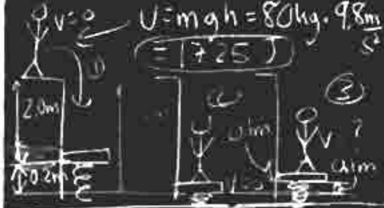


$U = mgh = 80\,kg \cdot 9.8\,\frac{m}{s^2} \cdot 2.2\,m$ at point 3

$= 1725\,J$

Find Spring Constant k

Max Spring Compression

Total Energy $= \frac{1}{2}kx^2 = 1725\,J$

= Energy at Start.

$k = \frac{2}{x^2}\,1725\,J = \frac{2}{(.2m)^2}\,1725\,J$

$= 8.63 \times 10^4\,N/m$

Spring compression: $x = 0.1\,m$

height: $y = 0.1\,m$

Don't know v. Determine $\boxed{E = 1725\,J}$

$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + mgy$

$= \frac{8.63\times10^4\,N/m}{2}(.1m)^2 + \frac{1}{2}80\,kg\,v^2$

$+ (80\,kg)(9.8\,\frac{m}{s^2})(.1m) =$

$1725\,J = 431.3\,J + 40\,kg\,v^2 + 78.4\,J$

$1215\,J = 40\,kg\,v^2 \Rightarrow v = \sqrt{30.4\,m^2/s^2} = 5.5\,m/s$

Conservation of Energy:

$\Delta U + \Delta K = 0$ (absence of Friction)

Total Mechanical Energy

$E = U + K = $ Constant

$K = \frac{1}{2}mv^2$, $U_{grav} = mgy$

$U_{spring} = \frac{1}{2}kx^2$ $(U = U_g + U_s)$

Work ⇒ Change in K.E. ⇒

$W = \Delta K$ but $\Delta U + \Delta K = 0$

$\Rightarrow \boxed{\Delta U = -W}$ since $\Delta K = \Delta U$

$\Delta U = -W = -\int_{x_0}^{x} \vec{F}(x) \cdot d\vec{x}$

Force acts over distance Δx

$\Delta U = -W = -F(x)\Delta x$

$\Rightarrow F(x) = -\frac{\Delta U}{\Delta x}$, take limit

$F(x) = \lim_{\Delta x \to 0} -\frac{\Delta U}{\Delta x} = \boxed{-\frac{dU}{dx} = F(x)}$

ex. $U = \frac{1}{2}kx^2$

$F = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$

Gravity, springs are Conservative forces

Change in energy is path independent → only depends on beginning and ending

Friction is not a Conservative Force

⇒ does depend on path

Separate work by Friction and by Conservative Forces

$W_{TOTAL} = \boxed{W_C + W_F = \Delta K}$

Work done by Cons. Forces

$W_C = -\Delta U \Rightarrow$

$-\Delta U + W_F = \Delta K$

$W_F = \Delta U + \Delta K$

Total $E = U + K$

$\Delta E = \Delta U + \Delta K$

$W_F = \Delta E$ = Change in total Mech. Energy

Impulse + Momentum

Particle moving at V_0 at $t = 0$
constant acceleration a
at time t $V(t) = V_0 + at$
$F = ma$

mult by $m \Rightarrow$

$mV = mV_0 + mat$

$mV = mV_0 + Ft$

$mV - mV_0 = Ft =$ Impulse of force
= Force x Application time
Contact time

Force starts at t_1, ends at t_2

$$\text{Impulse} = \vec{J} = \vec{F} \cdot (t_2 - t_1)$$

$$\vec{p} = m\vec{v} = \text{Momentum}$$

Force varying. $\vec{J} = \sum_i \vec{F}_i \Delta t_i$

$$\Rightarrow \vec{J} = \int_{t_1}^{t_2} \vec{F}(t)\,dt = \Delta\vec{p}$$

$= \vec{p}_2 - \vec{p}_1$, Force x (time interval) = Impulse

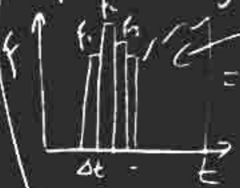
Force x (distance) = Work

Units of impulse = Newton-secs

Units of momentum = kgm/s = N·s

Work = Change in Energy

Impulse = change in Momentum



$J = F_1\Delta t + F_2\Delta t \ldots$

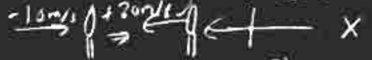
= area under Curve

Ex. Tennis game.

Tennis ball approaches racket at 10 m/s, leaves in opposite direction at 20 m/s, ball mass = .060 kg

Contact Time = 0.01 s

Find average force on ball



$\Delta\vec{p} = m\vec{v}_2 - m\vec{v}_1$ = Change in Momentum

$(0.060kg)(20\frac{m}{s}) - (0.060kg)(-10\frac{m}{s})$

$= 1.8\, kg\frac{m}{s} = 1.8\, N\cdot s$ = Impulse

$= F\Delta t = \Delta p \Rightarrow \boxed{F = \frac{\Delta p}{\Delta t}}$

$= \frac{1.8\, Ns}{.01\, s} = 180\, N$

Conservation of Momentum

2 bodies alone exert forces on each other, nothing else ⇒ Impulse ⇒ Momentum changes, but by Newton 3: Forces equal in magnitude, opposite in direction. AND [illegible] have same Contact time ⇒ impulse same but opposite ⇒ Δp's are equal & opposite ⇒ Total Momentum unchanged

TOTAL Momentum before Collision = Momentum after Collision

$P_b = P_a$ before = after

objects 1, 2 have indices before, after

$m_1\vec{V}_{1b} + m_2\vec{V}_{2b} = m_1\vec{V}_{1a} + m_2\vec{V}_{2a}$

2 dimensions 2 Eq'ns

$m_1V_{1bx} + m_2V_{2bx} = m_1V_{1ax} + m_2V_{2ax}$

$m_1V_{1by} + m_2V_{2by} = m_1V_{1ay} + m_2V_{2ay}$

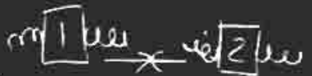
If I give masses, velocities before collision ⇒ enough info to find velocities after? NO (2 Eq'ns, 4 unknowns)

Additional Conditions (a variety of these)

① Total Kinetic Energy is Conserved (stays same)

Elastic Collision

$KE_b = KE_a$



Approach on frictionless surface have springs rebound

Another Case (KE $\neq$ Conserved)

Completely Inelastic Collision = They stick together

$\Rightarrow \vec{V}_{1a} = \vec{V}_{2a} = \vec{V}_a$

(2 unknowns gone!)

$$m_1\vec{V}_{b_1} + m_2\vec{V}_{b_2} = (m_1+m_2)\vec{V}_a$$

K_b = Initial KE $= \frac{1}{2}m_1V_{1b}^2 + \frac{1}{2}m_2V_{2b}^2$

K_a = Final KE $= \frac{1}{2}(m_1+m_2)V_a^2$

Assume body 2 is at rest

$\vec{V}_{2b} = 0$