Phys 201

Exam 2

Monday 10/25

5:45 - 6:45 pm

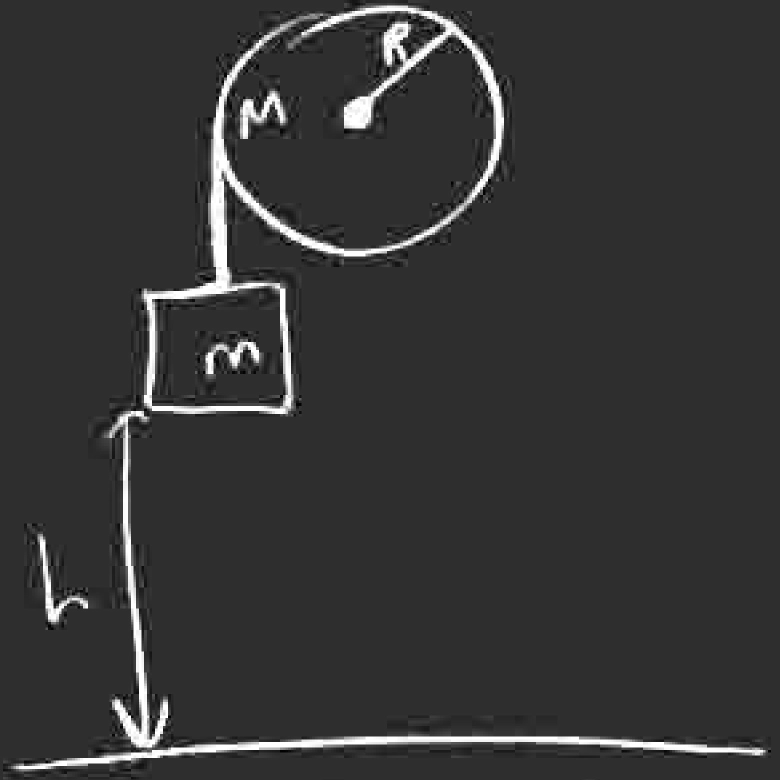
272 Bascom

Review Sessions

1300 Sterling

Friday 10/22 5-7pm

Sunday 10/24 2-4 pm



What is the velocity when $h=0$?

$a = \left(\frac{m}{m+M/2}\right) g$

$v = at$

Use energy

$K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

Rope $R\omega = v$

So $K = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{1}{2} M R^2\right)\left(\frac{v}{R}\right)^2$

$= \frac{1}{2} m v^2 + \frac{1}{4} M v^2$

$K = \frac{1}{2}\left(m + \frac{1}{2} M\right) v^2$

at start $K = 0$, $U = mgh$

at end ($h=0$) $U = 0$

$K = \frac{1}{2}\left(m + \frac{1}{2} M\right) v^2$

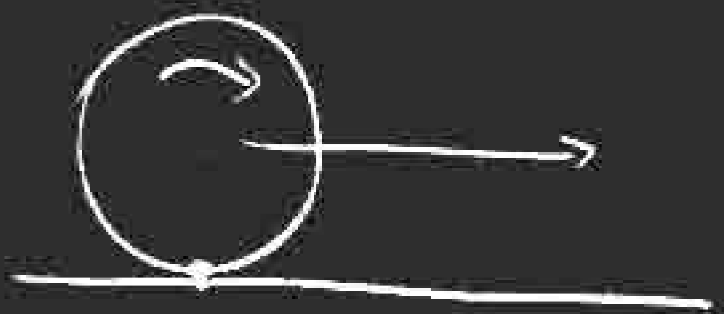
conservation of energy $\Rightarrow$

$mgh = \frac{1}{2}\left(m + \frac{1}{2} M\right) v^2$

So $v^2 = \dfrac{mgh}{\frac{1}{2}m + \frac{1}{4}M}$

and $v = \sqrt{\dfrac{2mgh}{m + \frac{1}{2}M}}$

Rolling motion



Center moves in straight line

If no slipping, contact is instantaneously stationary.

If no slipping, relate ω to v_{cm}

if rotate by θ, translate by θR

$$v_{cm} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

So $\boxed{v_{cm} = R\omega}$

$\boxed{a_{cm} = R\alpha}$

(Parallel axis theorem) total KE

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

translate at $v_{cm} = R\omega$

rotate at angular velocity ω

$$K = \frac{1}{2} m (R\omega)^2 + \frac{1}{2} I_{cm} \omega^2$$

e.g. cylinder $I_{cm} = \frac{1}{2} MR^2$

about contact point

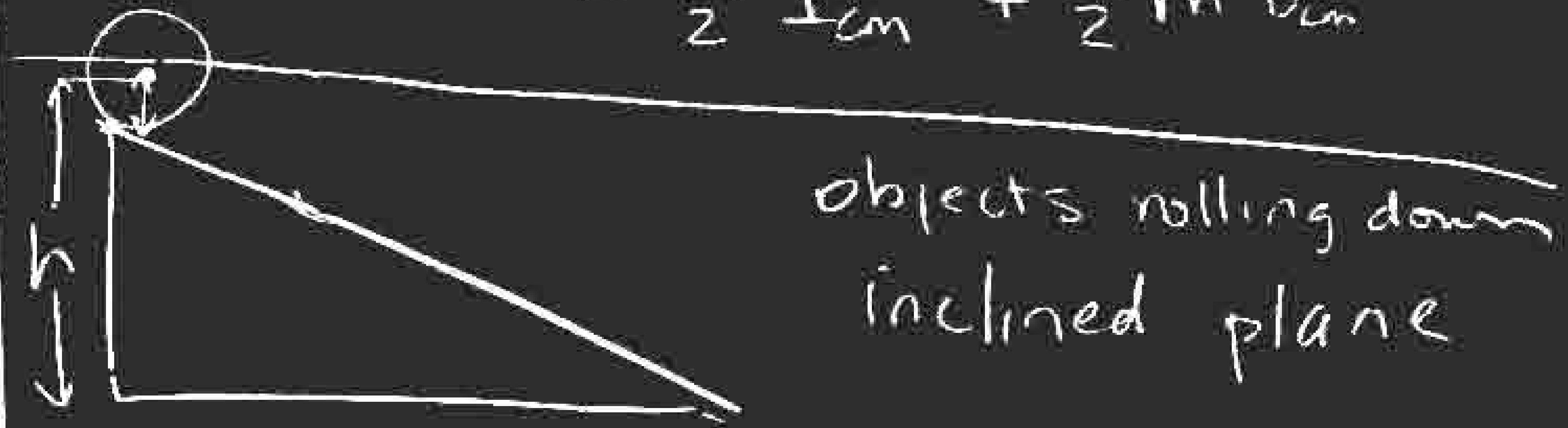
$$K = \frac{1}{2} I_p \omega^2$$

$$I_p = I_{cm} + MR^2$$

parallel axis theorem

$$K = \frac{1}{2}\left(I_{cm} + MR^2\right)\omega^2$$

$$= \frac{1}{2} I_{cm} + \frac{1}{2} m v_{cm}^2$$



objects rolling down inclined plane

$\boxed{\text{what is } v \text{ as function of } h?}$

no slipping $v = R\omega$

$$\text{energy} = mgh + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I(v/R)^2$$

is constant

initially $v = 0$, $E = mgh$

at end $h = 0$ $E = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

$$v^2 = \frac{mgh}{\frac{1}{2}(I/r^2 + m)}$$

$$\boxed{v = \sqrt{\frac{2gh}{(1 + I/mr^2)}}}$$

Ch 11 angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

angular momentum

cross product

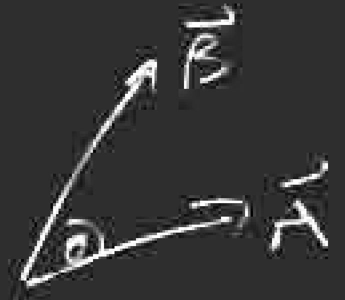
$$\vec{\tau} = \vec{r} \times \vec{F}$$

torque

$$\boxed{\frac{d\vec{L}_{net}}{dt} = \sum \vec{\tau}_i}$$

Cross product

$$\vec{C} = \vec{A} \times \vec{B}$$



magnitude of $\vec{C} = |\vec{A}||\vec{B}| \sin\theta$

direction of $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$

and given by right hand rule

$[\hat{i} \times \hat{j} = \hat{k}]$

Properties of cross product

1) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

2) $\vec{A} \times \vec{A} = 0$

3) if $\vec{A} \perp \vec{B}$, $|\vec{A} \times \vec{B}| = |A||B|$

4) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} + \vec{C}$

5) $\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{array}{l} \hat{i}(A_y B_z - A_z B_y) \\ -\hat{j}(A_x B_z - A_z B_x) \\ +\hat{k}(A_x B_y - A_y B_x) \end{array}$$

11.2 Angular momentum

def $\vec{L} = \vec{r} \times \vec{p}$

recall

$$\frac{d\vec{p}}{dt} = \vec{F}$$

so
$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$
$$= \left(\frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}\right)$$
$$= \left(\frac{\vec{p}}{m} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}\right)$$
$$= \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \sum \vec{F}_i = \vec{\tau}$$

$$\boxed{\frac{d\vec{L}}{dt} = \sum_i \vec{\tau}_i}$$

- must measure $\vec{L}$ and $\vec{\tau}$ about same origin

units of $\vec{L}$: (mass)(length)^2/(time)

MKS $kg\text{-}m^2/s$

- direction of $\vec{L}$ is perpendicular to plane formed by $\vec{r}$ and $\vec{p}$

for system of particles $\vec{L} = \vec{L}_1 + \vec{L}_2 + \ldots$

$$\sum \vec{F}_{ext} = \frac{d\vec{p}_{tot}}{dt} \Rightarrow \frac{d\vec{L}_{tot}}{dt} = \sum_i \frac{d\vec{L}_i}{dt} = \sum_i \vec{\tau}_i$$