

$$\text{Power} = P = \vec{F} \cdot \vec{v}$$

$$P = dW/dt$$

Ex: Car mass 1200kg  
Requires 20hp to drive  
at 50km/hr on level road

Find Force of Friction

$$F = \frac{P}{v} = \frac{20 \text{ hp} (0.746 \text{ kW/hp})}{50 \frac{\text{km}}{\text{hr}} \left( \frac{1}{3600} \frac{\text{hr}}{\text{sec}} \right)} = 1070 \text{ N}$$

Car goes up a 10%  
Grade (10m up in 100m)  
⇒ Speed up = 5 km/hr

if speed across remains 50 km/hr  
 $\left( \frac{50 \text{ km}}{\text{hr}} \right) \left( \frac{1}{3600} \frac{\text{hr}}{\text{sec}} \right) = 1.389 \frac{\text{m}}{\text{s}}$

$$\text{Force of gravity} = mg = 9.8 \frac{\text{m}}{\text{s}^2} \cdot 1200 \text{ kg} = 1.176 \times 10^4 \text{ N}$$

$$P = F \cdot v = (1.176 \times 10^4 \text{ N}) (1.389 \frac{\text{m}}{\text{s}}) = 1.63 \times 10^4 \text{ W} = 16.3 \text{ kW} \cdot \frac{1 \text{ hp}}{0.746 \text{ kW}} = 21.9 \text{ hp} = \text{Power to go up}$$

$$\text{Total Power} = 20 \text{ hp} + 21.9 \text{ hp} = \boxed{41.9 \text{ hp}} \approx \boxed{42 \text{ hp}}$$

Gravitational Potential Energy

Work done against gravity  
 Stored in GPE, released  
 as kinetic energy

$F_{grav} \uparrow y_2$   
 $\downarrow y_1$   
 $W_{grav} = F_{grav}(y_2 - y_1)$   
 $= -mg(y_2 - y_1)$

Gravitational Potential Energy  
 $= U = mgy \Rightarrow$

$W_{grav} = -\Delta U$

Let  $W' =$  Work by all forces except Gravity

$K =$  Kinetic Energy

$W' + W_{grav} = K_2 - K_1$

$W' - (mgy_2 - mgy_1) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$W' = (\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2) + (mgy_2 - mgy_1)$

$W = (\frac{1}{2}mv_2^2 + mgy_2) - (\frac{1}{2}mv_1^2 + mgy_1)$

If  $W' = 0$  (i.e. only gravity)

$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + mgy_1$

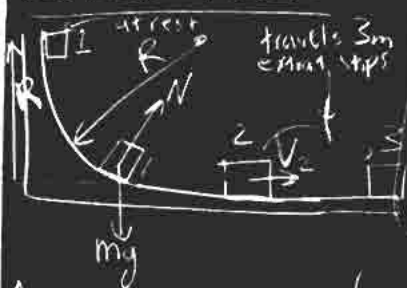
Start from  $y_1 = 0, v_1 = 0$

$\frac{1}{2}mv^2 + mgy = 0$   
 $\frac{1}{2}v^2 = -gy$

$v = \sqrt{-2gy} = \sqrt{2as}$

True for non  
 constant acceleration  
 and forces

# Curved Track



$$v_2 = \sqrt{2gR}$$

Now add friction to the track  
 $R = 1m, v_1 = 4 \frac{m}{s}$   
 $m = 2kg$

Work done by Friction

$$K_1 + U_1 = mgR$$

$$= K_2 + W_F$$

$$W_F = mgR - \frac{1}{2}mv_2^2$$

At start, assume no friction

$$K_1 + U_1 = K_2 + U_2$$

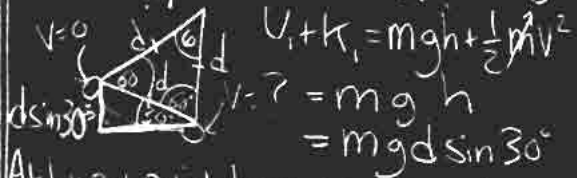
$$0 + mgR = \frac{1}{2}mv_2^2 + 0$$

$$= 2kg \cdot 9.8 \frac{m}{s^2} \cdot 1m - \frac{1}{2} \cdot 2kg \left(\frac{4m}{s}\right)^2 = 3.6 \text{ Joules}$$

Block continues from 2  $\rightarrow$  3, stops after 3m  
 What is coefficient of kinetic friction?  
 $W_F = \frac{1}{2}mv_2^2$  ← all of this KE is lost as work against friction

b.c.  $v_3 = 0$   
 $W_F = K_2 - K_3 = \frac{1}{2}mv_2^2 - 0$   
 $W_F = \frac{1}{2}mv_2^2 = \frac{1}{2} \cdot 2kg \left(\frac{4m}{s}\right)^2 = 16 \text{ N}\cdot\text{m}$   
 $W_F = F_f \cdot d = F_f \cdot 3m \Rightarrow F_f = \frac{16 \text{ N}\cdot\text{m}}{3m} = 5.33 \text{ N}$   
 $= \mu_k N = \mu_k mg = \mu_k \cdot 2kg \cdot 9.8 \frac{m}{s^2} = \mu_k \cdot 19.6 \text{ N}$   
 $\mu_k = 5.33 \text{ N} / 19.6 \text{ N} = 0.27 = \mu_k$

Sphere of mass  $m$  on weightless line  
 $\Rightarrow$  Pendulum, length =  $0.5m = d$   
 swings at max angle  $60^\circ$  wrt vertical  
 $\Rightarrow$  Velocity thru vertical?



At lowest point  $h=0 \Rightarrow U_2 + K_2 = \frac{1}{2}mv^2$   
 $U_1 + K_1 = U_2 + K_2 \Rightarrow \frac{1}{2}mv^2 = mgd \sin 30^\circ$   
 $v^2 = 2gd \sin 30^\circ$   
 $= 2 \cdot 9.8 \frac{m}{s^2} \cdot (0.5m) \cdot (0.5) = 4.9 \frac{m^2}{s^2} \Rightarrow v = 2.2 \frac{m}{s}$

## Elastic Potential Energy (Spring)



$F =$  elastic force  
 Work to stretch a spring distance  $x$

$$W_{sp} = \frac{1}{2}kx^2$$

Spring released  $\frac{1}{2}kx^2$  released

Total Energy of a spring + mass system

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$\frac{1}{2}kx^2 =$  Elastic PE  
 $mgh =$  Grav. P.E.

$$\frac{1}{2}mv^2 = K.E.$$

# Gravity, Spring, Kinetic Energy

80 kg person jumps from height of 2 m onto spring which compresses 0.2 m and rebounds. Find speed when compressed 0.1 m  
Set  $y = 0$  at lowest point



Find initial GPE

$$U = mgh =$$
$$80 \text{ kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 2.2 \text{ m}$$
$$= 1725 \text{ J}$$