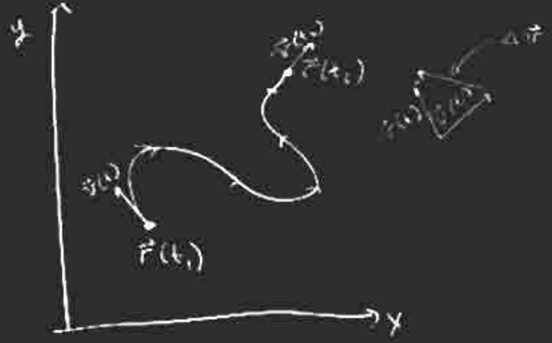
Motion in a plane

velocity — last time

acceleration

average acceleration vector
over some interval Δt

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x \hat{i} + \Delta v_y \hat{j}}{\Delta t}$$



what is $\Delta\vec{v}$?

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 \quad \left[\vec{v}(t_2) - \vec{v}(t_1)\right]$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = (a_x, a_y)$$

$$= a_x \hat{i} + a_y \hat{j}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t} = \frac{dv_y}{dt}$$

Motion of projectiles

path of object subject to earth's gravitational force

ignore air resistance

Only acceleration is $-g$ (gravity)

X axis = horizontal

y axis = vertical

$\vec{a} = -g\hat{j}$

$\vec{a} = (0, -g)$

given initial position (x, y)

initial Velocity (v_x, v_y)

say at $t = 0$

$(x, y) = (x_0, y_0)$

$(v_x, v_y) = (v_{x0}, v_{y0})$

$a_x = 0 = \frac{dv_x}{dt}$

$\Rightarrow v_x = \text{constant} = v_{x0}$

$a_y = -g = \frac{dv_y}{dt} \Rightarrow v_y = -gt + \text{constant}$

$\Rightarrow v_y = v_{y0} - gt$

$\frac{dx}{dt} = v_x = v_{x0}$

$\Rightarrow x = v_{x0}t + \text{constant}$

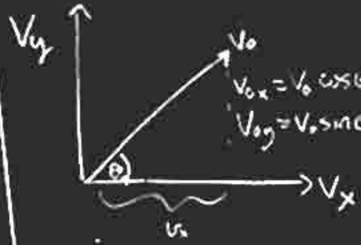
$\Rightarrow x = v_{x0}t + x_0$

$\frac{dy}{dt} = -gt + v_{y0}$

integrate.

$$\boxed{\begin{aligned} y(t) &= -\tfrac{1}{2}gt^2 + v_{y0}t + y_0 \\ x(t) &= v_{x0}t + x_0 \end{aligned}}$$

consider $x_0 = 0,\ y_0 = 0$



$V_{0x} = V_0 \cos\theta$

$V_{0y} = V_0 \sin\theta$

Special case $(x, y) = 0$

$x = (v_0 \cos\theta_0)t$

$y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$

$v_x = v_{x0}, \quad v_y = v_{y0} - gt$

Find shape of trajectory

$$x = (v_0 \cos\theta_0) t$$

$$\rightarrow t = \frac{x}{v_0 \cos\theta_0}$$

$$y = (v_0 \sin\theta_0)\left(\frac{x}{v_0\cos\theta_0}\right) - \frac{1}{2} g \left(\frac{x}{v_0 \cos\theta_0}\right)^2$$

$$y = \tan\theta_0 \, x - \frac{g}{2v_0^2\cos^2\theta_0} x^2$$

So $y = ax + bx^2$

with $a = \tan\theta_0$

$b = -g / 2v_0^2 \cos^2\theta_0$

$\vec{r}, \theta$ & vectors

distance $= \sqrt{x^2+y^2}$

speed $= \sqrt{v_x^2 + v_y^2}$

direction $\tan\theta = v_y / v_x$

Shooting monkey

ball

$x(t) = v_0 \cos\theta \, t$

$y(t) = v_0 \sin\theta \, t - \frac{1}{2} g t^2$

$x(0) \equiv x_0$

monkey

$X = D$

$y = D \tan\theta_0 - \frac{1}{2} g t^2$

look at time where $V_0 \cos\theta \, t = D$

$t = \frac{D}{V_0 \cos\theta}$ $\left(X_{ball} = X_{monkey} \right)$

$y_{ball} = \frac{v_0 \sin\theta_0 \, D}{V_0 \cos\theta_0} - \frac{1}{2} g t^2$

$= D \tan\theta_0 - \frac{1}{2} g t^2$

$= y_{monkey}$

Circular motion

uniform speed

radius R

speed v

$\boxed{a = v^2/R}$

if I go around circle 1 time velocity vector rotates by 2π

$|\vec{v}| = v$

$\Rightarrow$ total change in v is $2\pi v$

time for period $= \frac{2\pi R}{v}$

$$\frac{\Delta v}{\Delta t} = \frac{\text{change in } v}{\text{time}} = \frac{2\pi v}{(2\pi R/v)} = \frac{v^2}{R}$$

acceleration $\vec{a}$ is $\perp$ (perpendicular) to $\vec{v}$

$$|\Delta \vec{r}| = |\vec{v}\Delta t| = |\vec{r}|\delta\theta \Rightarrow \Delta t = \frac{r}{v}\delta\theta$$

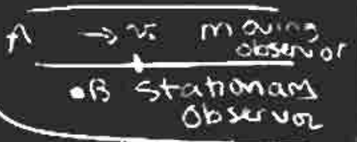
$$\Rightarrow \delta\theta = \frac{v}{r}\Delta t$$

$$|\Delta \vec{v}| = v\,\delta\theta$$

$$\Delta v = v\,\delta\theta = v\left(\frac{v}{r}\Delta t\right) = \frac{v^2}{r}\Delta t$$

$$\Rightarrow \boxed{\frac{\Delta v}{\Delta t} = \frac{v^2}{r}}$$

Relative velocity



if velocity in frame a is $(v, 0)$

velocity in frame b $= (v + v_0, 0)$