Vectors

Denote a vector $\vec{V}$

Something described by units = Scalar = time, temp mass

Magnitude + direction $\Rightarrow$ Vector

ex. Wind: # knots WNW

Displacement: Vector from point 1 $\rightarrow$ point 2

Displ = End − Beginning

Ex. NY $\leftrightarrow$ Cleveland

640 mi = Distance

Displacement = 640 mi West

Components of Vectors

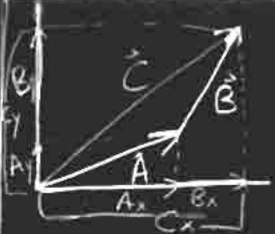
A_x, A_y are x y components of vector $\vec{A}$

$A_x = |\vec{A}| \cos\theta$, $A_y = |\vec{A}| \sin\theta$

$\tan\theta = A_y / A_x$, $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

Use to add 2 vectors $\Rightarrow$ $\vec{A} + \vec{B} = \vec{C}$



Add Vectors Head to tail

= Add

Sum of these vectors

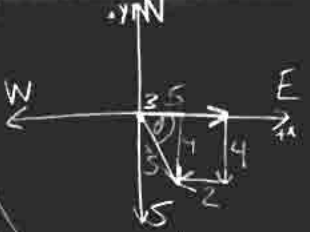
$\vec{C} = \vec{A} + \vec{B}$, $\vec{A} = (A_x, A_y)$, $\vec{B} = (B_x, B_y)$

$\vec{C} = (C_x, C_y)$

$C_x = A_x + B_x$

$C_y = A_y + B_y$

Car Drives
5 mi East
4 mi South
2 mi West
Find magnitude of displacement



x	y
+5	0
0	−4
−2	0
+3	−4

$\vec{S} = (S_x, S_y)$

$S_x = +3, S_y = -4$

$S = |\vec{S}| = \sqrt{S_x^2 + S_y^2}$

$= \sqrt{3^2 + 4^2} = 5\,mi$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°$
South of East

Multiplication of vectors ⇒

$C = \vec{a}\cdot\vec{B} = (a_x, a_y)\cdot(b_x, b_y)$

$= a_x b_x + a_y b_y$ = Scalar

number = Scalar

Product of 2 vectors

$\vec{a}\cdot\vec{b} = ab\cos\phi$

$a = |\vec{a}|,\ b = |\vec{b}|$

mag. $ab\cos\phi$

mag $(a\cos\phi)b$ $\quad a\cos\phi$

$\vec{a}\cdot\vec{b} = C$ = dot product

If $a \perp b \equiv \vec{a}$ perpendicular to $\vec{b}$

$C = ab\cos 90° = 0$

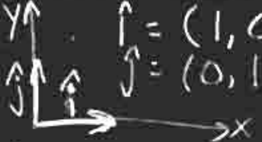
$\Rightarrow \vec{a}\cdot\vec{b} = 0$ if $a \perp b$

$\phi = 0°$

$\vec{a} \parallel \vec{b} = \vec{a}$ parallel to b $\Rightarrow \vec{a}\cdot\vec{b} = ab$

$\phi = 0 \quad \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}| = ab$

Unit Vector: Vector with magnitude 1: Used to specify direction



$\hat{i} = (1, 0)$

$\hat{j} = (0, 1)$

$\vec{A} = (A_x, A_y) = A_x\hat{i} + A_y\hat{j}$

$\vec{B} = (B_x, B_y) = B_x\hat{i} + B_y\hat{j}$

$= (B_x, 0) + (0, B_y)$

$\vec{C} = \vec{A} + \vec{B} = (A_x\hat{i} + A_y\hat{j})$

$+ (B_x\hat{i} + B_y\hat{j})$

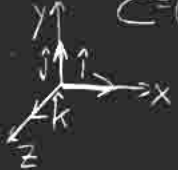
$$= (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j}$$

$$\vec{C} = \vec{A}+\vec{B} = (C_x, C_y) = C_x\hat{i} + C_y\hat{j}$$

$$C_x = A_x+B_x,\ C_y = A_y+B_y$$

$$C_x\hat{i} + C_y\hat{j} = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j}$$

Apply to 3-Dimensions



$$\vec{C} = (C_x, C_y, C_z)$$

$$\hat{i} = (1,0,0)$$

$$\hat{j} = (0,1,0)$$

$$\hat{k} = (0,0,1)$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{C} = \vec{A}+\vec{B} \Rightarrow$$

$$\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$

$$= (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} + (A_z+B_z)\hat{k} \Rightarrow 3\ \text{eq}_n$$

$$C_x = A_x+B_x,\ C_y = A_y+B_y$$

$$C_z = A_z+B_z$$

Find angle btw 2 Vectors in 3-D.

$$\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k},\ \vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{A} = (2,3,4),\ \vec{B} = (1,-2,3)$$

$$|\vec{A}||\vec{B}|\cos\theta = \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\Rightarrow \cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{|\vec{A}||\vec{B}|}$$

$$= \frac{(2)(1)+(3)(-2)+(3)(4)}{\sqrt{2^2+3^2+4^2}\sqrt{1^2+(-2)^2+3^2}}$$

$$= \frac{8}{\sqrt{29}\sqrt{14}} = .397 \Rightarrow \theta = 66.6^\circ$$

Motion in a plane

Displacement: $P \to Q$

$$\Delta\vec{S} = (\Delta X, \Delta Y)$$

Average Vector Velocity $\bar{\vec{V}}$

$$\bar{\vec{V}} = \frac{\Delta\vec{S}}{\Delta t} = \left(\frac{\Delta X}{\Delta t}, \frac{\Delta Y}{\Delta t}\right)$$

$$\bar{\vec{V}} = (\bar{V}_x, \bar{V}_y)$$

$$\bar{V}_x = \frac{\Delta X}{\Delta t}, \quad \bar{V}_y = \frac{\Delta Y}{\Delta t}$$

Instantaneous Velocity

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta\vec{S}}{\Delta t} = \frac{d\vec{S}}{dt} = (V_x, V_y)$$

$$V_x = \lim_{\Delta t \to 0} \frac{\Delta X}{\Delta t} = \frac{dX}{dt}$$

$$V_y = \lim_{\Delta t \to 0} \frac{\Delta Y}{\Delta t} = \frac{dY}{dt}$$

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2}, \quad \tan\theta = V_y / V_x$$

Average Acceleration

$$\bar{\vec{a}} = \frac{\Delta\vec{V}}{\Delta t}$$

$$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$$

$$\vec{V}_1 + \Delta\vec{V} = \vec{V}_2$$