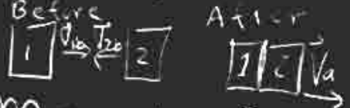
Inelastic Collisions

Completely inelastic
Stick together

$\vec{V}_{2a} = \vec{V}_{1a} = \vec{V}_a$



Momentum Cons:

$m_1\vec{V}_{1b} + m_2\vec{V}_{2b} = (m_1+m_2)\vec{V}_a$

Kinetic Energy: $K_b = \frac{1}{2}m_1V_{1b}^2 + \frac{1}{2}m_2V_{2b}^2$

$K_a = \frac{1}{2}(m_1+m_2)V_a^2$

If body 2 starts at rest.

$\vec{V}_{2b} = 0 \Rightarrow$ Look at $\frac{\text{after}}{\text{before}}$

$\frac{K_a}{K_b} = \frac{(m_1+m_2)V_a^2}{m_1V_{1b}^2}$

Mom Cons: $\vec{P}_b = \vec{P}_a$

$m_1\vec{V}_{1b} = (m_1+m_2)\vec{V}_a$

$\vec{V}_a = \frac{m_1}{m_1+m_2}\vec{V}_{1b} \Rightarrow V_a^2 = \left(\frac{m_1}{m_1+m_2}\right)^2 V_{1b}^2$

$\frac{K_a}{K_b} = \frac{(m_1+m_2)}{m_1V_{1b}^2}\left(\frac{m_1}{m_1+m_2}\right)^2 V_{1b}^2$

$\frac{K_a}{K_b} = \frac{m_1}{m_1+m_2} < 1$

Total KE decreases in an inelastic Collision

Energy is lost (to heat..)

Momentum Conserved

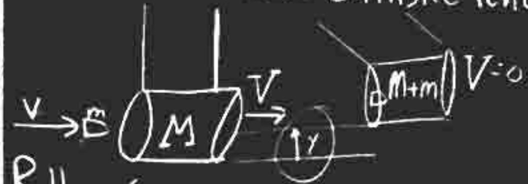
Completely Elastic

Energy Conserved

$$\frac{1}{2}m_1 v_{1b}^2 + \frac{1}{2}m_2 v_{2b}^2 = \frac{1}{2}m_1 v_{1a}^2 + \frac{1}{2}m_2 v_{2a}^2$$

$$m_1 \vec{v}_{1b} + m_2 \vec{v}_{2b} = m_1 \vec{v}_{1a} + m_2 \vec{v}_{2a}$$

Inelastic Collision. Ballistic Pendulum



Bullet of mass m strikes block mass M

block + bullet have speed V immediately after collision (then slows as it rises to a stop at height y)

Mom Cons.

$$mv = (m+M)V$$

$$\boxed{v = \left(\frac{m+M}{m}\right)V}$$

$$K_a = \frac{1}{2}(m+M)V^2$$

becomes all Grav. Pot. Egy

$$= U_{grav} = (m+M)gy$$

$$\frac{1}{2}(m+M)V^2 = (m+M)gy$$

$$\boxed{V = \sqrt{2gy}}$$

$$\frac{K_a}{K_b} = \frac{\frac{1}{2}(m+M)V^2}{\frac{1}{2}mv^2}$$

$$= \frac{\frac{1}{2}(m+M)V^2}{\frac{1}{2}m\left(\frac{m+M}{m}\right)^2 V^2}$$

$$= \frac{m}{m+M} = \frac{K_a}{K_b}$$

KE lost

$$\boxed{v = \left(\frac{m+M}{m}\right)\sqrt{2gy}}$$

2-D case Collisions at angles: Use Components

Do x, y separately

$y \uparrow \; \rightarrow x$

$\boxed{400g} \xrightarrow{125 \frac{cm}{s}}$

1 Mom Cons, 2

x: $m_{1b}V_{1bx} + m_2V_{2bx} = m_1V_{1ax} + m_2V_{2ax}$

$(400g)(125 \frac{cm}{s}) + 0 = 400g \; 80\frac{cm}{s} + 600g \; V_{2ax}$

$600g \; V_{2ax} = 18{,}000 \, g\frac{cm}{s} \Rightarrow V_{2ax} = 30 \, cm/s$

y: $m_1V_{1by} + m_2V_{2by} = m_1V_{1ay} + m_2V_{2ay}$

$0 + 0 = (400g)(60\frac{cm}{s}) + (600g)\, V_{2ay}$

$600g \; V_{2ay} = -24{,}000 \, g\frac{cm}{s} \Rightarrow V_{2ay} = -40 \, cm/s$

$V_{2a} = \sqrt{V_{2ax}^2 + V_{2ay}^2} = \sqrt{(30\frac{cm}{s})^2 + (-40\frac{cm}{s})^2} = 50 \, cm/s$

$\theta = \tan^{-1}\left(\frac{V_{2ay}}{V_{2ax}}\right) = \tan^{-1}\left(\frac{-40}{30}\right) = \tan^{-1}(-1.33)$

$= -53.1° = 53.1°$ below x axis

KE Lost! (Left for you)

Recoil

$\boxed{1}\boxed{2} \Rightarrow \leftarrow \boxed{1} \; \boxed{2} \rightarrow$

$\vec{P}_b = \vec{P}_a \Rightarrow 0 = m_1V_1 + m_2V_2 \Rightarrow \frac{V_1}{V_2} = -\frac{m_2}{m_1}$

K.E.: $K_b = 0$

After: $K_a = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$

from spring elastic P.E.

Ratio of KE of masses.

$$\frac{K_1}{K_2} = \frac{\frac{1}{2} m_1 V_1^2}{\frac{1}{2} m_2 V_2^2} = \frac{m_1}{m_2}\left(\frac{V_1}{V_2}\right)^2$$

$$= \frac{m_1}{m_2}\left(\frac{-m_2}{m_1}\right)^2 = \boxed{\frac{m_2}{m_1} = \frac{K_1}{K_2}}$$

Center of mass

Bunch of masses $m_1, m_2, \ldots$

w/ velocities $\vec{V}_1, \vec{V}_2, \ldots$

and locations in 3-D,

$(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$

$= \vec{r}_1, \vec{r}_2, \ldots$

Center of mass

$$\vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$M = \sum_i m_i$$

Total momentum of system of Particles

Total Momentum $\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 \ldots$

$(\vec{P}_i = m_i \vec{V}_i)$, Total Mass $M = m_1 + m_2 + \ldots$

Define a point which describes motion of entire system

has $M\vec{V} = m_1\vec{V}_1 + m_2\vec{V}_2 + \ldots$

$$\vec{V} = \frac{1}{M}(m_1\vec{V}_1 + m_2\vec{V}_2 + \ldots)$$

$$\vec{V} = \frac{d\vec{r}}{dt} \Rightarrow \frac{d\vec{r}}{dt} = \frac{1}{M}\left(m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt}\right.$$

$$\frac{d\vec{r}}{dt} = \frac{d}{dt}\left(\frac{1}{M}(m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots)\right)$$

$$= \frac{d}{dt}\vec{r}_{cm}$$ (see previous board)

Velocity that describes system of particles is velocity of center of mass

$$\vec{V}_{cm} = \frac{1}{M}(m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots)$$

$$\frac{d}{dt}\vec{V}_{cm} = \frac{d}{dt}\frac{1}{M}(m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots),$$

$$\frac{d\vec{V}_{cm}}{dt} = \vec{a}_{cm} = \frac{1}{M}\left(m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + \dots\right)$$

$$\vec{a}_{cm} = \frac{1}{M}(m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots)$$

$$M\vec{a}_{cm} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots$$

$$= \sum \text{Forces on } 1, 2, 3, \dots$$

In case of no external forces

$$M\vec{a}_{cm} = 0$$

Momentum Cons.