Phys 201
Exam 1
5:45-6:45pm
Wed 9/29
Bascom 272

Gravitational force between masses m_1 and m_2 separated by r is

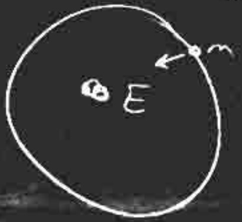
$$F_g = \frac{G m_1 M_2}{r^2}$$

(towards each other)

G = Newton's gravitational constant

6.673×10^{-11} N-m^2/kg^2

calculate circular orbit of small object about a big object



$$\vec{F} = m_m \vec{a}$$

(direction inward)

$$\frac{G M_E m_m}{r^2} = m \frac{v^2}{r}$$

Solve for v:

$$v = \sqrt{\frac{GM_E}{r}}$$

period $T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$

$M_E = 6\times10^{24}$ kg

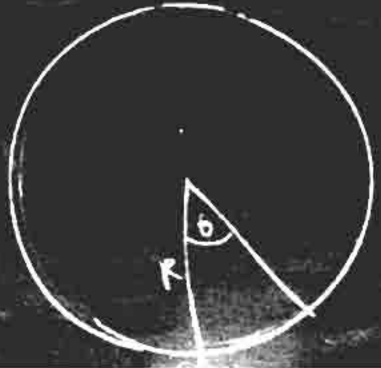
$r = 3.85\times10^{5}$ km

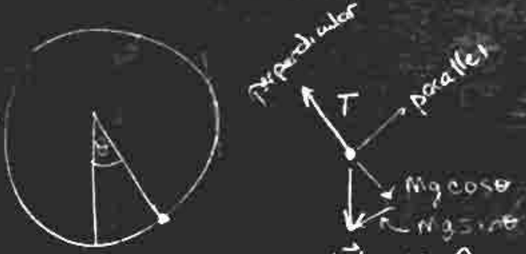
$$T = 2.37\times10^{6}\,s \times \frac{1\,\text{day}}{\left(24\frac{h}{\text{day}}\right)\left(3600\frac{s}{h}\right)} = 27\text{ days}$$

Non-uniform circular motion

e.g. Motion in a vertical circle

how fast to "loop the loop"





$\vec{W} = -Mg\hat{j}$

exactly at top

$\vec{w} = -Mg\hat{j}$

$\vec{T} = -T\hat{j}$

note T cannot be negative

$m a_{\parallel} = -mg\sin\theta$

$m a_{\perp} = mg\cos\theta$

$a_{\perp} = v^2/R$ ← this is true even when $|\vec{v}|$ is not constant

at top ($\vec{F} = m\vec{a}$) $\quad -Mg - T = -\dfrac{mv^2}{R}$

since $T \geq 0$ and mg fixed, only works if $\dfrac{v^2}{R} > g$

$\left(g + \frac{T}{m} = \frac{v^2}{R}\right)$

$\frac{T}{m} \geq 0$

to stay on circular path, need

$v > v_c = \sqrt{gR}$

same v_c for loop

Motion in presence of resistive forces

recall kinetic solid friction

$F_f = \mu_k N$ (N: Normal force)

liquids (empirical)

$F_r = -b\vec{v}$

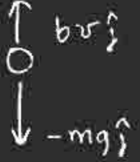
resistive force

(in air at high speeds get

$F_r = \frac{1}{2} D \rho A v^2$)

D: drag coefficient

Drop particle in liquid



$m\frac{dv}{dt} = F - \mu mg$

if $F > \mu mg$ keep accelerating

$$m\frac{d\vec{v}}{dt} = \vec{F} \quad \left[-mg - b(\vec{v})\right] \quad \vec{v} = -v\hat{j}$$

$$\Rightarrow -m\frac{dv}{dt} = bv - mg$$

note that v will increase until

$$bv = mg$$

$$\Rightarrow v_T = mg/b$$

v_T = terminal velocity

How long to reach velocity close to v_T?

$$-m\frac{dv}{dt} = bv - mg$$

$$\frac{dv}{dt} = -\frac{b}{m}v + g \quad \text{differential equation for } v$$

try solution of form

$$v(t) = A + Be^{\alpha t}$$

$$\alpha Be^{\alpha t} = -\frac{b}{m}\left(A + Be^{\alpha t}\right) + g$$

$e^{\alpha t}$ terms & cst terms must cancel separately for equality to hold at all times

$$\left(\alpha B + \frac{b}{m} B\right) e^{\alpha t} = 0$$

$$-\frac{b}{m} A + g = 0$$

$$\Rightarrow \alpha = -b/m$$

$$A = mg/B$$

so $v(t) = \frac{mg}{b} + Be^{-\frac{b}{m}t}$

$$\frac{mg}{b} \uparrow \; v_T$$

$$Be^{-\frac{b}{m}t} \uparrow$$ fix B using initial condition

$v(t=0) = 0 \Rightarrow B = -mg/b$

$$v(t) = \frac{mg}{b}\left(1 - e^{-\frac{b}{m}t}\right)$$

$e^{-\frac{b}{m}t} = e^{-t/(m/b)} = e^{-t/\tau}$ with $\tau = m/b$

how long until $v(t)$ is near v_T?

define "relaxation time $\tau = \frac{m}{b} = \frac{v_T}{g}$

$$v(t) = \frac{mg}{b}\left(1 - e^{-t/\tau}\right)$$

$$\tau = v_T/g$$

for thick shampoo

$$\tau = \frac{.003 \text{ m/s}}{9.8 \text{ m/s}^2}$$

$$\tau \sim 3 \text{ ms.} \quad (3 \times 10^{-3} \text{ s})$$