle when it collides with wall is

$$\Delta p = -2 m v_{xi} \hat{i}$$

$$\frac{\text{change in momentum}}{\text{Unit time}} = \overline{F_x} \leftarrow \text{force particles exerted by wall}$$

Unit time ← time between collisions $= \frac{(2d)}{v_x}$

$$\frac{(-2mv_{xi})}{(2d/v_{xi})}$$

average force on 1 particle colliding with right wall $= -\frac{m v_{xi}^2}{d}$

average force exerted by i^{th} particle on wall $= \frac{m v_{xi}^2}{d}$

If lots of particles average force is the force (basically)

$$\text{total force} = \frac{m}{d} \sum_i v_{xi}^2$$

$$= \frac{m}{d} N \overline{v_{xi}^2} \longleftarrow \text{average of } v_{xi}^2$$

to calculate $\overline{v_{xi}^2}$, note

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

$$+ \quad \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

$$\Rightarrow \overline{v_x^2} = \frac{1}{3} \overline{v^2}$$

$$F = \frac{m}{d} \frac{N}{3} \overline{v^2}$$

$$P = F/A = \left(\frac{2}{3}\right)\left(\frac{N}{Ad}\right)\left(\frac{1}{2} m \overline{v^2}\right)$$

$Ad = v_1 \Delta t$

$PV = \frac{2}{3} N (\frac{1}{2} m \overline{v^2})$

compare to $PV = nRT$

$\Rightarrow T = \frac{2}{3k_B} (\frac{1}{2} m \overline{v^2})$

relates T to $\overline{KE}$

$\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T \quad \left(k_B = \frac{R \leftarrow \text{gas cst}}{N} \right)$

note $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$

$\frac{1}{2} k_B T$ per translational degree of freedom.

ideal gas,

E_{int} depends only on T

(not true for non-ideal gas)

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related $\overline{v^2}$ to T

root mean square speed $v_{RMS} \equiv \sqrt{\overline{v^2}}$

$v_{RMS} = \sqrt{\overline{v^2}} = \sqrt{\frac{3 k_B T}{m}} = \sqrt{\frac{3RT}{M}}$

(m ↑ molecule mass in kg; M ↑ molar mass in kg.)

note:
lighter molecules move faster.

Calculate specific heat for ideal gas.

last time:

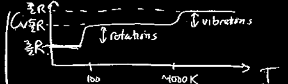
$$C_V = \frac{dE_{int}}{dT}$$

for monatomic atoms, $E_{int} = \frac{3}{2} nRT$

$\Rightarrow C_V = \frac{3}{2} R$.

now consider diatomic ideal gas

N_2, O_2 translations + 2 rotational d. of freedom
+ 2 vibrational degrees of freedom

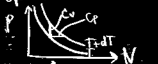


at room temperature, air has $C_V \sim \frac{5}{2}R$

translations - 3 d.of.f
rotations - 2 d.of.f

internal energy $\frac{3}{2}nRT \rightarrow \frac{5}{2}nRT \rightarrow \frac{7}{2}nRT$

Specific heat at constant pressure



1st law of thermo

$$dE_{int} = dQ + dW$$

$$dQ = dE_{int} - dW$$

fixed P:

$$nC_p \Delta T = nC_V \Delta T - (-P\Delta V)$$

For ideal gas, $PV = nRT$

for this path, $P\Delta V = nR\Delta T$

$$\Delta V = \frac{nR}{P}\Delta T$$

So $nC_p\Delta T = nC_V\Delta T + P\left(\frac{nR}{P}\right)\Delta T$

$$\therefore \boxed{C_p = C_V + R}$$

$$\gamma \equiv \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$$

For monatomic ideal gas

$C_V = \frac{3}{2}R$,

$$\gamma = \frac{5/2 R}{3/2 R} = \frac{5}{3}$$

For diatomic ideal gas,

$$\gamma = \frac{7/2 R}{5/2 R} = 7/5$$

So far have related $\frac{3}{2}kT$ to $\overline{\frac{1}{2}mv^2}$

now discuss probability distribution

Boltzmann distribution

number density ≡ # of $\frac{\text{molecules}}{\text{volume}}$ with energy between E and $E + dE$ $\equiv n_V(E) = n_0(V)\, e^{-E/k_B T}$

(V ↑ volume)